CAS CS 131 - Combinatorial Structures

Spring 2013

PROBLEM SET #4 (INDUCTION)
OUT: THURSDAY, FEBRUARY 28
DUE: THURSDAY, FEBRUARY 7

NO LATE SUBMISSIONS WILL BE ACCEPTED

To be completed individually.

1. Use Induction to prove that for all $n \geq 1$,
$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots + \frac{1}{2n-1} - \frac{1}{2n} = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \ldots + \frac{1}{2n}.$$

2. Use Induction to prove that for all $n \geq 1$,
$$2.2^1 + 3.2^2 + 4.2^3 + \ldots + (n+1)2^n = n2^{n+1}.$$

3. Use induction to prove that every integer $n \geq 2$ is divisible by a prime.

4. Use induction to prove that n cents of postage can be formed using 3 and 8 cent stamps for all $n \geq 15$.

5. The harmonic number H_n is the sum of the first n values of the harmonic series: $H_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$. For example, $H_1 = 1$, $H_2 = 1 + \frac{1}{2}$, and $H_3 = 1 + \frac{1}{2} + \frac{1}{3}$. Use induction to prove that
$$\sum_{i=1}^{n} H_i = (n+1)H_n - n.$$