CAS CS 131 - Combinatorial Structures

Spring 2011

Problem Set #1 (Logic & Proofs, Proofs by Induction)
Out: Tuesday, January 25
Due: Tuesday, February 1

NO LATE SUBMISSIONS WILL BE ACCEPTED

To be completed individually.

1. Classify each of the following statements as true, false, or not a proposition. Explain your answers: If the statement is a proposition, prove or disprove it. If it is not a proposition, state why.

   (a) An integer is a rational number.

   (b) 5 is an even number and $16^{-\frac{1}{4}} = \frac{1}{2}$.

   (c) Let n denote a positive integer.

   (d) If a and b are integers with $a - b \geq 0$ and $b - a \geq 0$, then $a = b$.

2. Determine whether the following implication is true. If it is true, provide a proof. If it is not true, provide a counterexample.

   "x is an even integer $\Longrightarrow x + 2$ is an even integer."

3. Prove that the equations

   $2x + 3y - z = 5$

   $x - 2y + 3z = 7$

   $x + 5y - 4z = 0$

   have no solution. (Give a proof by contradiction.)

4. Prove that the sum of a rational and an irrational number is irrational. (Use proof by contradiction.)

5. Use (weak) induction to prove that, for all real values a and d:

   $$a + (a + d) + (a + 2d) + (a + 3d) \cdots + (a + (n-1)d) = an + \frac{d(n-1)n}{2}$$

6. Use (weak) induction to prove that

   $(1 - \frac{1}{2})(1 - \frac{1}{3})(1 - \frac{1}{4}) \cdots (1 - \frac{1}{n}) = \frac{1}{n}$

   for all $n \geq 2$.