CAS CS 131 - Combinatorial Structures

Spring 2012

Problem Set #4 (Asymptotic Notation and Recurrences)
Out: Thursday, March 1
Due: Thursday, March 8

NO LATE SUBMISSIONS WILL BE ACCEPTED

To be completed individually.

1. Find a closed-form expression equal to the following product. Show your work.

$$\prod_{i=1}^{n} 2^{i^2+i}$$

2. Find a closed-form expression equal to the following summation. Show your work.

$$\sum_{i=3}^{\infty} (\frac{-1}{2})^i$$

3. Prove that the following relationships hold.
   (a) $3x^2 + 2x + 1 = O(x^2)$.
   (b) $\ln x = O(\log_b x)$, where the base b is any positive real number other than 1.
4. Prove that if $f(x) = O(g(x))$, and $g(x) = O(h(x))$, then $f(x) = O(h(x))$.
5. Show that the following asymptotic relationships hold:
   (a) $2n + \log n = \Theta(n)$
   (b) $n^2 = o(1.01^n)$
   (c) $\log n = o(n)$
   (d) $2^{n/2} = o(2^n)$
   (e) $\frac{n^2+2n-3}{n^2-7} \sim 1$
   (f) $\sum_{i=0}^{n} 2^{2i+1} = \Theta(4^n)$.
6. Solve the recurrence equation $a_n = a_{n-1} + 2^n$ for $n \geq 1$, given $a_0 = 5$.
7. Suppose we modify the traditional rules for the Towers of Hanoi Problem (that we have seen in class) by requiring that one moves discs only to an **adjacent** peg.
   (a) Solve the puzzle when there are $n = 2$ discs and show your moves in a diagram of three columns representing the three pegs, and discs numbered $1, 2, \cdots$ representing smallest to larger discs.

(b) Give a recurrence relation for T_n, the number of moves required to transfer n discs from one peg to another.

(c) Find an explicit formula for T_n.

(d) Suppose we can move a disc a second. Estimate the time required to transfer the discs if $n = 8, 16, 32, 64$.