CAS CS 131 - Combinatorial Structures

Spring 2012

Problem Set #1 (Logic & Proofs, Proofs by Induction)
Out: Thursday, January 26
Due: Thursday, February 2 at 2pm

NO LATE SUBMISSIONS WILL BE ACCEPTED

To be completed individually.

1. Classify each of the following statements as true, false, or not a proposition. Explain your answers: If the statement is a proposition, prove or disprove it. If it is not a proposition, state why.

   (a) An integer is a rational number.

   (b) 5 is an even number and $16^{-\frac{1}{4}} = \frac{1}{2}$.

   (c) Let n denote a positive integer.

   (d) If a and b are integers with $a - b \geq 0$ and $b - a \geq 0$, then $a = b$.

2. Determine whether the statement is true or false. Explain your answers.

   (a) $5 \subset \{1, 2, 3, 4, 5\}$

   (b) $3 \in \{1, 2, 3, 4, 5\}$

   (c) $\{3, 5\} \in \{1, 2, 3, 4, 5\}$

   (d) $\{3, 5\} \subset \{x | (x = 2 * k + 1) \wedge k \in \mathbf{N}\}$

3. Prove that the equations

   $2x + 3y - z = 5$

   $x - 2y + 3z = 7$

   $x + 5y - 4z = 0$

   have no solution. (Give a proof by contradiction.)

4. Prove that the sum of a rational and an irrational number is irrational. (Use proof by contradiction.)

5. Use induction to prove that

$$1^2 + 2^2 + \ldots + n^2 = \frac{n^3}{3} - \frac{n^2}{2} + \frac{n}{6},$$

   for every natural number n.