CAS CS 131 - Combinatorial Structures

Spring 2011

PROBLEM SET #8 (PROBABILITY)
OUT: TUESDAY, APRIL 26
DUE: TUESDAY, MAY 3

NO LATE SUBMISSIONS WILL BE ACCEPTED

To be completed individually.

1. There are three coins: a penny, a nickel, and a quarter. When these coins are flipped:

   - The penny comes up heads with probability 2/3 and tails with probability 1/3.
   - The nickel comes up heads with probability 1/4 and tails with probability 3/4.
   - The quarter comes up heads with probability 3/5 and tails with probability 2/5.

   Assume that the way one coin lands is unaffected by the way the other coins land. The goal of this problem is to determine the probability that an *even* number of coins come up heads. Your solution must include a tree diagram.

   (a) What is the sample space for this experiment?

   (b) What subset of the sample space is the event that an even number of coins come up heads?

   (c) What is the probability of each outcome in the sample space?

   (d) What is the probability that an even number of coins come up heads?

2. (From an old final exam.) *Finalphobia* is a rare disease in which the victim has the delusion that he or she is being subjected to an intense mathematical examination.

   - A person selected at random has finalphobia with probability 1/100.
   - A person with finalphobia has shaky hands with probability 9/10.
   - A person without finalphobia has shaky hands with probability 1/20.

   (a) What is the probability that a person selected at random has finalphobia, given that he or she does not have shaky hands?

   (b) Show whether or not the two events "a person has finalphobia" and "a person does not have shaky hands" are independent?

3. There is a set P consisting of 10,000 people.

   - The favorite color of 10% of the people is blue.
   - The favorite color of 40% is green.
   - The favorite color of 50% is red.

   (a) Suppose we select a set of two people $\{p_1, p_2\} \subseteq P$ at random. Let the variables C_1 and C_2 denote their favorite colors. Are C_1 and C_2 independent? Justify your answer.

(b) Suppose we select a sequence of two people $(p_1, p_2) \in P \times P$ at random. Let the variables C_1 and C_2 denote their favorite colors. Now are C_1 and C_2 independent? Justify your answer.

4. Justify your answers to the following questions about independence.

   (a) Suppose that you roll a fair die that has six sides, numbered $1, 2, \cdots, 6$. Is the event that the number on top is a multiple of 2 independent of the event that the number on top is a multiple of 3?

   (b) Now suppose that you roll a fair die that has *four* sides, numbered 1, 2, 3, 4. Is the event that the number on top is a multiple of 2 independent of the event that the number on top is a multiple of 3?

   (c) Now suppose that you roll a fair die that has *ten* sides, numbered $1, 2, \cdots, 10$. Again, is the event that the number on top is a multiple of 2 independent of the event that the number on top is a multiple of 3?

5. Suppose you flip n fair, independent coins. Let the variable X be the number of heads that come up.

   (a) What is the exact value of $\Pr(X \leq k)$, the probability of getting k or fewer heads? Your answer need not be in closed form.

   (b) Suppose $k < n/2$. Prove that:

$$\Pr(X \leq k) \leq \frac{n-k+1}{n-2k+1} \Pr(X = k)$$

   (Upper bound your previous answer with an infinite geometric sum and then evaluate the sum.)

   (c) If you flip a coin 100 times, the probability of getting exactly 30 heads is approximately 23 out of a million. Give an upper bound on the probability of getting *30 or fewer* heads.