MA/CS 109
DISCUSSION 4

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Problem 4

How many people are in a given generation?

2, 4, 8, 16, ...

$2^m$ (where $m$ is the generation)
Problem 4

• So the pattern that emerges is this:

<table>
<thead>
<tr>
<th>M (# of generations)</th>
<th>N (# of people in fam)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

• Your task is to find two equations:
  • One that goes from N -> M
  • Another that goes from M -> N
Finding N given M

• Given:

<table>
<thead>
<tr>
<th>M (# of generations)</th>
<th>N (# of people in fam)</th>
<th>N + 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>4 = 2^2</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>8 = 2^3</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
<td>16 = 2^4</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>32 = 2^5</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

• We notice that every N is consistently 2 away from the form: \(2^i\)

• We can then notice that the exponent is always one more than the generation

• This gives us: \(N = 2^{M+1} - 2\)
Finding M given N

- Now that we know: \( N = 2^{M+1} - 2 \), all we need to do is solve for \( M \)

\[
N = 2^{M+1} - 2
\]
\[
N + 2 = 2^{M+1} \quad \text{(add 2 to both sides)}
\]
\[
\log(N + 2) = \log(2^{M+1}) \quad \text{(take the log of both sides)}
\]
\[
\log(N + 2) = M + 1 \quad \text{(simplify)}
\]

\[
M = \log(N + 2) - 1
\]
\[
= \log\left(\frac{N + 2}{2}\right)
\]
Summing it all up

- How do we compare these functions?
  - By how they grow
Summing it all up

• How do we compare these functions?
  • By how they grow

![Graph comparing different functions: $N^2$, $(N^2)/2$, $x$, and $\log(x)$ over the range [0,10].]