CAS CS 131 - Combinatorial Structures

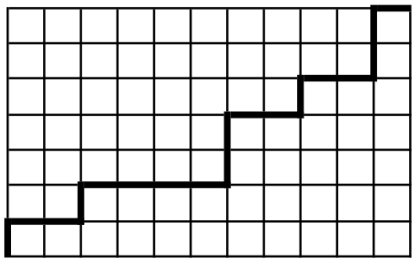
Spring 2012

Problem Set #9 (More Counting)
Out: Thursday, April 26
Due: Thursday, May 3

NO LATE SUBMISSIONS WILL BE ACCEPTED

To be completed individually.

1. Prove that $2^n = \binom{n}{0}\binom{n}{1}\binom{n}{2}\cdots\binom{n}{n}$.
   (a) First, Give a *combinatorial* argument proving that the total number of subsets of set $A = \{x_1, x_2, x_3, \cdots, x_n\}$ is 2^n.
   (b) Then, Give another *combinatorial* argument proving that the total number of subsets of set A is also $\binom{n}{0}\binom{n}{1}\binom{n}{2}\cdots\binom{n}{n}$.
   (c) Use parts (a) and (b) to prove that $2^n = \binom{n}{0}\binom{n}{1}\binom{n}{2}\cdots\binom{n}{n}$.
2. Two of the sqaures of a 7×7 checkerboard are going to be painted yellow, and the rest are going to be painted green.
   (a) In how many possible ways the mentioned checkerboard can be painted?
   (b) Two color schemes are equivalent if one can be obtained from the other by applying a rotation in the plane of the board. How many inequivalent color schemes are possible?
3. In how many different ways can Blockbuster arrange 64 copies of *Despicable Me*, 96 copies of *Toy Story 3*, 55 copies of *Robin Hood* and 1 copy of *Shrek Goes Fourth* on a shelf? What if they are to be arranged in 5 shelves?
4. in a 2D grid, determine the number of paths from $(0, 0)$ to (m, n) following the gridlines and moving only in the up or right direction.



5. A jar contains 100 marbles, identical except that 30 are red, 20 black, 5 green and the rest w hite. If a marble is taken from the jar at random, w hat is the probability that the marble is:

   (a) red ?

   (b) black or green?

   (c) not red?

   (d) multicolor?

6. There are three coins: a penny, a nickel, and a quarter. When these coins are flipped:

   - The penny comes up heads with probability 2/3 and tails with probability 1/3.
   - The nickel comes up heads with probability 1/4 and tails with probability 3/4.
   - The quarter comes up heads with probability 3/5 and tails with probability 2/5.

   Assume that the way one coin lands is unaffected by the way the other coins land. The goal of this problem is to determine the probability that an *even* number of coins come up heads. Your solution must include a tree diagram.

   (a) What is the sample space for this experiment?

   (b) What subset of the sample space is the event that an even number of coins come up heads?

   (c) What is the probability of each outcome in the sample space?

   (d) What is the probability that an even number of coins come up heads?