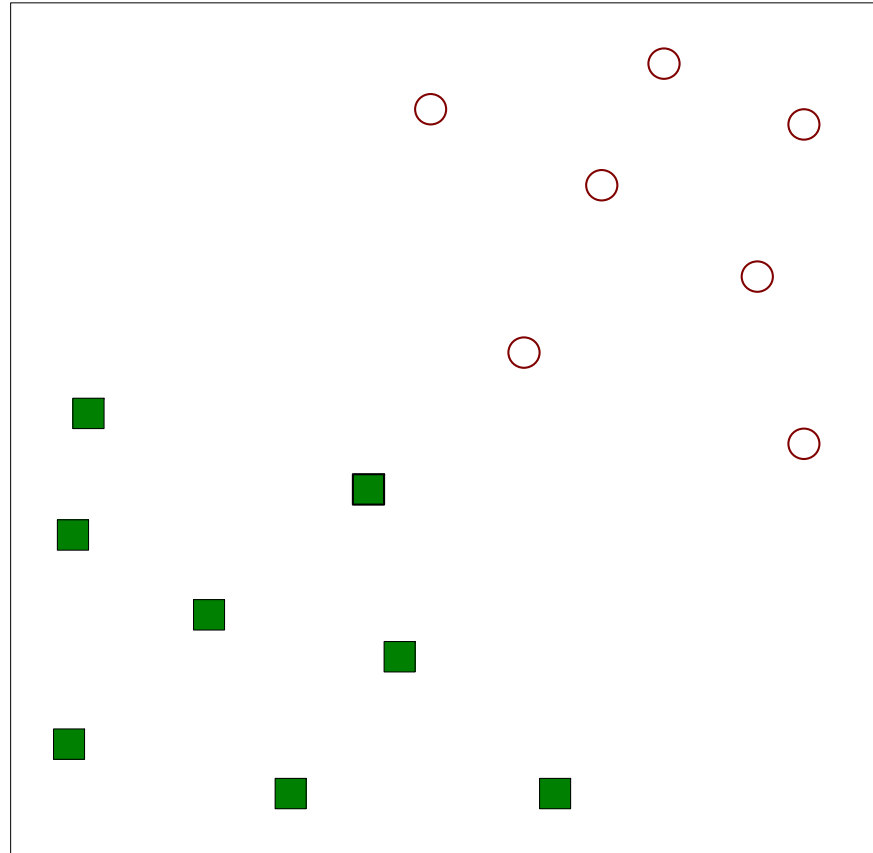


Lecture outline

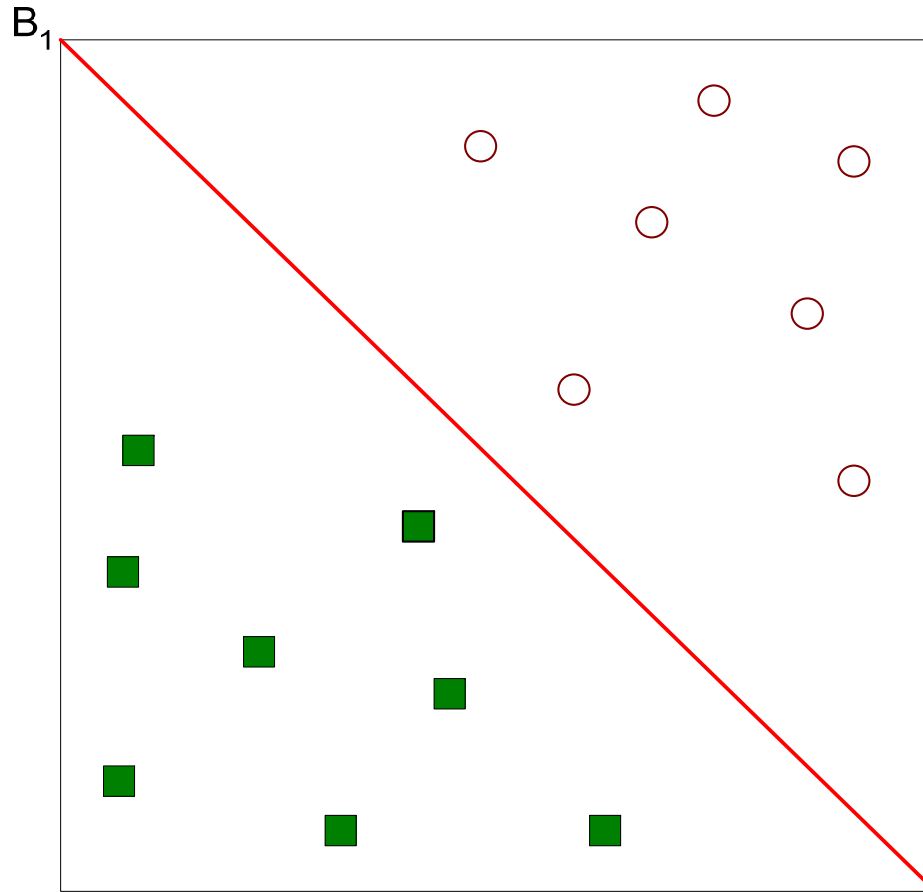
- Support vector machines

Support Vector Machines



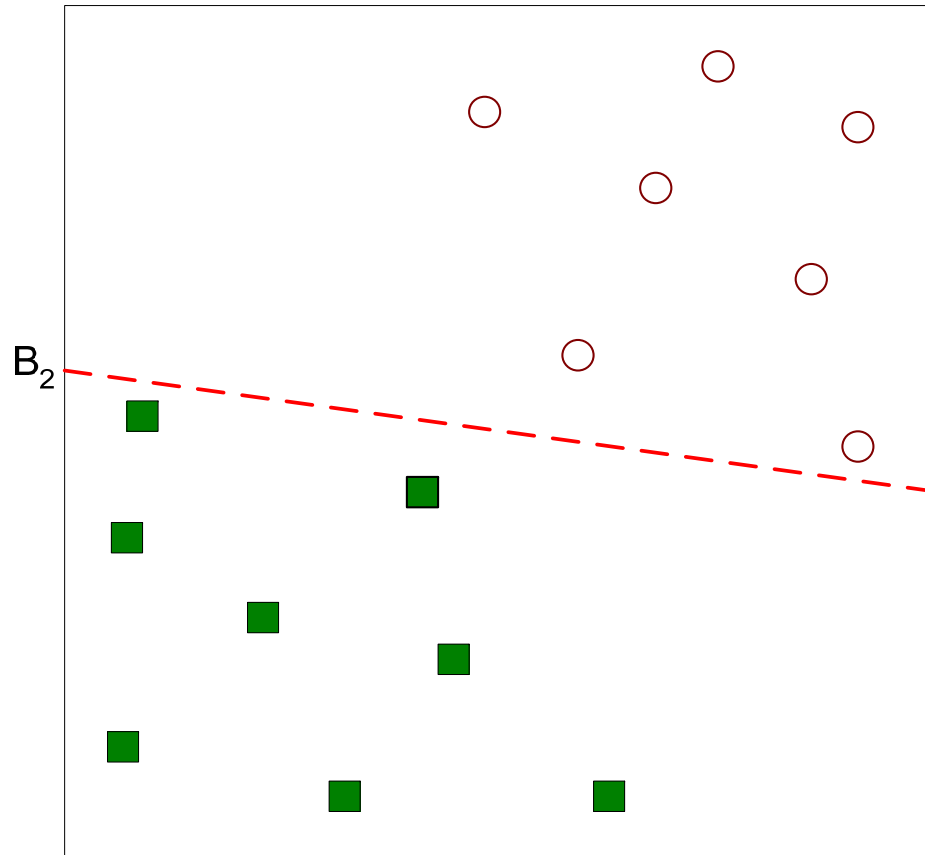
- Find a linear hyperplane (decision boundary) that will separate the data

Support Vector Machines



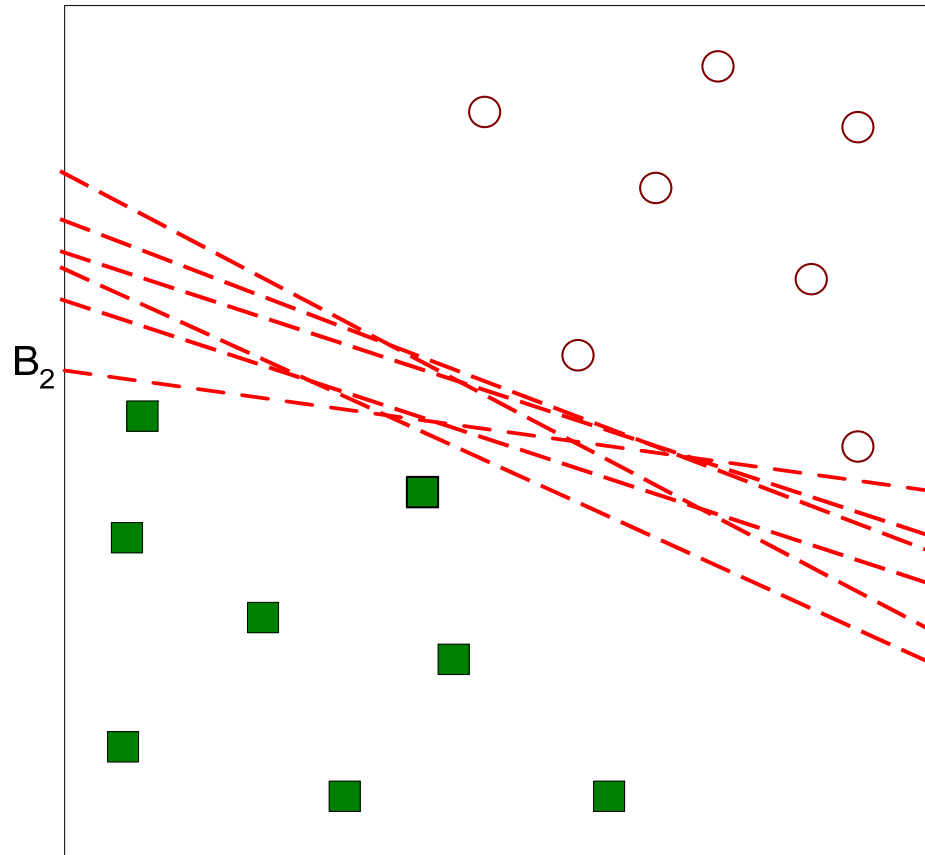
- One Possible Solution

Support Vector Machines



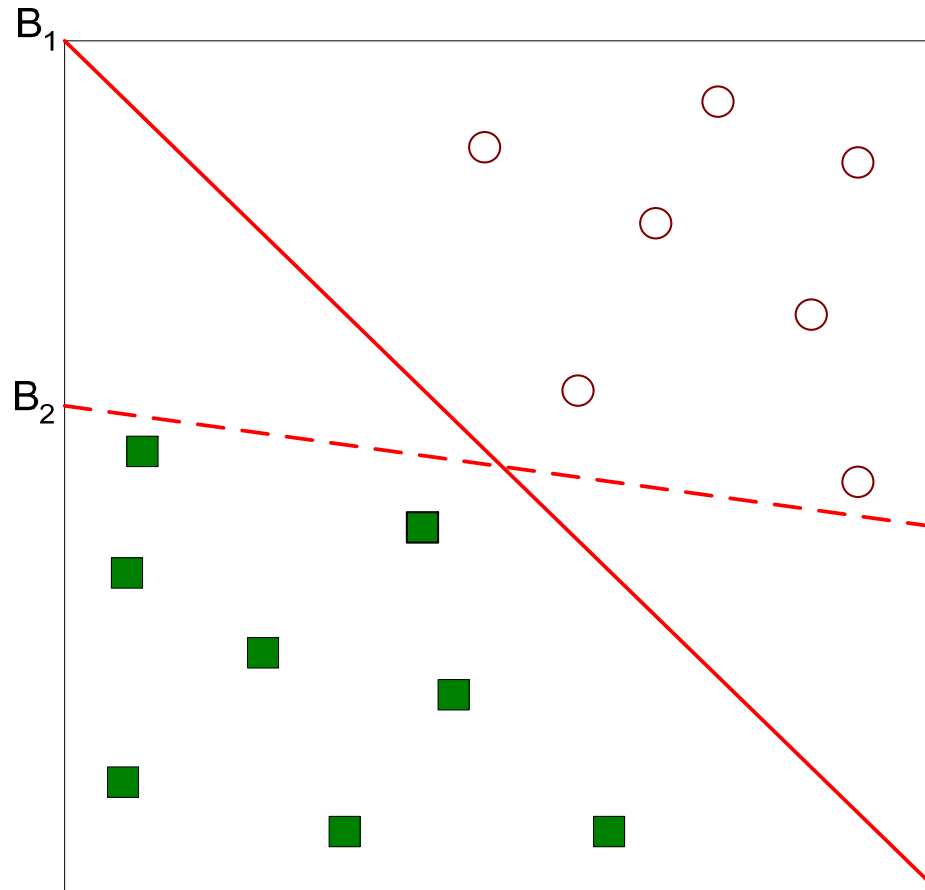
- Another possible solution

Support Vector Machines



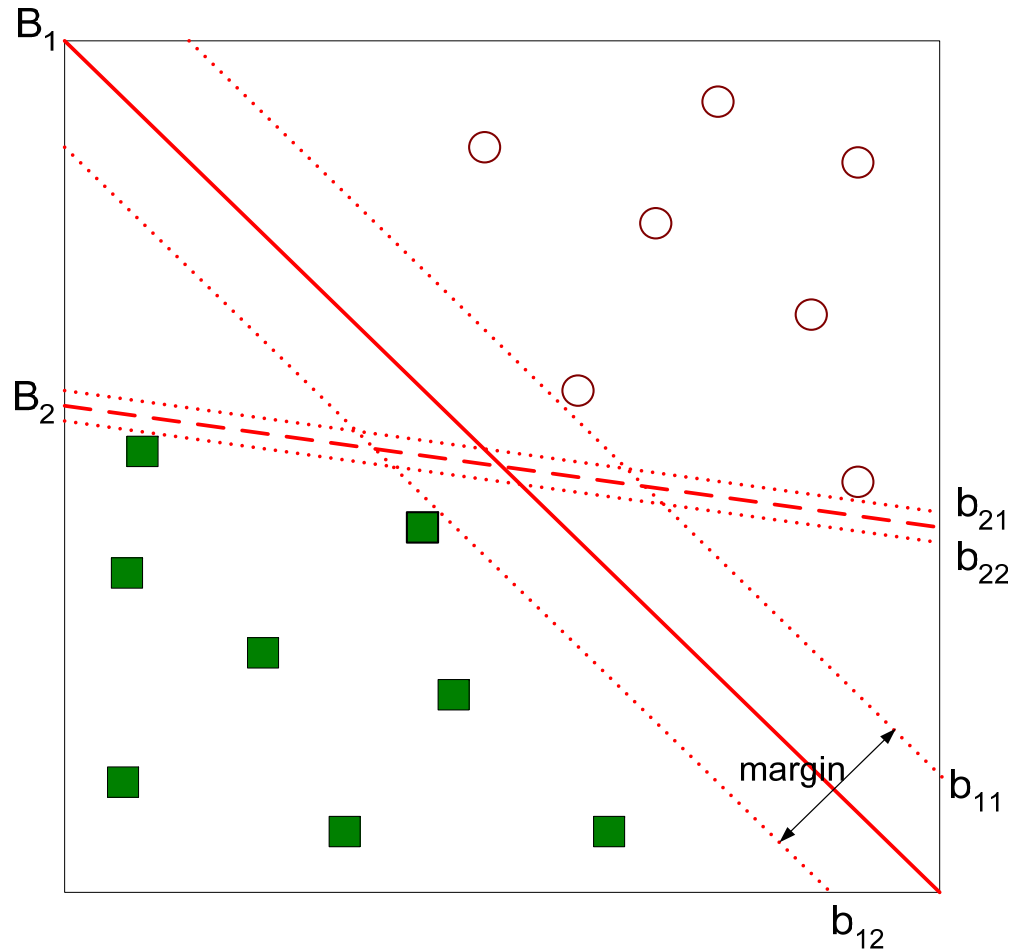
- Other possible solutions

Support Vector Machines



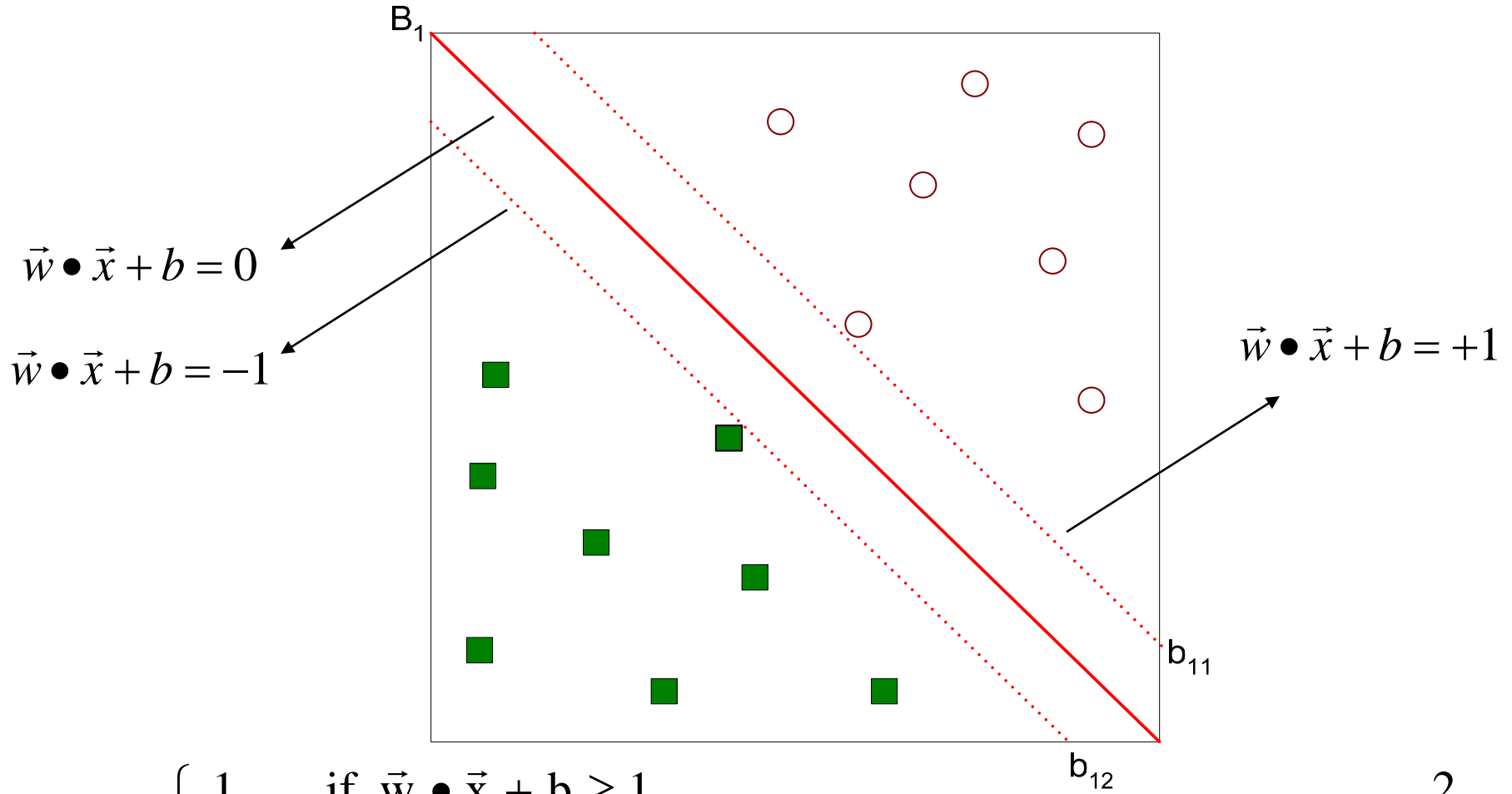
- Which one is better? B_1 or B_2 ?
- How do you define better?

Support Vector Machines



- Find hyperplane **maximizes** the margin => B1 is better than B2

Support Vector Machines



$$f(\vec{x}) = \begin{cases} 1 & \text{if } \vec{w} \bullet \vec{x} + b \geq 1 \\ -1 & \text{if } \vec{w} \bullet \vec{x} + b \leq -1 \end{cases}$$

$$\text{Margin} = \frac{2}{\|\vec{w}\|^2}$$

Support Vector Machines

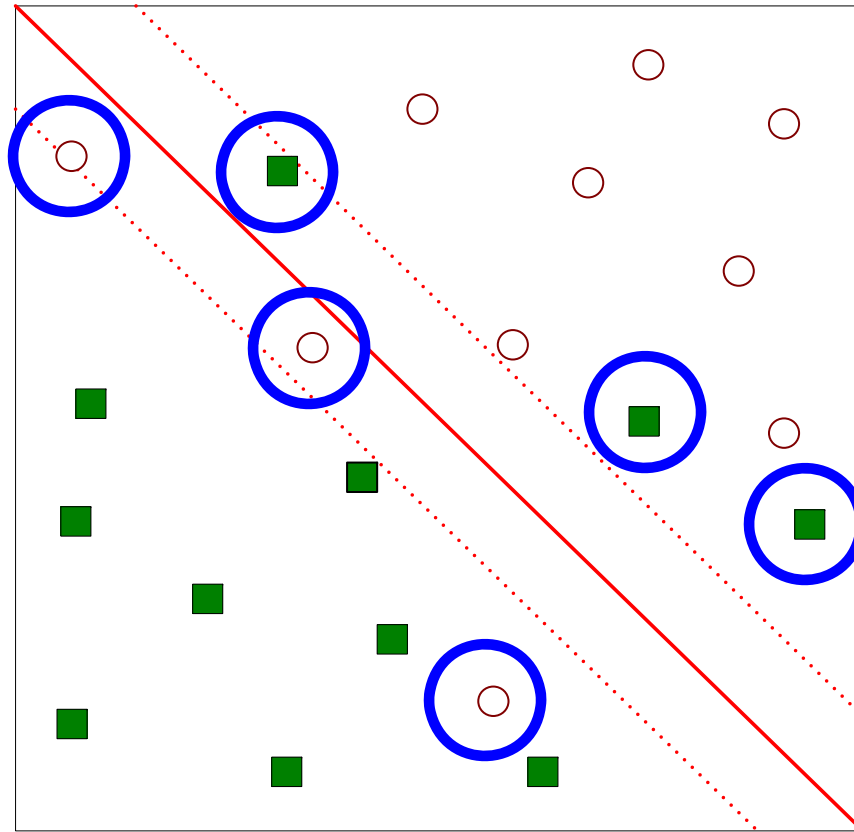
- We want to maximize: $\text{Margin} = \frac{2}{\|\vec{w}\|^2}$
 - Which is equivalent to minimizing: $L(w) = \frac{\|\vec{w}\|^2}{2}$
 - But subjected to the following constraints:

$$f(\vec{x}_i) = \begin{cases} 1 & \text{if } \vec{w} \bullet \vec{x}_i + b \geq 1 \\ -1 & \text{if } \vec{w} \bullet \vec{x}_i + b \leq -1 \end{cases}$$

- This is a constrained optimization problem
 - Numerical approaches to solve it (e.g., quadratic programming)

Support Vector Machines

- What if the problem is not linearly separable?



Support Vector Machines

- What if the problem is not linearly separable?
 - Introduce slack variables

- Need to minimize:

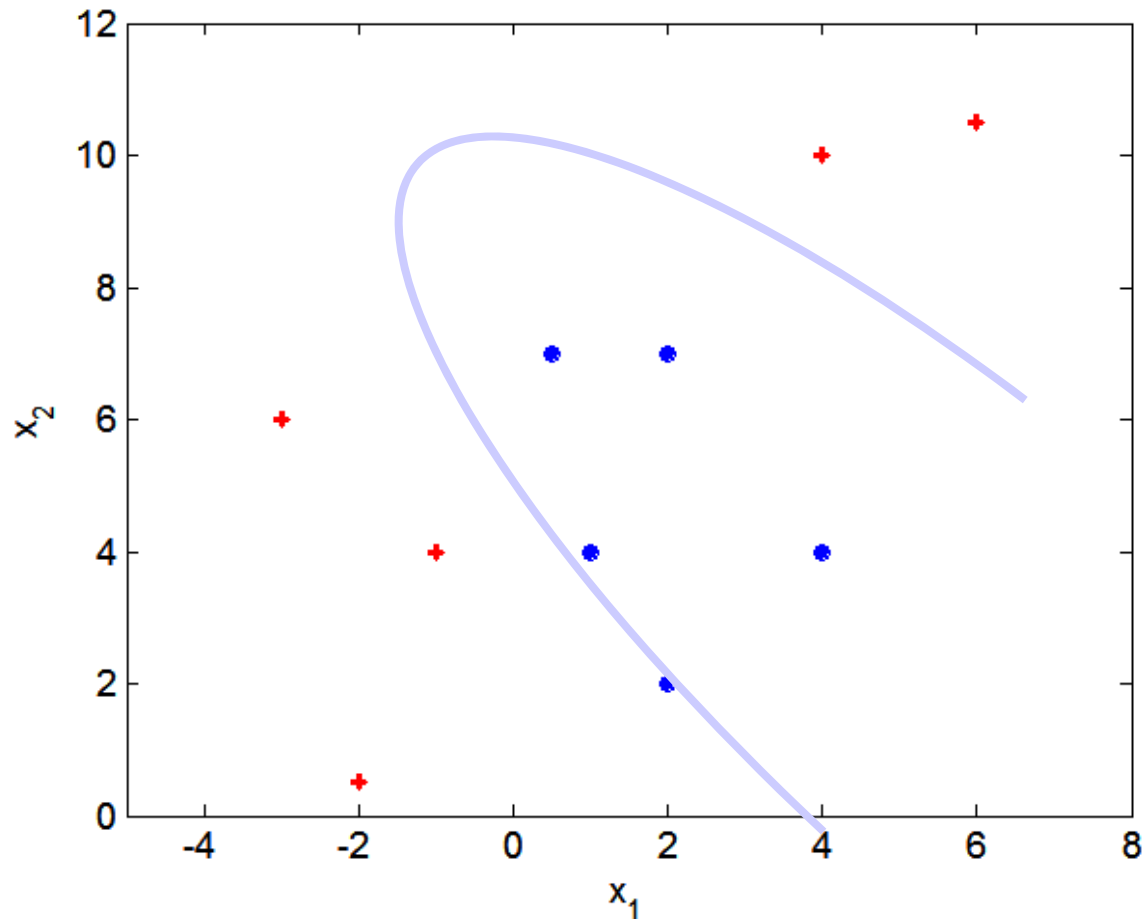
$$L(w) = \frac{\|\vec{w}\|^2}{2} + C \left(\sum_{i=1}^N \xi_i \right)$$

- Subject to:

$$f(\vec{x}_i) = \begin{cases} 1 & \text{if } \vec{w} \cdot \vec{x}_i + b \geq 1 - \xi_i \\ -1 & \text{if } \vec{w} \cdot \vec{x}_i + b \leq -1 + \xi_i \end{cases}$$

Nonlinear Support Vector Machines

- What if decision boundary is not linear?



Nonlinear Support Vector Machines

- Transform data into higher dimensional space

