CAS CS 131 - Combinatorial Structures
Spring 2012

Problem Set #1 (Logic & Proofs, Proofs by Induction)
Out: Thursday, January 26
Due: Thursday, February 2 at 2pm

NO LATE SUBMISSIONS WILL BE ACCEPTED

To be completed individually.

1. Classify each of the following statements as true, false, or not a proposition. Explain your answers: If the statement is a proposition, prove or disprove it. If it is not a proposition, state why.

   (a) An integer is a rational number.
   (b) 5 is an even number and $16^{-\frac{3}{4}} = \frac{1}{2}$.
   (c) Let $n$ denote a positive integer.
   (d) If $a$ and $b$ are integers with $a - b \geq 0$ and $b - a \geq 0$, then $a = b$.

2. Determine whether the statement is true or false. Explain your answers.

   (a) $5 \subset \{1, 2, 3, 4, 5\}$
   (b) $3 \in \{1, 2, 3, 4, 5\}$
   (c) $\{3, 5\} \in \{1, 2, 3, 4, 5\}$
   (d) $\{3, 5\} \subset \{x | (x = 2 \cdot k + 1) \land k \in \mathbb{N}\}$

3. Prove that the equations

   \begin{align*}
   2x + 3y - z &= 5 \\
   x - 2y + 3z &= 7 \\
   x + 5y - 4z &= 0
   \end{align*}

   have no solution. (Give a proof by contradiction.)

4. Prove that the sum of a rational and an irrational number is irrational. (Use proof by contradiction.)

5. Use induction to prove that

   \[ 1^2 + 2^2 + \ldots + n^2 = \frac{n^3}{3} - \frac{n^2}{2} + \frac{n}{6}, \]

   for every natural number $n$. 

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