

**CAS CS 131 - Combinatorial Structures**  
*Spring 2011*

PROBLEM SET #2 (LOGIC & PROOFS)

OUT: TUESDAY, FEBRUARY 1

DUE: TUESDAY, FEBRUARY 8

**NO LATE SUBMISSIONS WILL BE ACCEPTED**

**To be completed individually.**

1. Write the negation of this statement in clear simple English. Do not use the expression “It is not the case” in your answers.

If she works, she will earn money.

2. Write down the contrapositive of the following implication:

If  $\frac{a}{b}$  and  $\frac{b}{c}$  are integers, then  $\frac{a}{c}$  is an integer.

3. Determine whether this argument is valid; prove or disprove its validity.

If it rains, Erik will be sick.

It did not rain.

Erik was not sick.

4. Use (strong) induction to prove that every integer  $n \geq 2$  is divisible by a prime.
5. Use (weak) induction to prove that  $n$  cents of postage can be formed using 3 and 8 cent stamps for all  $n \geq 15$ .
6. The harmonic number  $H_n$  is the sum of the first  $n$  values of the harmonic series:  $H_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$ . For example,  $H_1 = 1$ ,  $H_2 = 1 + \frac{1}{2}$ , and  $H_3 = 1 + \frac{1}{2} + \frac{1}{3}$ . Use (weak) induction to prove that

$$\sum_{i=1}^n H_i = (n+1)H_n - n.$$