CS 235: Algebraic Algorithms **Assignment 7**

Due: April 3, 2008

1 Group actions on the hexagon

In class we looked at what happens when we permute the vertices of a pentagon by applying the group S_5. Let's look at a slightly bigger case, by applying S_6 to the vertices of a hexagon.

1.1 Size (10 points)

How big is S_6? Explain.

1.2 Subgroups (20 points)

Prove that the set of elements H in S_6 mapping the hexagon to itself is a subgroup of S_6. (Note this set is *not* just the identity permutation: we can rearrange the vertices, and still get a shape that looks like a hexagon.) How big is this subgroup? Prove your answer.

1.3 Number of configurations (10 points)

Using your previous answer, compute how many different graphs we can get by permuting the hexagon (that is, how many different sets of edges).

1.4 Mapping between shapes (15 points)

Consider Figure 1. Again using 1.2, how many permutations are there that map the hexagon (left) to the graph on the right?

1.5 Non-commutative (10 points)

Give an example showing that the behavior of S_6 on the hexagon is not commutative, even if we just look at the edges. That is, give examples of permutations σ_1 and σ_2 such that applying them to the hexagon in different orders produces different sets of edges.

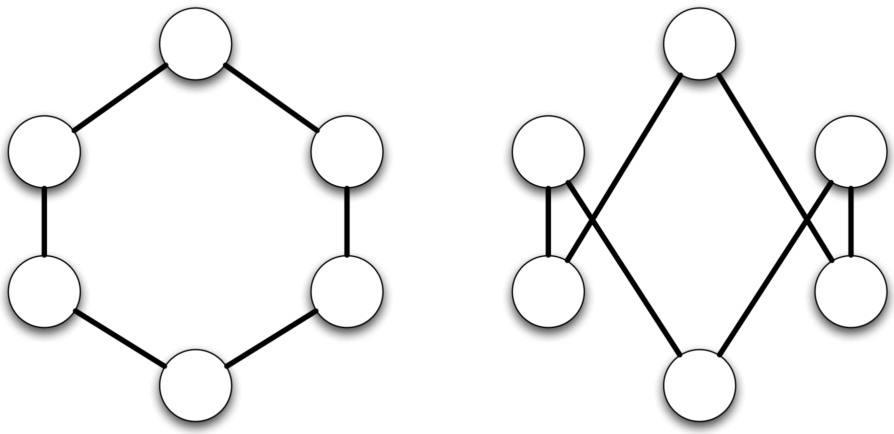

Figure 1: Two different configurations of the hexagon graph.

2 Distinct necklaces (35 points)

Suppose we are making 21-bead necklaces using two different colors of beads. How many distinct necklaces are there if you can rotate each necklace but not flip it over? (That is, our "identity" group is just $\mathbb{Z}_{21}$, not D_{21}.) Prove your answer.

3 Really distinct necklaces (40 points extra credit)

What if, in the previous problem, we also allow the necklaces to be flipped over (so now our identity group is D_{21})? How many distinct necklaces are there on 21 beads?