To be completed individually.

1. Suppose $a$ and $b$ are real numbers. Prove that if $a < b < 0$ then $a^2 > b^2$.

2. Suppose $A \setminus B \subseteq C \cap D$ and $x \in A$. Prove that if $x \notin D$ then $x \in B$.

3. Suppose that $A \setminus B$ is disjoint from $C$ and $x \in A$. Prove that if $x \in C$ then $x \in B$.

4. Suppose that $y + x = 2y - x$, and $x$ and $y$ are not both zero. Prove that $y \neq 0$.

5. Suppose that $x$ and $y$ are real numbers. Prove that if $x^2y = 2x + y$, then if $y \neq 0$ then $x \neq 0$.

6. Suppose that $x$ is a real number.
   - Prove that if $x \neq 1$ then there is a real number $y$ such that $y^2 + y + 1 = x$.
   - Prove that if there is a real number $y$ such that $y^2 + y - 2 = x$, then $x \neq 1$.

7. • Prove that for all real numbers $x$ and $y$ there is a real number $z$ such that $x + z = y - z$.
   - Would the statement in part (a) be correct if “real number” were changed to “integer”? Justify your answer.

8. Consider the following putative theorem:
   
   **Theorem?** For all real numbers $x$ and $y$, $x^2 + xy - 2y^2 = 0$.
   - What’s wrong with the following proof of the theorem?
     
     **Proof:** Let $x$ and $y$ be equal to some arbitrary real number $r$. Then
     
     $$x^2 + xy - 2y^2 = r^2 + r.r - 2r^2 = 0.$$ 
     
     Since $x$ and $y$ were both arbitrary, this shows that for all real numbers $x$ and $y$, we have $x^2 + xy - 2y^2 = 0$.
   - Is this theorem correct? Justify your answer with either a proof or a counterexample.