CAS CS 131 - Combinatorial Structures

Spring 2012

PROBLEM SET #6 (ASYMPTOTIC NOTATION AND RECURRENCES)
OUT: THURSDAY, MARCH 22
DUE: THURSDAY, MARCH 29

NO LATE SUBMISSIONS WILL BE ACCEPTED

To be completed individually.

1. Solve the following linear recurrences:
   - $f(n) = f(n-1) + 6f(n-2), f(0) = 3, f(1) = 6$
   - $f(n) = 2f(n-1) - f(n-2), f(0) = 4, f(1) = 1$
   - $f(n) = 2f(n-1) + 2n, f(0) = 3$
   - $f(n) = 4f(n-2) + 3, f(0) = 3, f(1) = 2$

2. In how many different ways can a $2 \times n$ rectangular board be tiled using 1×2 and 2×2 pieces?
   Hint: Start by writing the recurrence equation, and then solve the linear recurrence using what we studied in class.

3. Use the master theorem to give tight asymptotic bounds for the following recursions:
   (a) $T(n) = 4T(n/2) + n$.
   (b) $T(n) = 4T(n/2) + n^2$.
   (c) $T(n) = 4T(n/3) + n^3$.

4. The recurrence $T(n) = 7T(n/2) + n^2$ describes the running time of an algorithm A. A competing algorithm A' has a running time of $T'(n) = aT'(n/4) + n^2$. What is the largest integer value for a such that A' is asymptotically faster than A?

5. Remember the modified version of binary search which we discussed in the previous homework (Problem 7). Write down the recurrence equation for both part (a) and (b) and this time find out the asymptotic complexity of them using master theorem.