CAS CS 131 - Combinatorial Structures

Spring 2011

PROBLEM SET #4 (ASYMPTOTIC NOTATION AND RECURRENCES)
OUT: TUESDAY, MARCH 1
DUE: TUESDAY, MARCH 8

NO LATE SUBMISSIONS WILL BE ACCEPTED

To be completed individually.

1. Prove that the following relationships hold.
   (a) $3x^2 = O(x^2)$.
   (b) $\ln x = O(\log_b x)$, where the base b is any positive real number other than 1.
2. Prove that if $f(x) = O(g(x))$, and $g(x) = O(h(x))$, then $f(x) = O(h(x))$.
3. Solve the recurrence equation $a_n = a_{n-1} + 2^n$ for $n \geq 1$, given $a_0 = 5$.
4. Assume you are given a *sorted* list of numbers. These numbers may represent keys to personal records. We want to search for a particular key, whether it exists or not, and if it exists, we return its position in the list.

   A binary search is an algorithm for locating the position of an element in a sorted list. It inspects the middle element of the sorted list: if equal to the sought value, then the position has been found; otherwise, the upper half or lower half is chosen for further searching based on whether the sought value is greater than or less than the middle element. The method reduces the number of elements needed to be checked by a factor of two each time, and finds the sought value if it exists in the list or if not determines "not present".

   Describing the running time in terms of number of comparisons, show that in the worst case, it is $O(\log n)$, where n is the size of the sorted list. For this, you should write the corresponding recurrence equation and then solve it.
5. Suppose we modify the traditional rules for the Towers of Hanoi Problem (that we have seen in class) by requiring that one moves discs only to an **adjacent** peg.
   (a) Solve the puzzle when there are $n = 2$ discs and show your moves in a diagram of three columns representing the three pegs, and discs numbered $1, 2, \cdots$ representing smallest to larger discs.
   (b) Give a recurrence relation for T_n, the number of moves required to transfer n discs from one peg to another.
   (c) Find an explicit formula for T_n.
   (d) Suppose we can move a disc a second. Estimate the time required to transfer the discs if $n = 8, 16, 32, 64$.