

Lecture outline

- Classification
- Naïve Bayes classifier
-

Bayes Theorem

- X, Y random variables
- Joint probability: $\Pr(X=x, Y=y)$
- Conditional probability: $\Pr(Y=y \mid X=x)$
- Relationship between joint and conditional probability distributions

$$\Pr(X, Y) = \Pr(X \mid Y) \times \Pr(Y) = \Pr(Y \mid X) \times \Pr(X)$$

- **Bayes Theorem:**

$$\Pr(Y \mid X) = \frac{\Pr(X \mid Y) \Pr(Y)}{\Pr(X)}$$

Bayes Theorem for Classification

- X : attribute set
- Y : class variable
- Y depends on X in a **non-deterministic** way
- We can capture this dependence using
 $\Pr(Y|X)$: Posterior probability
vs
 $\Pr(Y)$: Prior probability

Building the Classifier

- Training phase:
 - Learning the posterior probabilities $\Pr(Y|X)$ for every combination of X and Y based on training data
- Test phase:
 - For test record X' , compute the class Y' that maximizes the posterior probability $\Pr(Y'|X')$

Bayes Classification: Example

	binary	categorical	continuous	class
Tid	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Figure 4.6. Training set for predicting borrowers who will default on loan payments.

$X' = (\text{Home Owner} = \text{No}, \text{Marital Status} = \text{Married}, \text{Annual Income} = 120\text{K})$

Compute: $\Pr(\text{Yes} | X')$, $\Pr(\text{No} | X')$ pick No or Yes with max Prob.

How can we compute these probabilities??

Computing posterior probabilities

- Bayes Theorem

$$\Pr(Y | X) = \frac{\Pr(X | Y) \Pr(Y)}{\Pr(X)}$$

- **P(X)** is constant and can be ignored
- **P(Y)**: estimated from training data; compute the fraction of training records in each class
- **P(X|Y)?**

Naïve Bayes Classifier

$$\Pr(X | Y = y) = \prod_{i=1}^d \Pr(X_i | Y = y)$$

- Attribute set $\mathbf{X} = \{X_1, \dots, X_d\}$ consists of d attributes
- Conditional independence:
 - \mathbf{X} conditionally independent of \mathbf{Y} , given \mathbf{X} :
 $\Pr(\mathbf{X} | \mathbf{Y}, \mathbf{Z}) = \Pr(\mathbf{X} | \mathbf{Z})$
 - $\Pr(\mathbf{X}, \mathbf{Y} | \mathbf{Z}) = \Pr(\mathbf{X} | \mathbf{Z}) \times \Pr(\mathbf{Y} | \mathbf{Z})$

Naïve Bayes Classifier

$$\Pr(X|Y = y) = \prod_{i=1}^d \Pr(X_i|Y = y)$$

- Attribute set $\mathbf{X} = \{X_1, \dots, X_d\}$ consists of d attributes

$$\Pr(X|Y) = \frac{\Pr(Y) \prod_{i=1}^d \Pr(X_i|Y)}{\Pr(X)}$$

Conditional probabilities for categorical attributes

- Categorical attribute X_i
- $\Pr(X_i = x_i | Y = y)$: fraction of training instances in class y that take value x_i on the i -th attribute

$$\Pr(\text{homeOwner} = \text{yes} | \text{No}) = 3/7$$

$$\Pr(\text{MaritalStatus} = \text{Single} | \text{Yes}) = 2/3$$

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Figure 4.6. Training set for predicting borrowers who will default on loan payments.

Estimating conditional probabilities for continuous attributes?

- Discretization?
- How can we discretize?

Naïve Bayes Classifier: Example

- $X' = (\text{HomeOwner} = \text{No}, \text{MaritalStatus} = \text{Married}, \text{Income} = 120\text{K})$
- Need to compute $\Pr(Y|X')$ or $\Pr(Y) \times \Pr(X'|Y)$
- But $\Pr(X'|Y)$ is
 - $Y = \text{No}$:
 - $\Pr(\text{HO} = \text{No} | \text{No}) \times \Pr(\text{MS} = \text{Married} | \text{No}) \times \Pr(\text{Inc} = 120\text{K} | \text{No}) = 4/7 \times 4/7 \times 0.0072 = 0.0024$
 - $Y = \text{Yes}$:
 - $\Pr(\text{HO} = \text{No} | \text{Yes}) \times \Pr(\text{MS} = \text{Married} | \text{Yes}) \times \Pr(\text{Inc} = 120\text{K} | \text{Yes}) = 1 \times 0 \times 1.2 \times 10^{-9} = 0$

Naïve Bayes Classifier: Example

- $X' = (\text{HomeOwner} = \text{No}, \text{MaritalStatus} = \text{Married}, \text{Income} = 120\text{K})$
- Need to compute $\Pr(Y|X')$ or $\Pr(Y) \times \Pr(X'|Y)$
- But $\Pr(X'|Y = \text{Yes})$ is 0?
- Correction process:

$$\Pr(X_i = x_i | Y = y_j) = \frac{n_c + mp}{n + m}$$

n_c : number of training examples from class y_j that take value x_i

n : total number of instances from class y_j

m : equivalent sample size (balance between prior and posterior)

p : user-specified parameter (prior probability)

Characteristics of Naïve Bayes Classifier

- Robust to isolated noise points
 - noise points are averaged out
- Handles missing values
 - Ignoring missing-value examples
- Robust to irrelevant attributes
 - If X_i is irrelevant, $P(X_i|Y)$ becomes almost uniform
- Correlated attributes degrade the performance of NB classifier