

HW2 due now

HW3

- Due 9pm Wed Sept 27, 2006
- Entirely online—no paper, no email
- In lab yesterday we added question #6:
Add a post to your blog that contains an image directly hyperlinked to another website

Publicity

Publicity

- **First to submit HW1**
 1. Meghan Madore
 2. John Garawecki-Maxwell
 3. Nicole Edine

Publicity

- **First to submit HW2 blog bookmark**
 1. Vicki Hall
 2. Natalie Fava
 3. Kristin Cicarella

Publicity

- **Most popular bookmarks so far**
 - 1.
 - 2.
 - 3.

Reading

- Six Degrees Ch 2, 3
- Wikipedia
 - http://en.wikipedia.org/wiki/Social_Bookmarking
 - http://en.wikipedia.org/wiki/Web_feed

Last time: Small “worlds”

- Consider some domain and call that the “world” (e.g., Boston, BU, CS-103, all websites on the Internet)
- That “world” is small if it has the following three defining traits:
 1. I know only a small fraction of the “world”
 2. My friends know the same people I do
 3. Any two people in the “world” are connected by a relatively short path of relationships

Formalizing Small World Network

- Your books do not use mathematical notation
- We will use notation from <http://planetmath.org/encyclopedia/Graph.html>

The screenshot shows a web browser window with the URL <http://planetmath.org/encyclopedia/Graph.html>. The page title is "PlanetMath: graph". The main content area is titled "graph" and contains the following text:

A *graph* G is an ordered pair of disjoint sets (V, E) such that E is a subset of the set $V^{(2)}$ of unordered pairs of V . V and E are always assumed to be finite, unless explicitly stated otherwise. The set V is the set of *vertices* (sometimes called *nodes*) and E is the set of *edges*. If G is a graph, then $V = V(G)$ is the vertex set of G , and $E = E(G)$ is the edge set. Typically, $V(G)$ is defined to be nonempty. If x is a vertex of G , we sometimes write $x \in G$ instead of $x \in V(G)$.

An edge $\{x, y\}$ (with $x \neq y$) is said to *join* the vertices x and y and is denoted by xy . This xy and yx are said to be *equivalent*; the vertices x and y are the *endvertices* of this edge. If $xy \in E(G)$, then x and y are *adjacent*, or *neighboring*, vertices of G , and the vertices x and y are *incident* with the edge xy . Two edges are *adjacent* if they have exactly one common endvertex. Also, $x \sim y$ means that the vertex x is adjacent to the vertex y .

Notice that this definition allows pairs of the form $\{x, x\}$, which would correspond to a node joining to itself. Some authors explicitly disallow this in their definition of a graph.

Below the text are three diagrams illustrating different types of graphs:

- A complete bipartite graph $K_{3,3}$ with two sets of three vertices and all possible edges between them.
- A complete graph K_5 with five vertices arranged in a pentagon, with all possible edges between them.
- A star graph $K_{1,4}$ with one central vertex connected to four peripheral vertices.

The caption below the diagrams reads "Some graphs."

Important set notation

We can list a set explicitly

$\{2,3,5,7\}$

Or implicitly

$\{x: x \text{ is a prime number} < 10\}$

Or by name

P = set of prime numbers < 10

1a. How many people do I know?

Mathematically speaking:

We choose our world by defining a **graph** with a set of **vertices** V and a set of **edges** E

Example:

V = registered members of Screen Actors' Guild

E = all pairs of actors who co-participated in any movie

In this context, “How many people do I know” becomes
How many vertices are adjacent to me in the graph

For any vertex x in V ,

degree(x) = the number of vertices adjacent to x

1b. I know a small bit of the world

For any set V ,

$|V|$ = the number of elements in V

Also note that “ \ll ” means “is much less than”

“Each person knows a small fraction of the world” becomes

$\text{degree}(x) \ll |V|$ for any vertex x in V

1c. How many people do I know?

We just said:

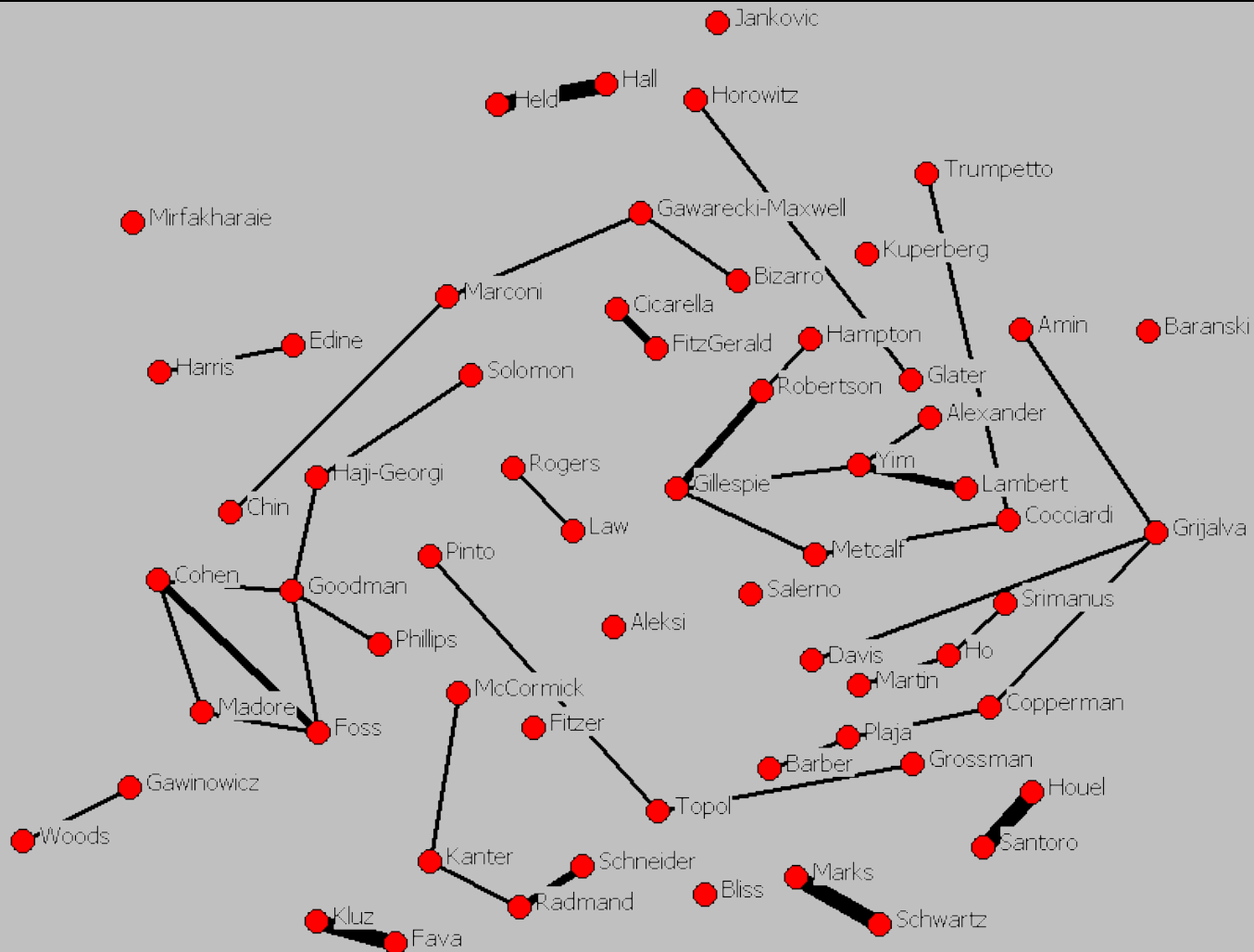
$\text{degree}(x)$ = the number of vertices adjacent to x

For any vertex x in V , think about this:

$\{y: \text{there is an edge } (x,y) \text{ in the graph}\}$

How are these related?

Examples of degree



2a. Who are my friends

For any vertex x in a graph

neighborhood(x) = x and the set of vertices adjacent to x

Using set notation, we can say

neighborhood(x) = $\{x\} \cup \{y: \text{there is an edge } (x,y) \text{ in the graph}\}$

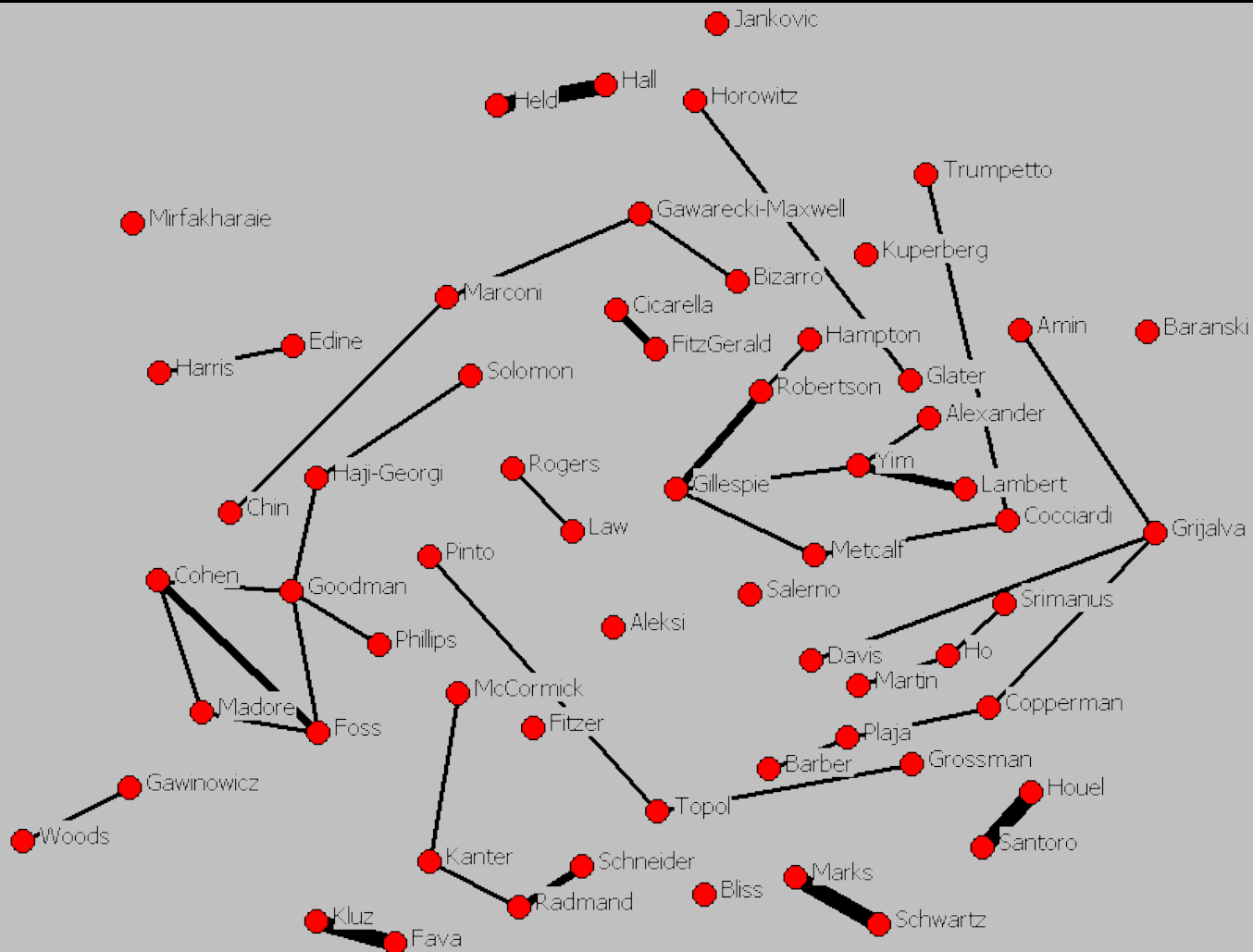
2b. Mutual friends

Notice that **neighborhood(x)** is a set of vertices

For any vertex y in $\text{neighborhood}(x)$, what is...

- $\text{neighborhood}(y)$
- $\text{neighborhood}(x) \cap \text{neighborhood}(y)$
- $$\frac{|\text{neighborhood}(x) \cap \text{neighborhood}(y)|}{|\text{neighborhood}(x) \cup \text{neighborhood}(y)|}$$

Examples of neighborhood



3a. What is a path

A **path** in a graph is a finite sequence of alternating vertices and edges, beginning and ending with a vertex

$$v_1 e_1 v_2 e_2 v_3 e_3 \dots e_{n-1} v_n$$

Such that

- every consecutive pair of vertices v_x, v_{x+1} , is adjacent
- e_x is incident with v_x and v_{x+1}

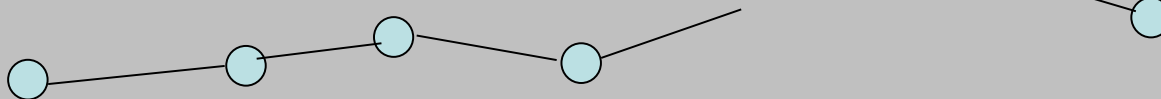
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v_1 e_1 v_2 e_2 v_3 $e_3 \dots$

e_{n-1} v_n

Strength of Weak Ties

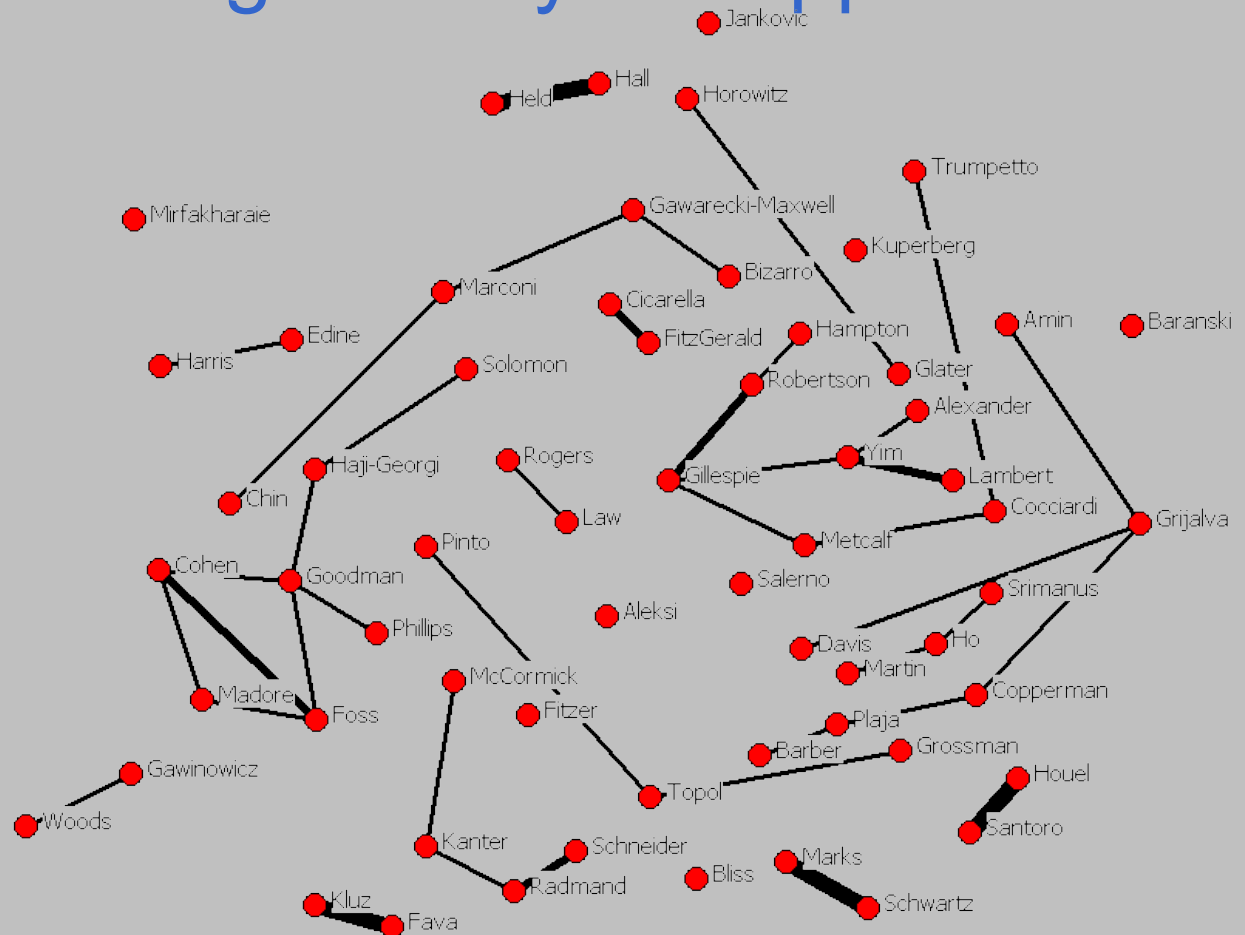
Two ways of thinking about weak ties:

3. A relationship between two people who don't know each other very well
4. A relationship between two people who don't have many mutual friends

“Weak ties” are much more important to job-hunters than “strong ties”

Network Dynamics

Where are new edges likely to happen?

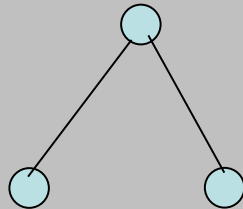


Triadic closure

- Some social triads are stable



- Some social triads are unstable



Triadic Social Network Table of the Elements

