CAS CS 131 - Combinatorial Structures

Spring 2013

Problem Set #2 (Quantificational Logic)
Out: Thursday, January 31
Due: Thursday, February 7

NO LATE SUBMISSIONS WILL BE ACCEPTED

To be completed individually.

1. Analyze the logical forms of the following statements. The universe of discourse is $\mathbb{R}$.
   (a) Every number that is larger than x is larger than y.
   (b) For every number a, the equation $ax^2 + 4x - 2 = 0$ has at least one solution iff $a \geq -2$.
   (c) All solutions of the inequality $x^3 - 3x < 3$ are smaller than 10.
   (d) If there is a number s such that $x^2+5x = w$ and there is a number y such that $4-y^2 = w$, then w is between -10 and 10.

2. Are these statements true or false? The universe of discourse is the set of all people, and $P(x, y)$ means "x is a parent of y."
   (a) $\exists x \forall y P(x, y)$.
   (b) $\forall x \exists y P(x, y)$.
   (c) $\neg \exists x \exists y P(x, y)$.
   (d) $\exists x \neg \exists y P(x, y)$.
   (e) $\exists x \exists y \neg P(x, y)$.

3. Negate these statements and then re-express the results as equivalent positive statements.
   (a) There is someone in the freshman class who doesn't have a roommate.
   (b) Everyone likes someone, but no one likes everyone.
   (c) $\forall y > 0 \exists x(ax^2 + bx + c = y)$.

4. Are these statements true or false? The universe of discourse is $\mathbb{N}$.
   (a) $\forall x(x < 7 \rightarrow \exists a \exists b \exists c(a^2 + b^2 + c^2 = x))$.
   (b) $\exists x \exists y((x - 4)^2 = 25 \wedge (y - 4)^2 = 25)$.

5. Are these statements true of false? The universe of discourse is $\mathbb{R}$.
   (a) $\forall x \exists y(2x - y = 0)$.
   (b) $\exists y \forall x(2x - y = 0)$.
   (c) $\forall x \exists y(x - 2y = 0)$.
   (d) $\forall x(x < 10 \rightarrow \forall y(y < x \rightarrow y < 9))$.
   (e) $\exists y \exists z(y + z = 100)$.
   (f) $\forall x \exists y(y > x \wedge \exists z(y + z = 100))$.