

CAS CS 131 - Combinatorial Structures

Spring 2011

PRACTICE PROBLEMS ON COUNTING AND PROBABILITY

OUT: THURSDAY, MAY 5

1. Consider a two-player game where a fair coin is repeatedly tossed. Each player chooses a length-3 sequence of heads (H) and tails (T). Whenever a chosen sequence comes up, the corresponding player wins the game.
 - (a) Assume player A chooses sequence (H,H,T) and player B chooses sequence (H,T,T). What's the probability that player A wins? [*Hint*: show the tree diagram (or enough of it) in solving this problem.]
 - (b) If you play that game and your opponent chooses sequence (H,H,H), can you choose a sequence that makes it more likely for you to win? If so, what's the probability that you win using that sequence?
2. In how many different ways can the letters in the name of the popular 1980s band *BANANARAMA* be arranged?
3. In how many different ways can one choose n out of $2n$ objects, given that n of the $2n$ objects are identical and the other n are all unique? [*Hint*: find ways to first select a subset from the unique objects.]
4. In how many different ways can the numbers shown on a red die, a green die, and a blue die total up to 15? Assume that these are ordinary 6-sided dice. [*Hint*: enumerate all possible cases.]
5. Jellybeans of 6 different flavors are stored in 5 jars. There are 11 jellybeans of each flavor. Prove that some jar contains at least three jellybeans of one flavor and also at least three jellybeans of some other flavor. [*Hint*: use the pigeonhole principle.]
6. There are five buttons numbered 1, 2, 3, 4, and 5. One may press a sequence of buttons; however, the same button can not be pressed twice in a row. How many different sequences of n button-presses are possible?
7. Suppose you have five 6-sided dice, which are colored red, blue, green, white, and black. A *roll* is a sequence specifying a value for each die. For example, one roll is: (3,1,4,1,5), where the first value comes from the red die, second value from the blue die, etc.
 - (a) For how many rolls is the value on every die different?
Example: (1,2,3,4,5) is a roll of this type, but (1,1,2,3,4) is not.
 - (b) For how many rolls do two dice have the same value and the remaining three dice all have different values?
Example: (6,1,6,2,3) is a roll of this type, but (1,1,2,2,3) and (4,4,4,5,6) are not.

8. Two young soccer players (teammates) are trying to learn new moves before their next game. Micheal is trying to learn the Rainbow move and John is trying to learn the Matthews-Scissors combo. The coach will decide whether they have been successful in doing the move right!

Suppose that Michael's probability of succeeding is $1/6$, John's chance of succeeding is $1/4$, and the success of one does not alter the other's chances.

- (a) If at least one of them succeeds, what is the probability that Michael succeeds in doing his move right before the next game?
- (b) If at most one of them succeeds, what is the probability that John succeeds in doing his move right before the next game?
- (c) If exactly one of them succeeds, what is the probability that it is Michael?