CAS CS 131 - Combinatorial Structures

Spring 2013

Problem Set #3 (Proofs)
Out: Thursday, February 13
Due: Thursday, February 21

NO LATE SUBMISSIONS WILL BE ACCEPTED

To be completed individually.

1. Suppose a and b are real numbers. Prove that if $a < b < 0$ then $a^2 > b^2$.

2. Suppose $A \setminus B \subseteq C \cap D$ and $x \in A$. Prove that if $x \notin D$ then $x \in B$.

3. Suppose that $A \setminus B$ is disjoint from C and $x \in A$. Prove that if $x \in C$ then $x \in B$.

4. Suppose that $y + x = 2y - x$, and x and y are not both zero. Prove that $y \neq 0$.

5. Suppose that x and y are real numbers. Prove that if $x^2 y = 2x + y$, then if $y \neq 0$ then $x \neq 0$.

6. Suppose that x is a real number.
   - Prove that if $x \neq 1$ then there is a real number y such that $\frac{y+1}{y-2} = x$.
   - Prove that if there is a real number y such that $\frac{y+1}{y-2} = x$, then $x \neq 1$.

7.
   - Prove that for all real numbers x and y there is a real number z such that $x + z = y - z$.
   - Would the statement in part (a) be correct if "real number" were changed to "integer"? Justify your answer.

8. Consider the following putative theorem:

   Theorem? For all real numbers x and y, $x^2 + xy - 2y^2 = 0$.

   - What's wrong with the following proof of the theorem?

     Proof: Let x and y be equal to some arbitrary real number r. Then

     $$x^2 + xy - 2y^2 = r^2 + r.r - 2r^2 = 0.$$

     Since x and y were both arbitrary, this shows that for all real numbers x and y, we have $x^2 + xy - 2y^2 = 0$.
   - Is this theorem correct? Justify your answer with either a proof or a counterexample.