CAS CS 131 - Combinatorial Structures

Spring 2013

Problem Set #1 (Logic)
Out: Thursday, January 24
Due: Thursday, January 31

NO LATE SUBMISSIONS WILL BE ACCEPTED

To be completed individually.

1. Let S stand for the statement "Steve is happy" and G for "George is happy." What English sentences are represented by the following expressions?

   (a) $(S \vee G) \wedge (\neg S \vee \neg G)$.

   (b) $[S \vee (G \wedge \neg S)] \vee \neg G$.

   (c) $S \vee [G \wedge (\neg S \vee \neg G)]$.

2. Identify the premises and conclusions of the following deductive arguments and analyze their logical forms. Do you think the reasoning is valid?

   (a) Jane and Pete won't both win the math prize. Pete will win either the math prize or the chemistry prize. Jane will win the math prize. Therefor, Pete will win the chemistry prize.

   (b) The main course will be either beef or fish. The vegetable will be either peas or corn. We will not have both fish as a main course and corn as a vegetable. Therefore, we will not have both beef as a main course and peas as a vegetable.

   (c) Either John or Bill is telling the truth. Either Sam or Bill is lying. Therefor, either John is telling the truth or Sam is lying.

   (d) Either sales will go up and the boss will be happy, or expenses will go up and the boss won't be happy. Therefor, sales and expenses will not both go up.

3. Use the first DeMorgan's law and the double negation law to derive the second DeMorgan's law.

4. Note that the associative laws say only that parentheses are unnecessary when combining *three* statements with $\wedge$ or $\vee$. In fact, these laws can be used to justify leaving parentheses out when more than three statements are combined. Use associative laws to show that $[P \wedge (Q \wedge R)] \wedge S$ is equivalent to $(P \wedge Q) \wedge (R \wedge S)$.

5. (a) Show that $P \leftrightarrow Q$ is equivalent to $(P \wedge Q) \vee (\neg P \wedge \neg Q)$.
   (b) Show that $(P \rightarrow Q) \vee (P \rightarrow R)$ is equivalent to $P \rightarrow (Q \vee R)$.

6. Show that $(P \rightarrow Q) \wedge (Q \rightarrow R)$ is equivalent to $(P \rightarrow R) \wedge [(P \leftrightarrow Q) \vee (R \leftrightarrow Q)]$.

7. Which of the following formulas are equivalent?

   (a) $P \rightarrow (Q \rightarrow R)$.
   (b) $Q \rightarrow (P \rightarrow R)$.
   (c) $(P \rightarrow Q) \wedge (P \rightarrow R)$.
   (d) $(P \wedge Q) \rightarrow R$.
   (e) $P \rightarrow (Q \wedge R)$.