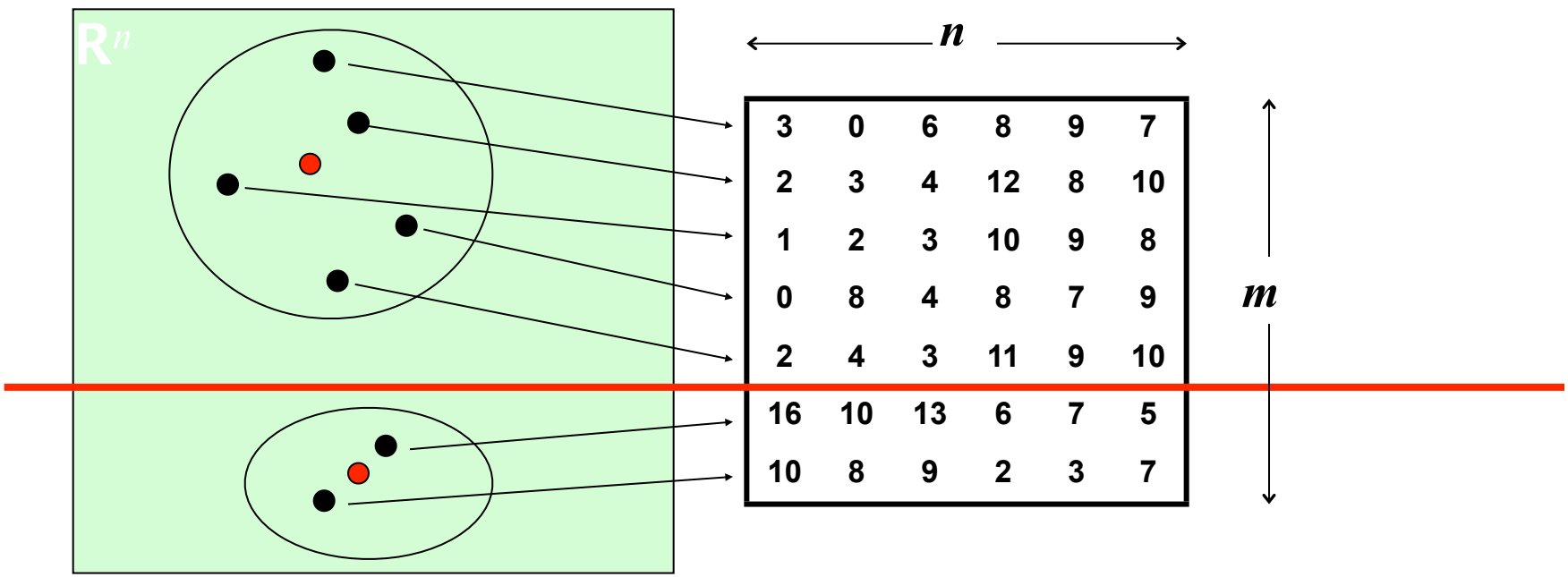
# Co-clustering or Bi-clustering

- References:
  - A. Anagnostopoulos, A. Dasgupta and R. Kumar: Approximation Algorithms for co-clustering, PODS 2008.
  - K. Puolamaki. S. Hanhijarvi and G. Garriga: An approximation ratio for biclustering, Information Processing Letters 2008.

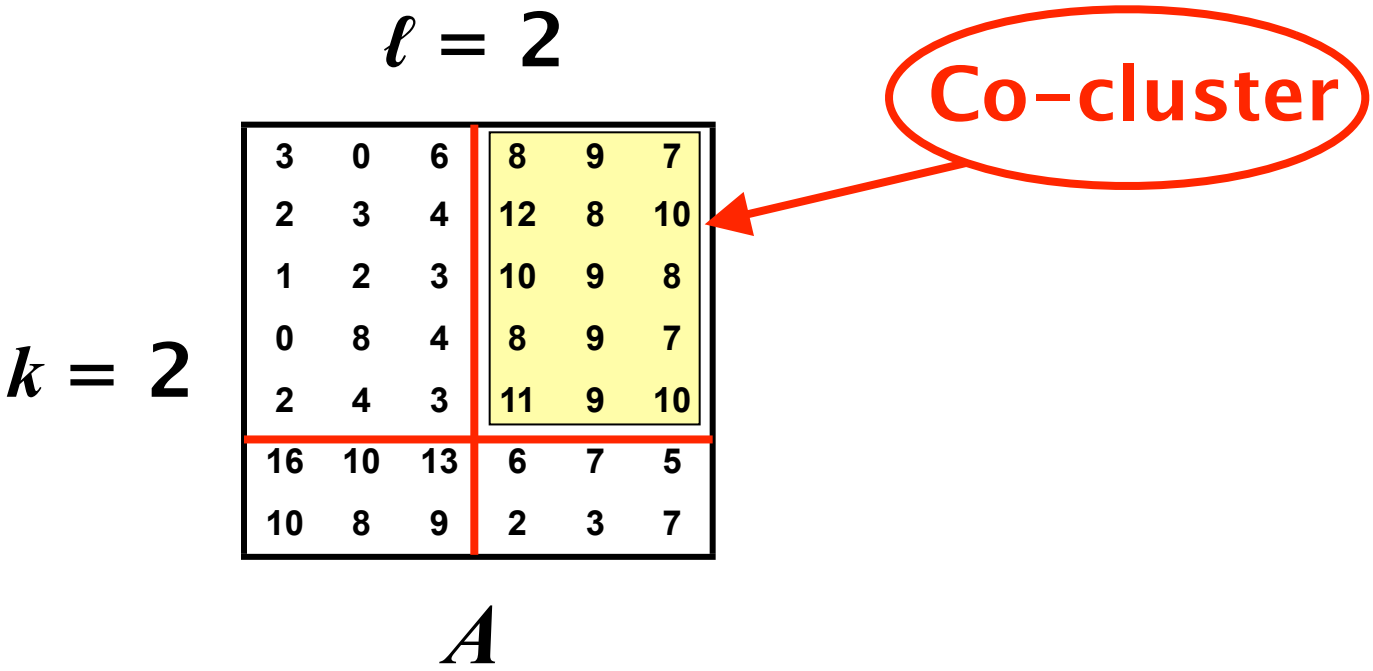
# Clustering

- $m$ points in $\mathbf{R}^n$
- Group them to $k$ clusters
- Represent them by a matrix $A \in \mathbf{R}^{m \times n}$
  - A point corresponds to a row of $A$
- **Cluster:** Partition the rows to $k$

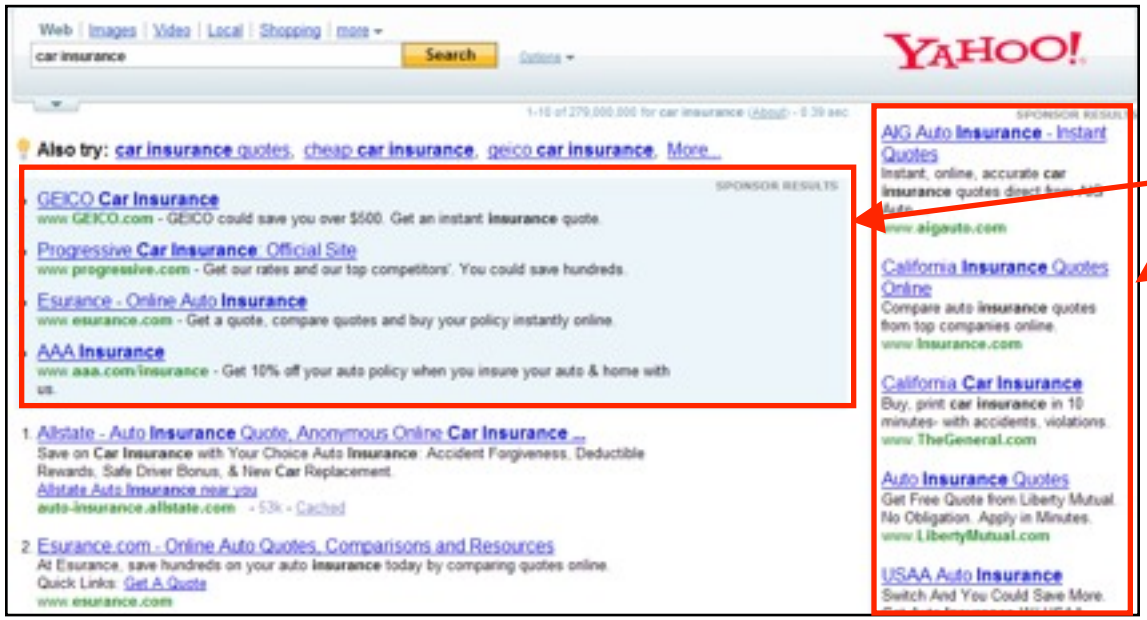


# Co-Clustering

- **Co-Clustering: Cluster rows and columns of $A$ simultaneously:**



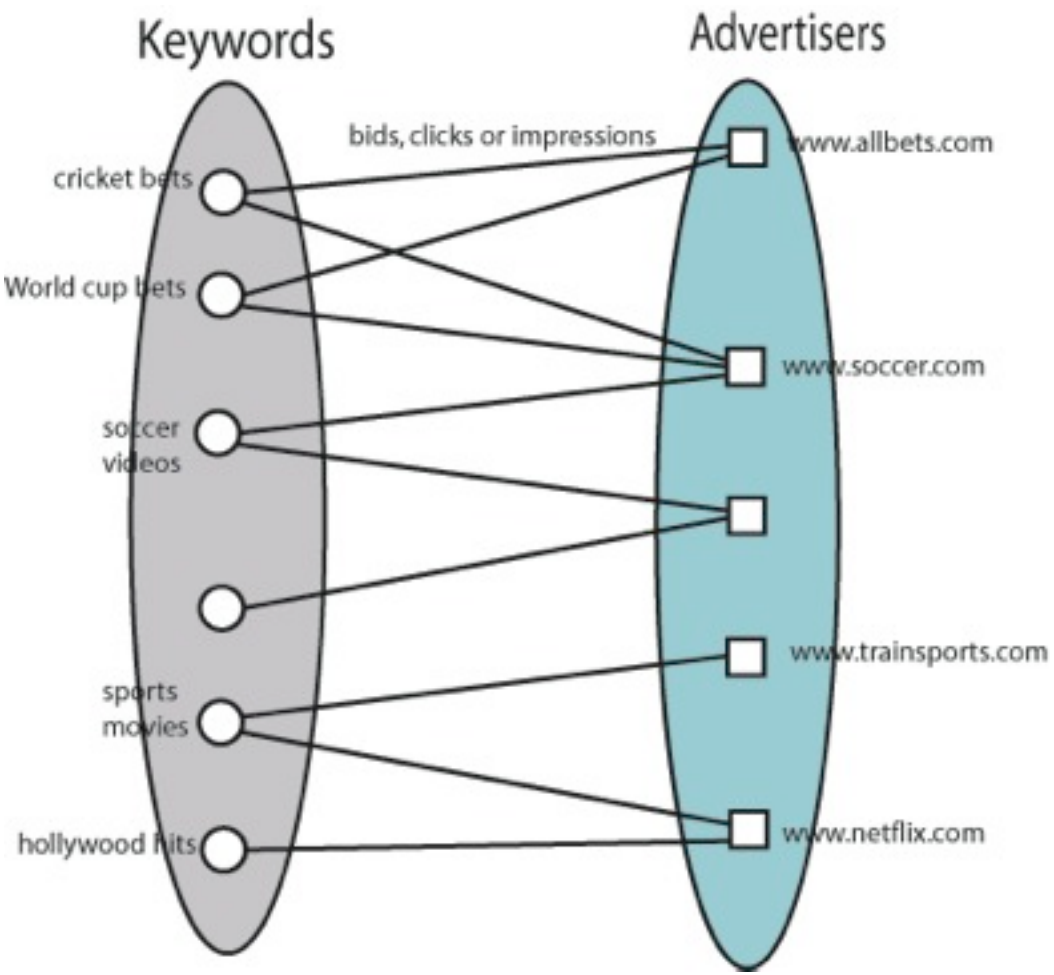
# Motivation: Sponsored Search



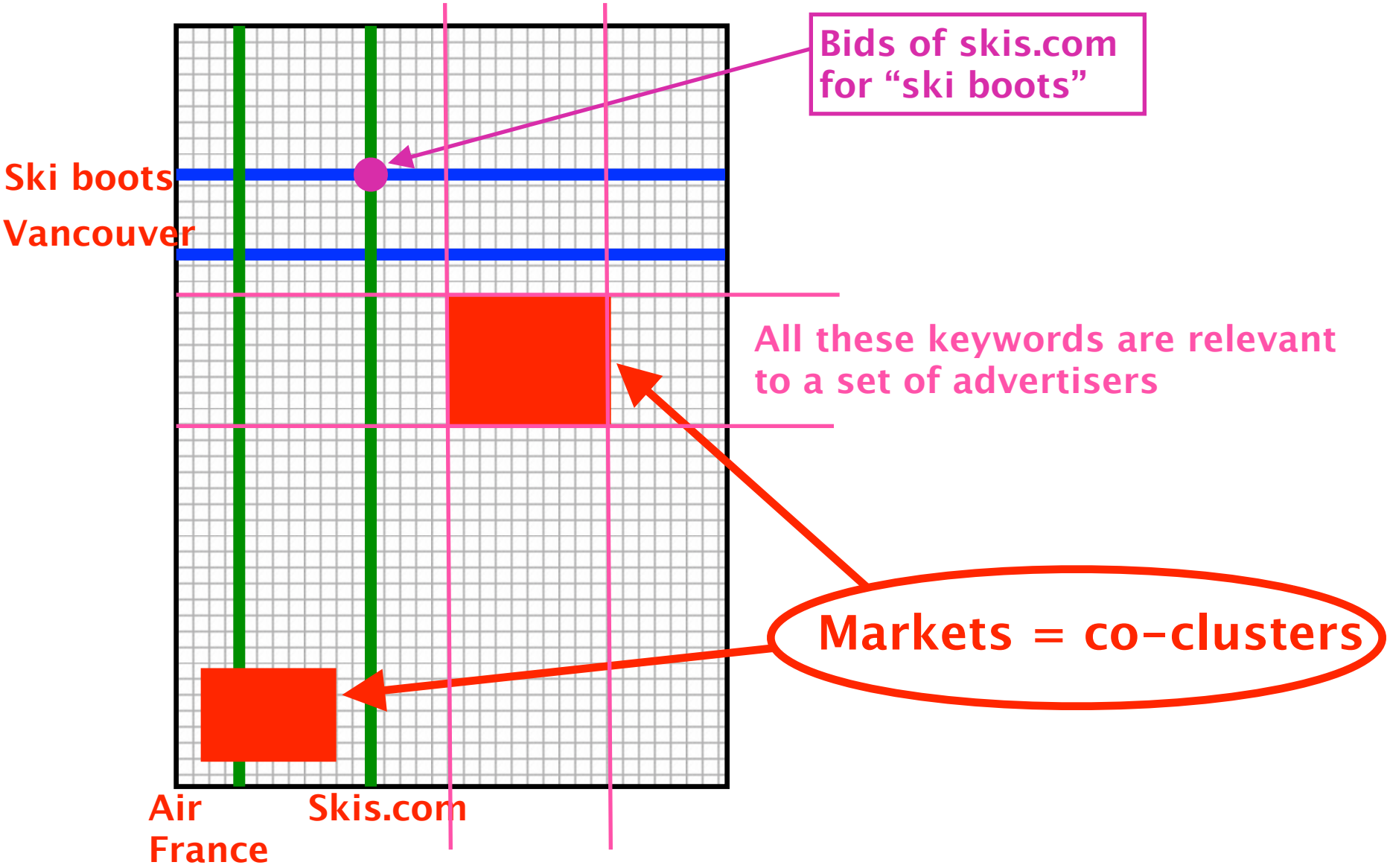

- Advertisers bid on keywords
- A user makes a query
- Show ads of advertisers that are relevant and have high bids
- User clicks or not an ad

# Motivation: Sponsored Search

- For every (advertiser, keyword) pair we have:
  - Bid amount
  - Impressions
  - # clicks
- Mine information at query time
  - Maximize # clicks / revenue



# Co-Clusters in Sponsored Search



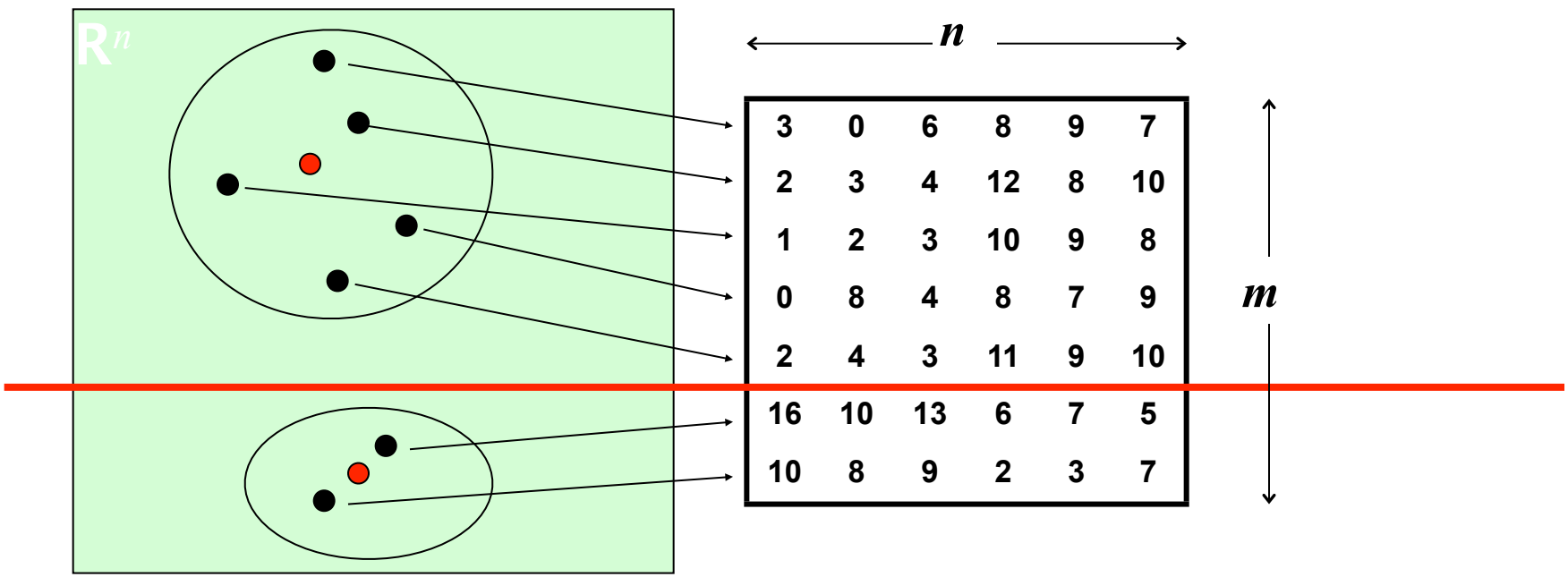
# Co-Clustering in Sponsored Search

**Applications:**

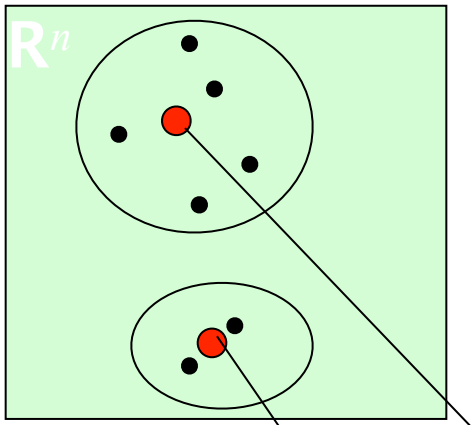
- Keyword suggestion
  - Recommend to advertisers other relevant keywords
- Broad matching / market expansion
  - Include more advertisers to a query
- Isolate submarkets
  - Important for economists
  - Apply different advertising approaches
- Build taxonomies of advertisers / keywords

# Clustering of the rows

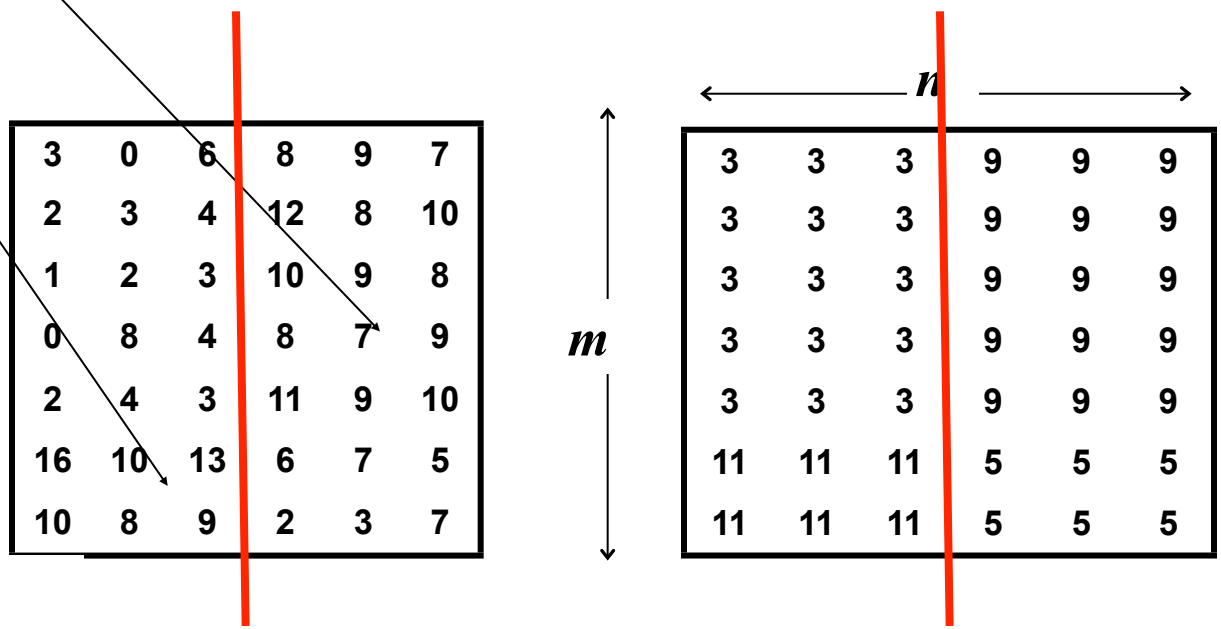
- $m$ points in $\mathbf{R}^n$
- Group them to $k$ clusters
- Represent them by a matrix $A \in \mathbf{R}^{m \times n}$
  - A point corresponds to a row of $A$
- **Clustering:** Partitioning of the rows into $k$ groups



# Clustering of the columns




- $n$ points in $\mathbf{R}^m$
- Group them to $k$ clusters
- Represent them by a matrix $A \in \mathbf{R}^{m \times n}$
  - A point corresponds to a column of $A$
- **Clustering:** Partitioning of the columns into $k$ groups



# Cost of clustering

| | | | | | |
|---|---|---|---|---|---|
| 3 | 0 | 6 | 8 | 9 | 7 |
| 2 | 3 | 4 | 12 | 8 | 10 |
| 1 | 2 | 3 | 10 | 9 | 8 |
| 0 | 8 | 4 | 8 | 7 | 9 |
| 2 | 4 | 3 | 11 | 9 | 10 |
| 16 | 10 | 13 | 6 | 7 | 5 |
| 10 | 8 | 9 | 2 | 3 | 7 |

Original data points A

| | | | | | |
|---|---|---|---|---|---|
| 1.6 | 3.4 | 4 | 9.8 | 8.4 | 8.8 |
| 1.6 | 3.4 | 4 | 9.8 | 8.4 | 8.8 |
| 1.6 | 3.4 | 4 | 9.8 | 8.4 | 8.8 |
| 1.6 | 3.4 | 4 | 9.8 | 8.4 | 8.8 |
| 1.6 | 3.4 | 4 | 9.8 | 8.4 | 8.8 |
| 13 | 9 | 11 | 4 | 5 | 6 |
| 13 | 9 | 11 | 4 | 5 | 6 |

Data representation A'

- In A' every point in A (row or column) is replaced by the corresponding representative (row or column)
- The quality of the clustering is measured by computing distances between the data in the cells of A and A'.
- **k-means clustering**:

$$\text{cost} = \sum_{i=1\ldots n} \sum_{j=1\ldots m} (A(i,j)-A'(i,j))^2$$

- **k-median clustering**:

$$\text{cost} = \sum_{i=1\ldots n} \sum_{j=1\ldots m} |A(i,j)-A'(i,j)|$$

# Co-Clustering

- **Co-Clustering:** Cluster rows and columns of $A \in \mathbf{R}^{m \times n}$ simultaneously
- $k$ row clusters, $\ell$ column clusters

- Every cell in **A** is represented by a cell in **A'**
- All cells in the same co-cluster are represented by the same value in the cells of **A'**

| | | | | | |
|---|---|---|---|---|---|
| 3 | 0 | 6 | 8 | 9 | 7 |
| 2 | 3 | 4 | 12 | 8 | 10 |
| 1 | 2 | 3 | 10 | 9 | 8 |
| 0 | 8 | 4 | 8 | 9 | 7 |
| 2 | 4 | 3 | 11 | 9 | 10 |
| 16 | 10 | 13 | 6 | 7 | 5 |
| 10 | 8 | 9 | 2 | 3 | 7 |

Original data **A**

| | | | | | |
|---|---|---|---|---|---|
| 3 | 3 | 3 | 9 | 9 | 9 |
| 3 | 3 | 3 | 9 | 9 | 9 |
| 3 | 3 | 3 | 9 | 9 | 9 |
| 3 | 3 | 3 | 9 | 9 | 9 |
| 3 | 3 | 3 | 9 | 9 | 9 |
| 11 | 11 | 11 | 5 | 5 | 5 |
| 11 | 11 | 11 | 5 | 5 | 5 |

Co-cluster representation **A'**

# Co-Clustering Objective Function

| | | | | | |
|---|---|---|---|---|---|
| 3 | 0 | 6 | 8 | 9 | 7 |
| 2 | 3 | 4 | 12 | 8 | 10 |
| 1 | 2 | 3 | 10 | 9 | 8 |
| 0 | 8 | 4 | 8 | 7 | 9 |
| 2 | 4 | 3 | 11 | 9 | 10 |
| 16 | 10 | 13 | 6 | 7 | 5 |
| 10 | 8 | 9 | 2 | 3 | 7 |

| | | | | | |
|---|---|---|---|---|---|
| 3 | 3 | 3 | 9 | 9 | 9 |
| 3 | 3 | 3 | 9 | 9 | 9 |
| 3 | 3 | 3 | 9 | 9 | 9 |
| 3 | 3 | 3 | 9 | 9 | 9 |
| 3 | 3 | 3 | 9 | 9 | 9 |
| 11 | 11 | 11 | 5 | 5 | 5 |
| 11 | 11 | 11 | 5 | 5 | 5 |

- In A' every point in A (row or column) is replaced by the corresponding representative (row or column)
- The quality of the clustering is measured by computing distances between the data in the cells of A and A'.
- **k-means Co-clustering**:

$$\text{cost} = \sum_{i=1\ldots n} \sum_{j=1\ldots m} (A(i,j)-A'(i,j))^2$$

- **k-median Co-clustering**:

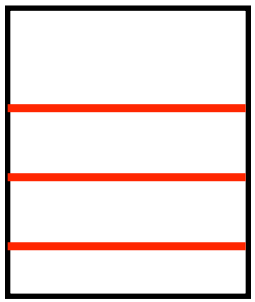
$$\text{cost} = \sum_{i=1\ldots n} \sum_{j=1\ldots m} |A(i,j)-A'(i,j)|$$

# Some Background

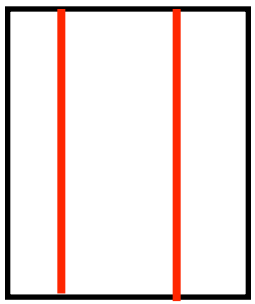
- A.k.a.: biclustering, block clustering, …
- Many objective functions in co-clustering
  - This is one of the easier
  - Others factor out row-column average (priors)
  - Others based on information theoretic ideas (e.g. KL divergence)
- A lot of existing work, but mostly heuristic
  - $k$-means style, alternate between rows/columns
  - Spectral techniques

# Algorithm

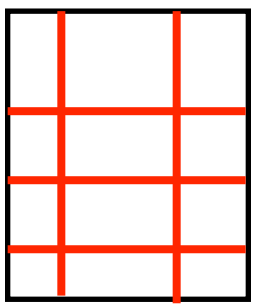
1. Cluster rows of $A$



2. Cluster columns of $A$



3. Combine



# Properties of the algorithm

**Theorem 1.** Algorithm with optimal row/column clusterings is 3-approximation to co-clustering optimum.

**Theorem 2.** For $L_2$ distance function, the algorithm with optimal row/column clusterings is a 2-approximation.

# Algorithm--details

- Clustering of the **n** rows of **A** assigns every row to a cluster with cluster name **{1,…,k}**
  - **R(i)= $r_i$** with **$1 \le r_i \le k$**
- Clustering of the **m** columns of **A** assigns every column to a cluster with cluster name **{1,…,$\ell$}**
  - **C(j)=$c_j$** with **$1 \le c_j \le \ell$**
- **A'(i,j) = {$r_i$,$c_j$}**
- **(i,j)** is in the same co-cluster as **(i',j')** if **A'(i,j)=A'(i',j')**