Link Analysis Ranking

How do search engines decide how to rank your query results?

- Guess why Google ranks the query results the way it does
- How would you do it?

Naïve ranking of query results

- Given query **q**
- Rank the web pages **p** in the index based on **sim(p,q)**

- **Scenarios where this is not such a good idea?**

Why Link Analysis?

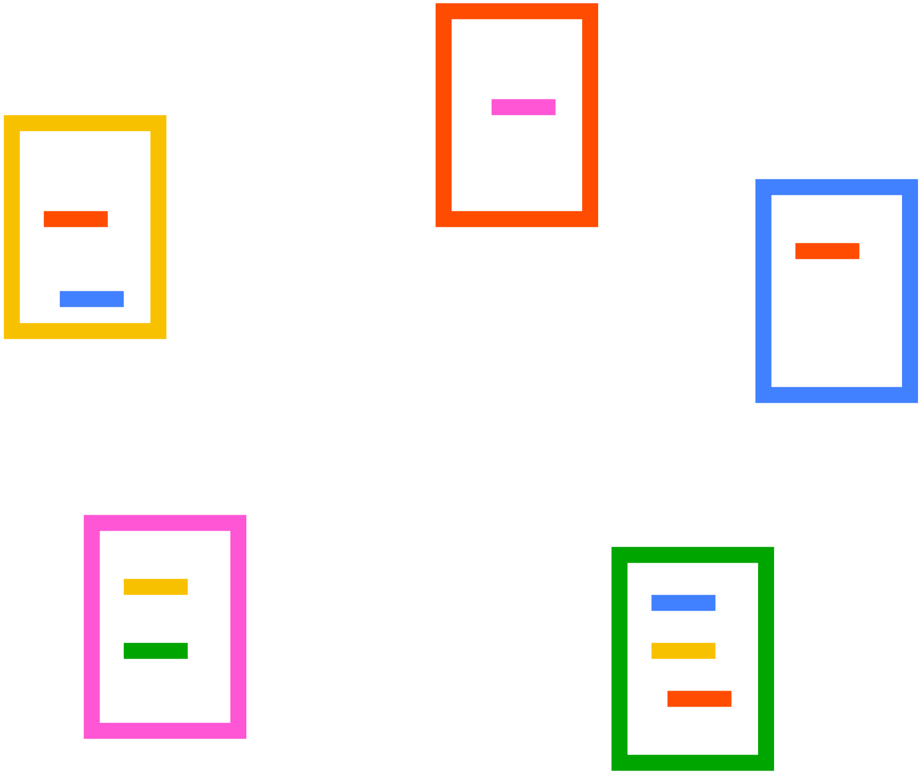
- First generation search engines
  - view documents as flat text files
  - could not cope with size, spamming, user needs
    - Example: Honda website, keywords: automobile manufacturer
- Second generation search engines
  - Ranking becomes critical
  - use of Web specific data: Link Analysis
  - shift from relevance to authoritativeness
  - a success story for the network analysis

Link Analysis: Intuition

- A link from page p to page q denotes endorsement
  - page p considers page q an authority on a subject
  - mine the web graph of recommendations
  - assign an authority value to every page

Link Analysis Ranking Algorithms

- Start with a collection of web pages
- Extract the underlying hyperlink graph
- Run the LAR algorithm on the graph
- Output: an authority weight for each node



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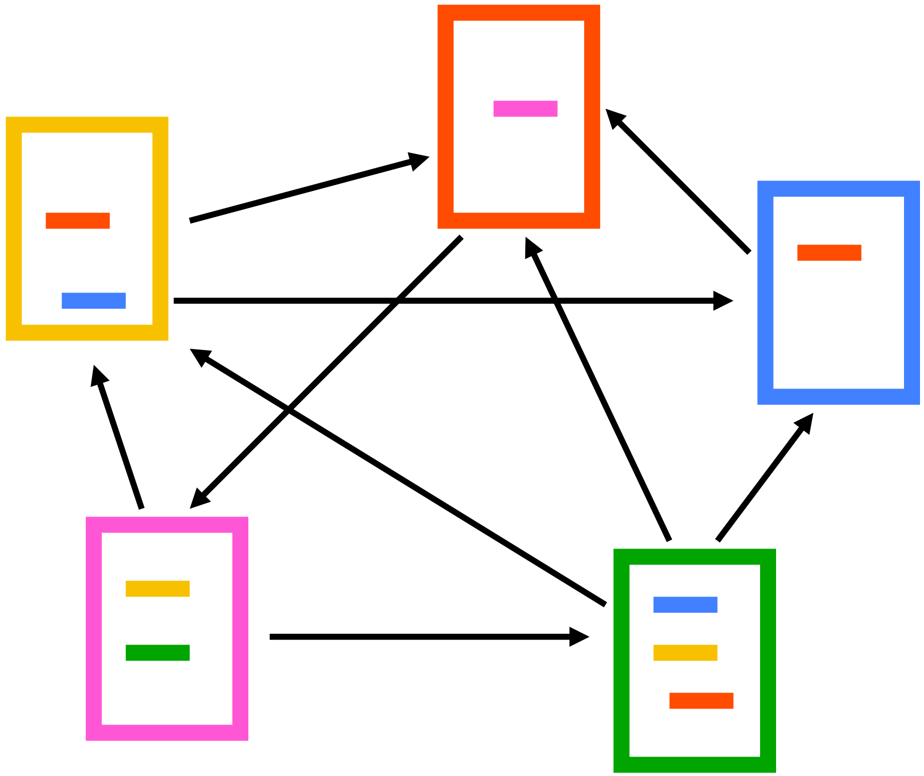

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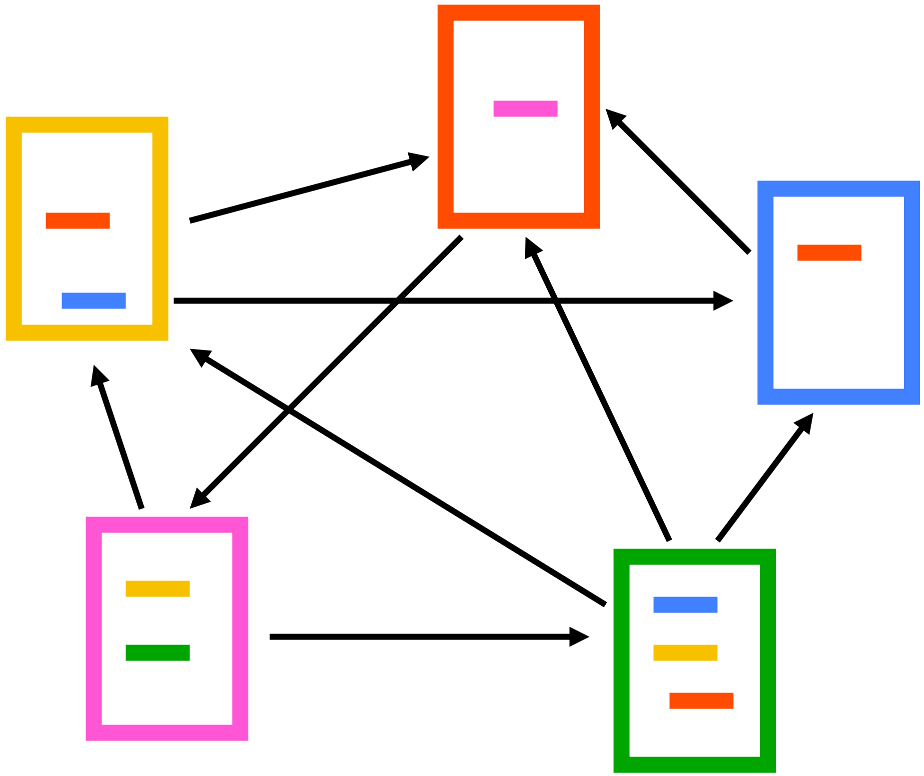

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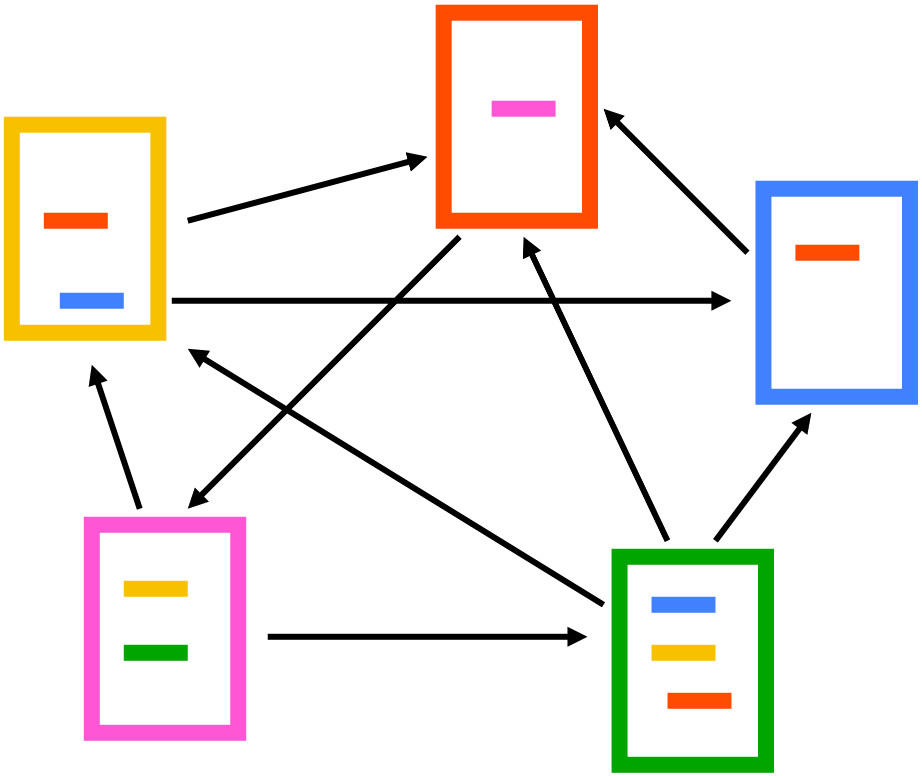

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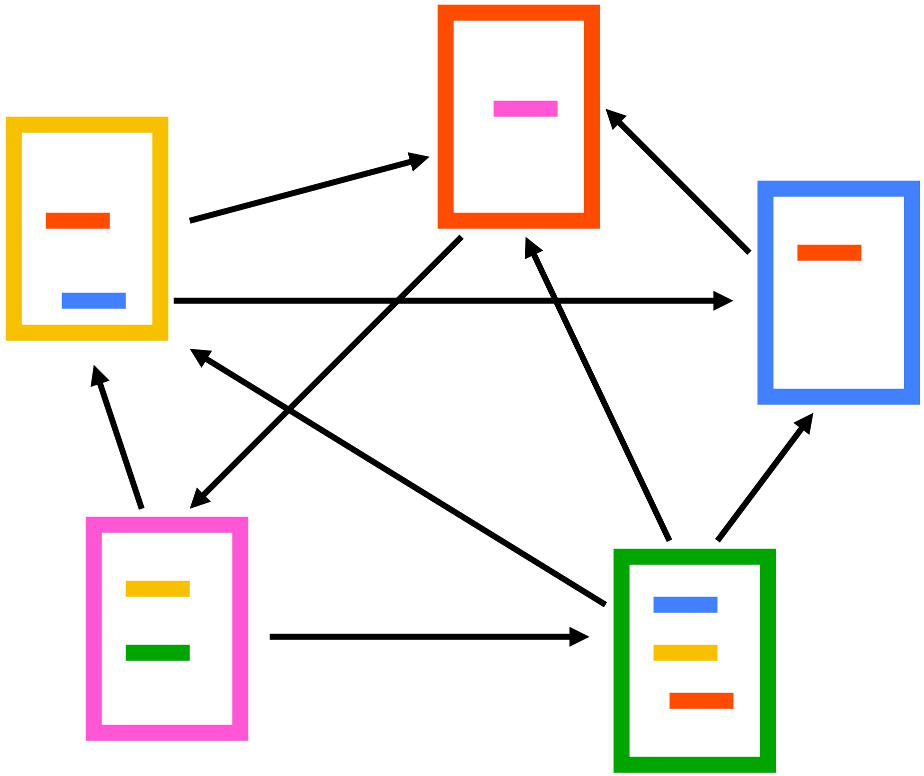

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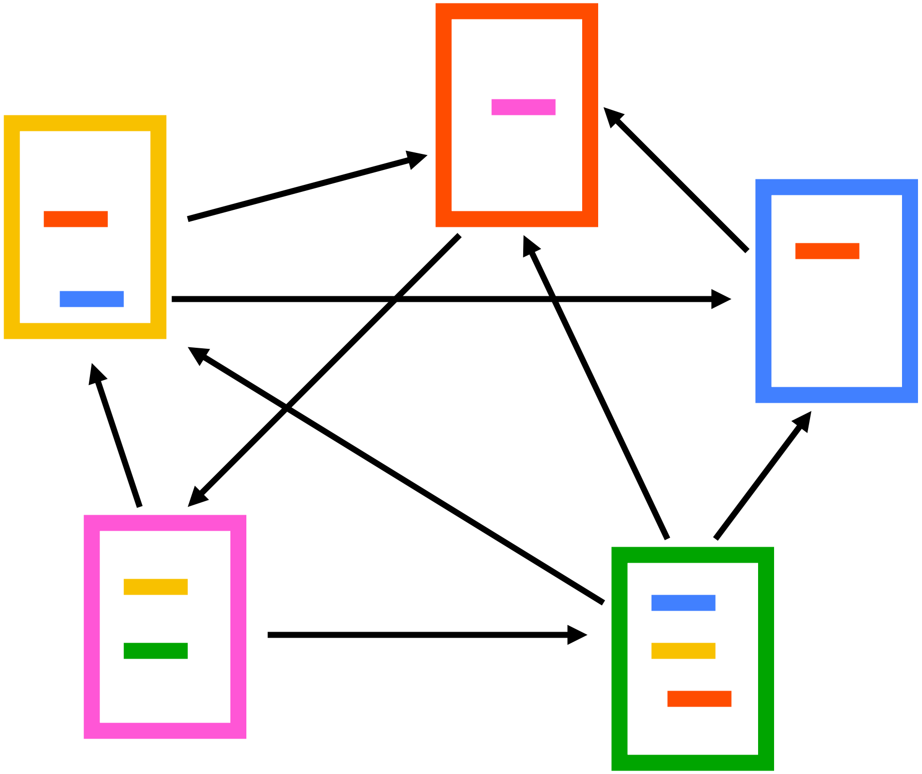

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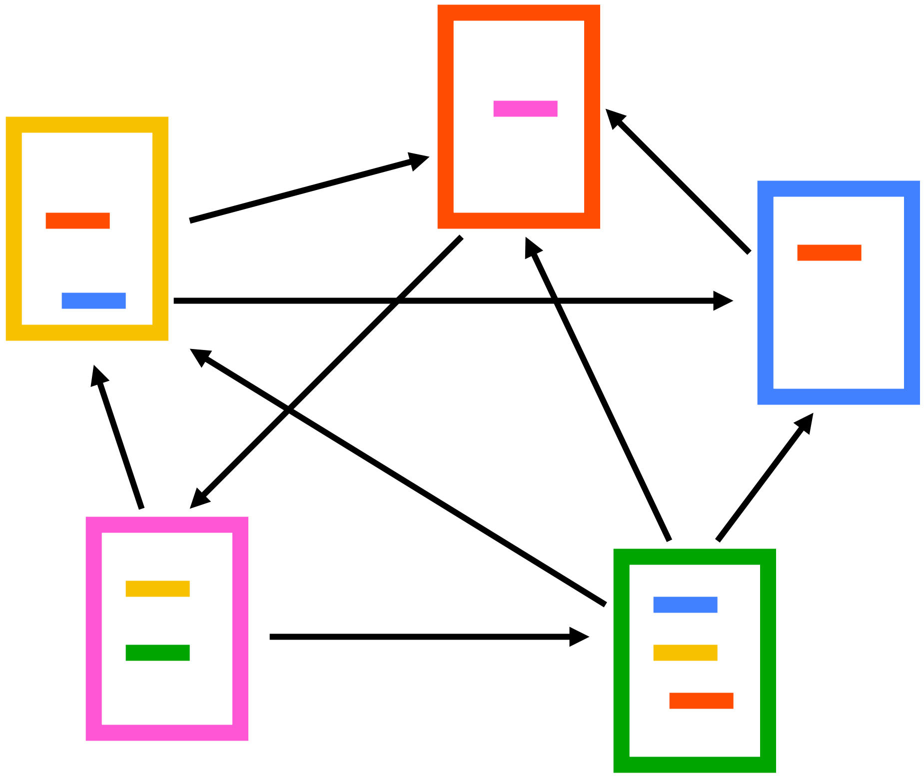

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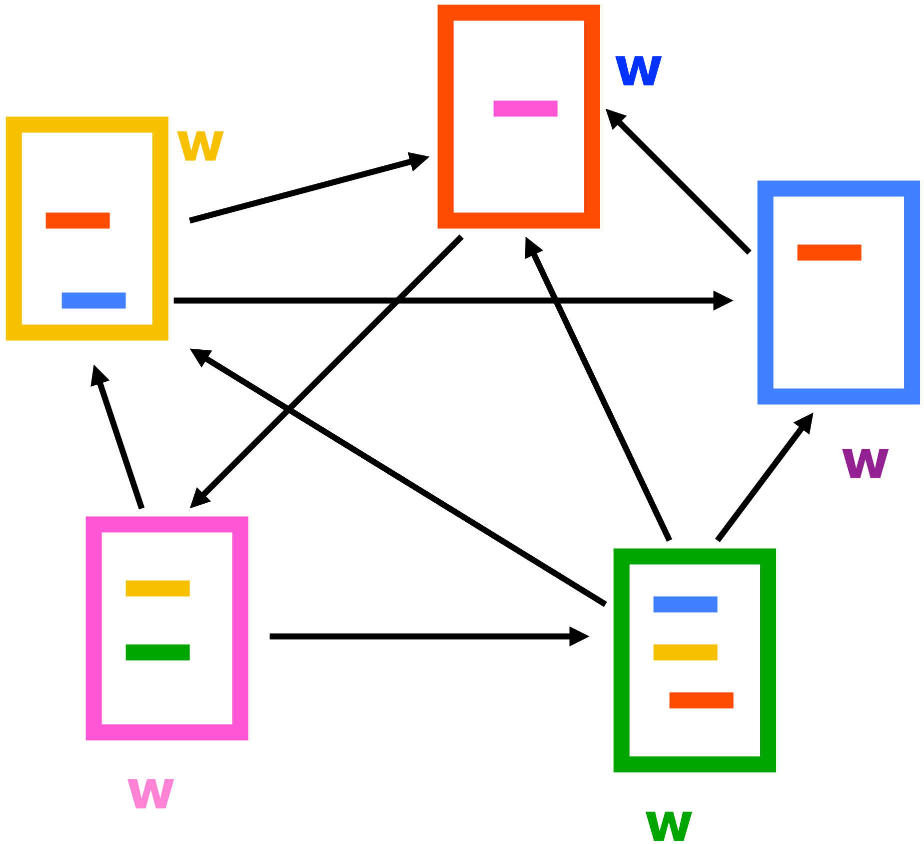


Algorithm input

- **Query dependent**: rank a small subset of pages related to a specific query
  - HITS (Kleinberg 98) was proposed as query dependent

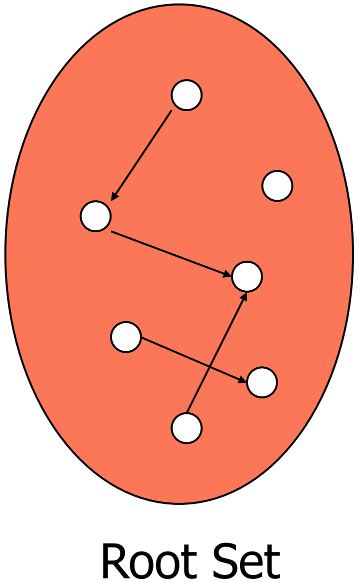
- **Query independent**: rank the whole Web
  - PageRank (Brin and Page 98) was proposed as query independent

Query-dependent LAR

- Given a query **q**, find a subset of web pages **S**

  that are related to **S**
- Rank the pages in **S** based on some ranking criterion

Query-dependent input



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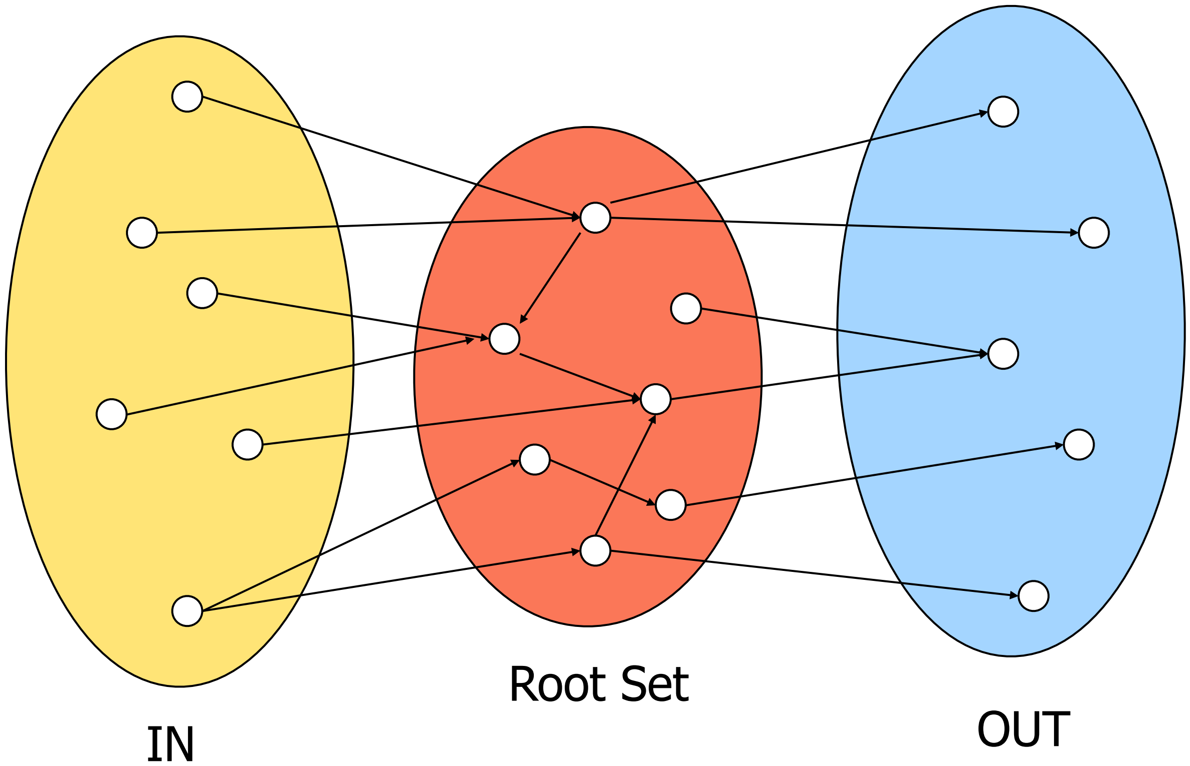

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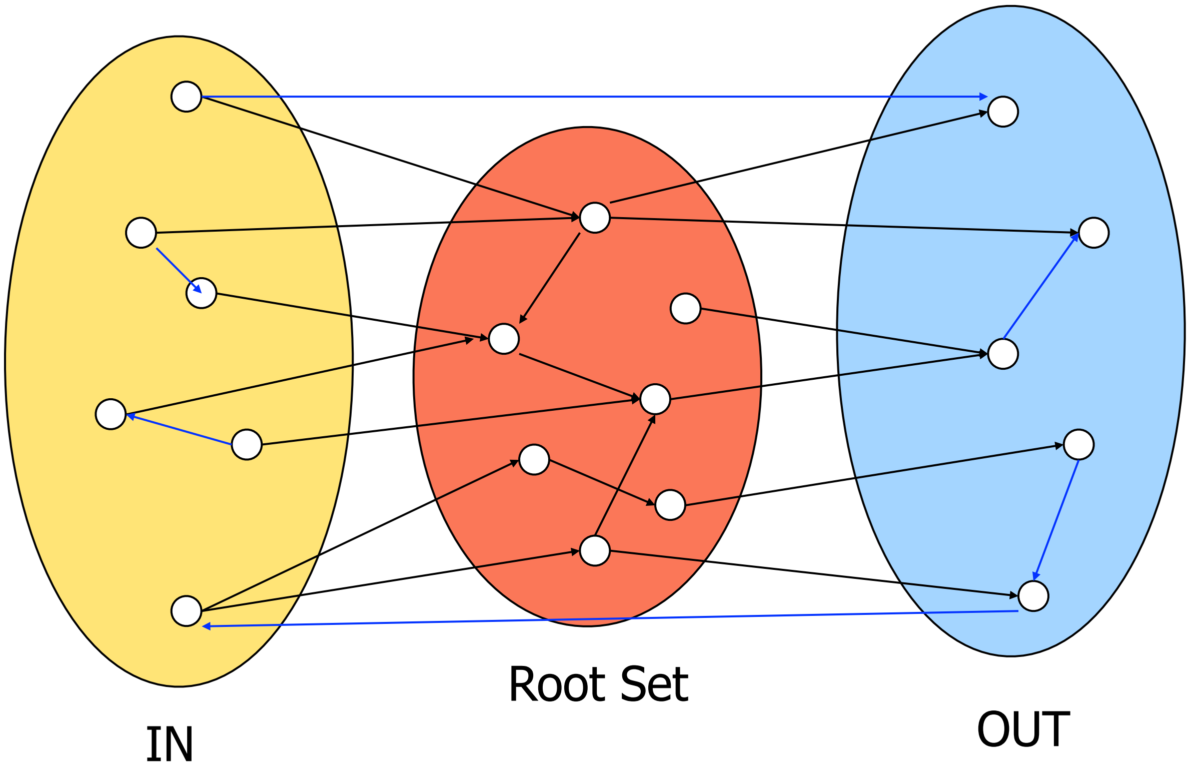

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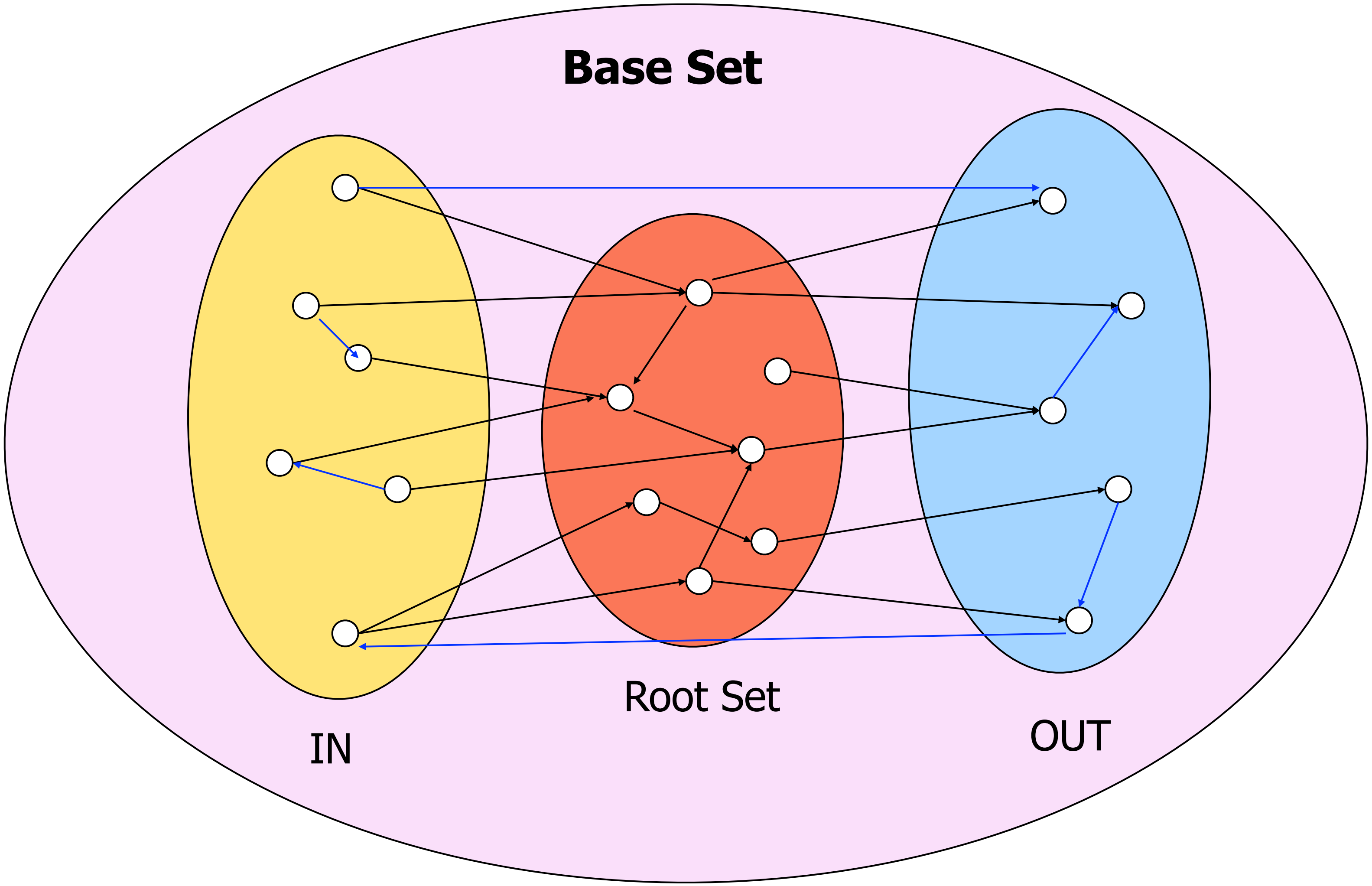


Properties of a good seed set S

- S is relatively small.
- S is rich in relevant pages.
- S contains most (or many) of the strongest authorities.

How to construct a good seed set **S**

- For query **q** first collect the **t** highest-ranked pages for **q** from a text-based search engine to form set **Γ**
- **S = Γ**
- Add to **S** all the pages pointing to **Γ**
- Add to **S** all the pages that pages from **Γ** point to

Link Filtering

- Navigational links: serve the purpose of moving within a site (or to related sites)
    - **www.espn.com → www.espn.com/nba**
    - **www.yahoo.com → www.yahoo.it**
    - **www.espn.com → www.msn.com**
- Filter out navigational links
    - – same domain name
    - – same IP address

How do we rank the pages in seed set **S?**

- In degree?
- Intuition
- Problems

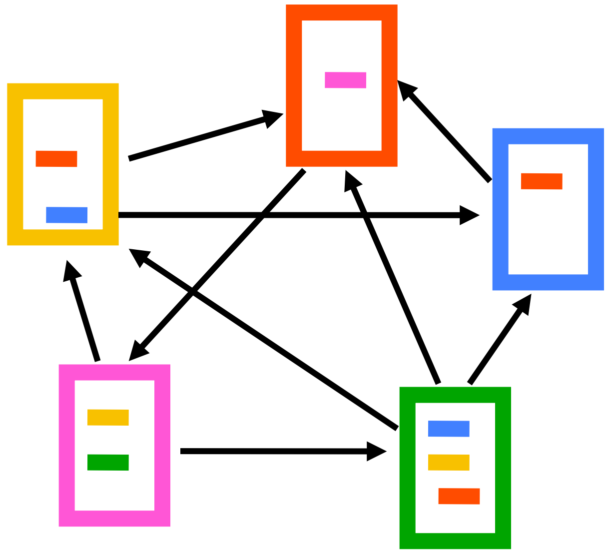
Hubs and Authorities [K98]

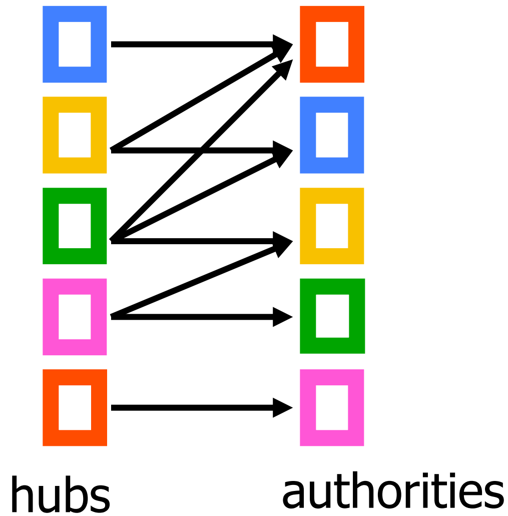
- Authority is not necessarily transferred directly between authorities

- Pages have double identity
  - hub identity
  - authority identity

- Good hubs point to good authorities

- Good authorities are pointed by good hubs





HITS Algorithm

- Initialize all weights to 1.
- Repeat until convergence
  - **O** operation : hubs collect the weight of the authorities

$$h_i = \sum_{j:i \to j} a_j$$

  - **I** operation: authorities collect the weight of the hubs

$$a_i = \sum_{j:j \to i} h_j$$

  - Normalize weights under some norm

HITS and eigenvectors

- The HITS algorithm is a power-method eigenvector computation
  - in vector terms $a^t = A^T h^{t-1}$ and $h^t = A a^{t-1}$
  - so $a^t = A^T A a^{t-1}$ and $h^t = A A^T h^{t-1}$
  - The authority weight vector a is the eigenvector of $A^T A$ and the hub weight vector h is the eigenvector of $A A^T$
  - Why do we need normalization?
- The vectors a and h are singular vectors of the matrix A

Singular Value Decomposition

$$A = U\Sigma V^T$$

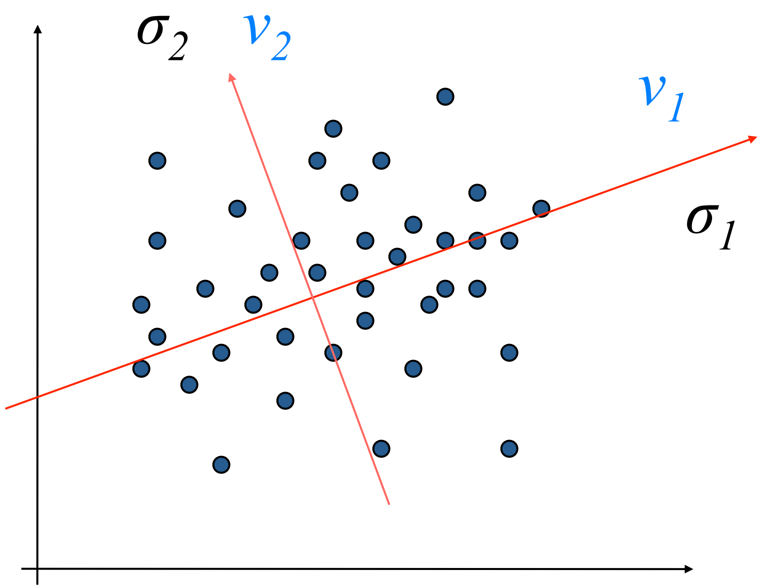
$$U = [\vec{u}_1, \ldots \vec{u}_r] \qquad V = [\vec{v}_1 \vec{v}_2 \ldots \vec{v}_r]$$

$$\Sigma = \mathrm{diag}(\sigma_1, \ldots, \sigma_r)$$

- **r** : rank of matrix A
- $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_r$: singular values (sq. roots of eig-vals AA^T, A^TA)
- $\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_r$: left singular vectors (eig-vectors of AA^T)
- $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_r$:right singular vectors (eig-vectors of A^TA)
- $$A = \sigma_1 \vec{u}_1 \vec{v}_1^T + \sigma_2 \vec{u}_2 \vec{v}_2^T + \ldots + \sigma_r \vec{u}_r \vec{v}_r^T$$

Singular Value Decomposition

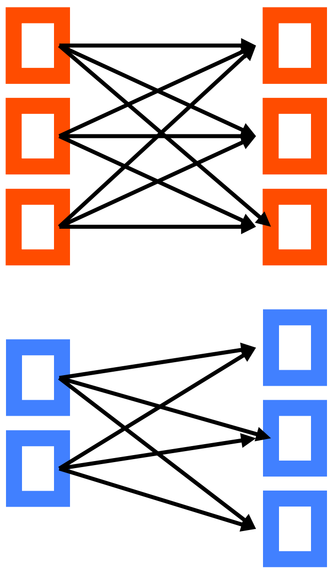
- Linear trend **v** in matrix A:
  - the tendency of the row vectors of A to align with vector **v**
  - strength of the linear trend: A**v**
- SVD discovers the linear trends in the data
- **u_i , v_i** : the i-th strongest linear trends
- σ_i : the strength of the i-th strongest linear trend



- HITS discovers the strongest linear trend in the authority space

HITS and the TKC effect

- The HITS algorithm favors the most dense community of hubs and authorities
  - Tightly Knit Community (TKC) effect



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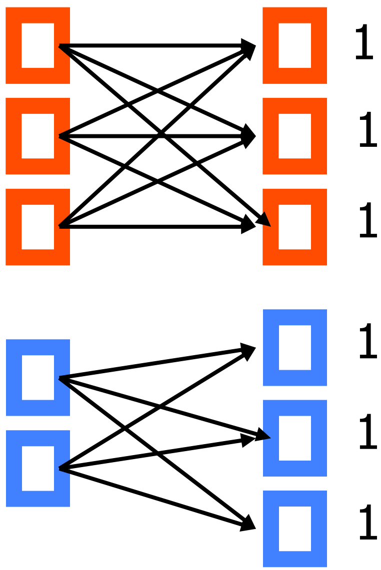

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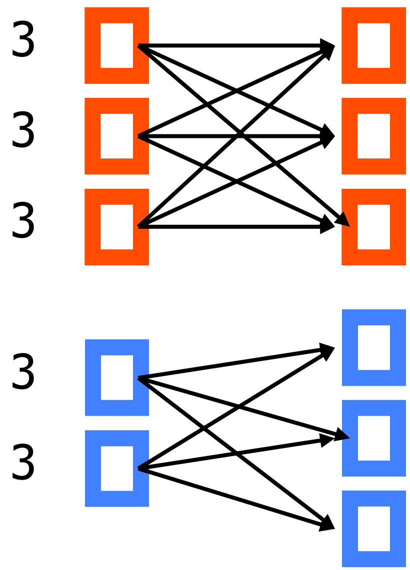

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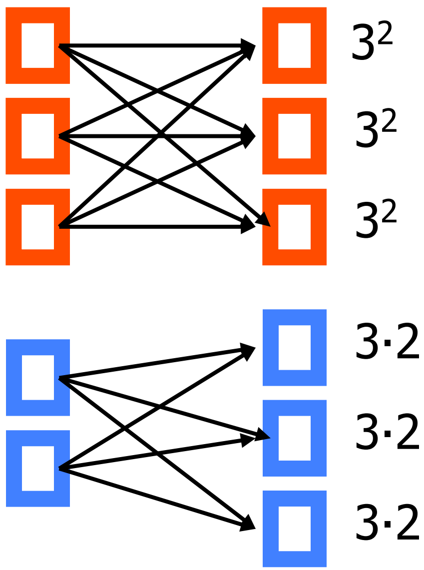

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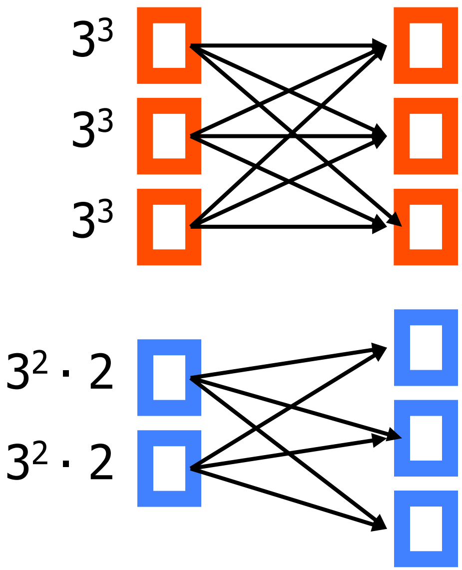

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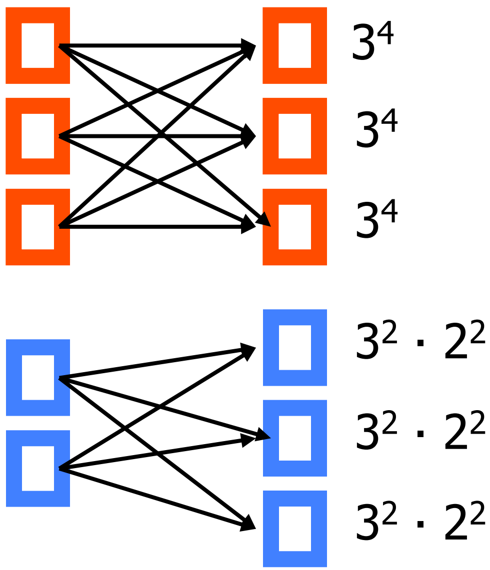

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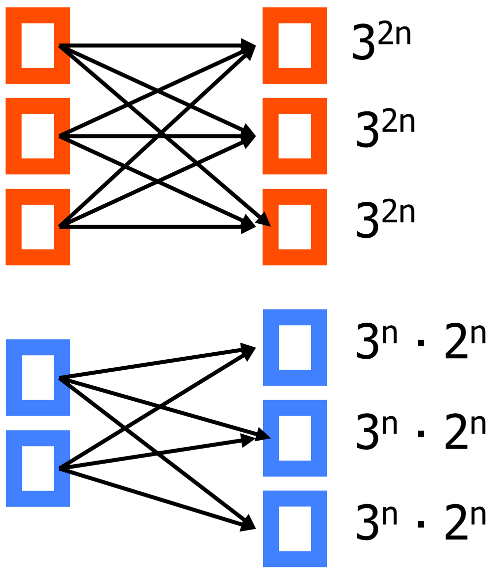
weight of node p is proportional to the number of $(BF)^n$ paths that leave node p



after n iterations

HITS and the TKC effect

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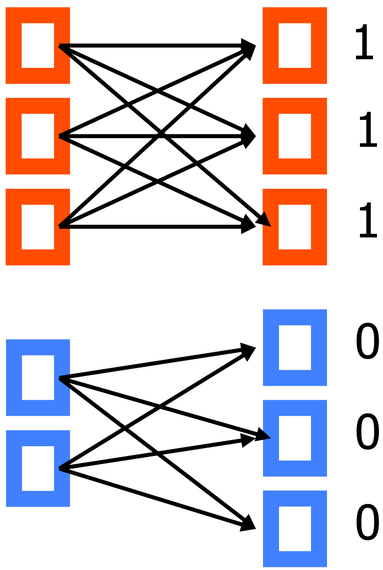


after normalization with the max element as $n \rightarrow \infty$

Query-independent LAR

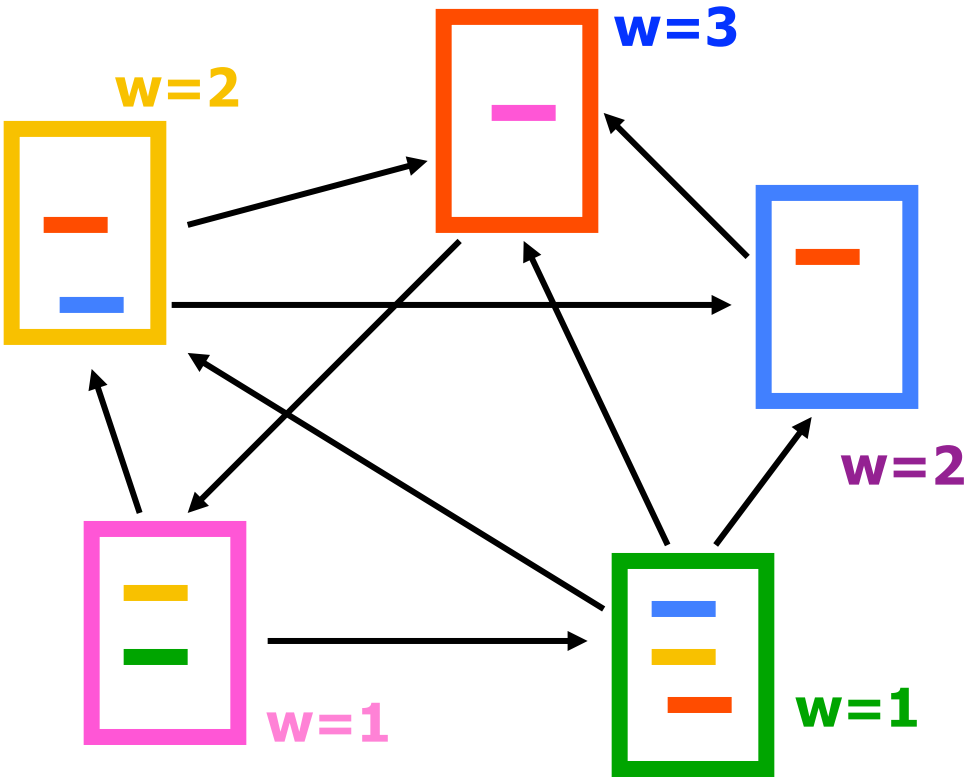
- Have an a-priori ordering of the web pages

- **Q**: Set of pages that contain the keywords in the query **q**
- Present the pages in Q ordered according to order π

- **What are the advantages of such an approach?**

InDegree algorithm

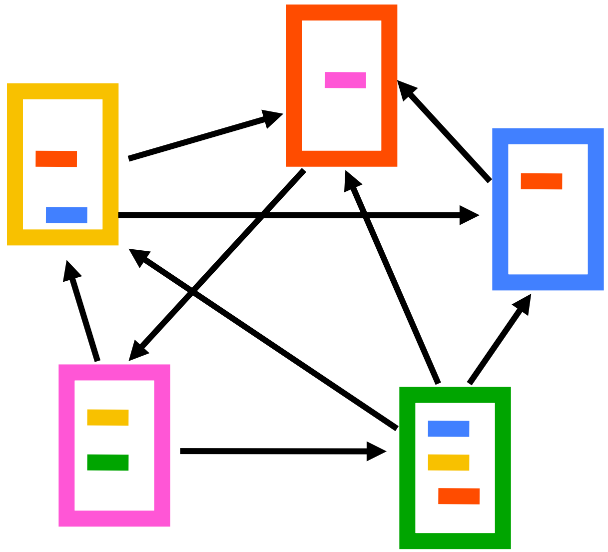
- Rank pages according to in-degree
  - $w_i = |B(i)|$



1. Red Page
2. Yellow Page
3. Blue Page
4. Purple Page
5. Green Page

PageRank algorithm [BP98]

- Good authorities should be pointed by good authorities
- Random walk on the web graph
  - pick a page at random
  - with probability 1- α jump to a random page
  - with probability α follow a random outgoing link
- Rank according to the stationary distribution
- $$\mathrm{PR}(p) = \alpha \sum_{q \to p} \frac{\mathrm{PR}(q)}{|F(q)|} + (1-\alpha)\frac{1}{n}$$



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Markov chains

- A Markov chain describes a discrete time stochastic process over a set of states

$$S = \{s_1, s_2, \ldots s_n\}$$

according to a transition probability matrix

$$P = \{P_{ij}\}$$

  - P_{ij} = probability of moving to state j when at state i
    - $\sum_j P_{ij} = 1$ (stochastic matrix)

- Memorylessness property: The next state of the chain depends only at the current state and not on the past of the process (first order MC)
  - higher order MCs are also possible

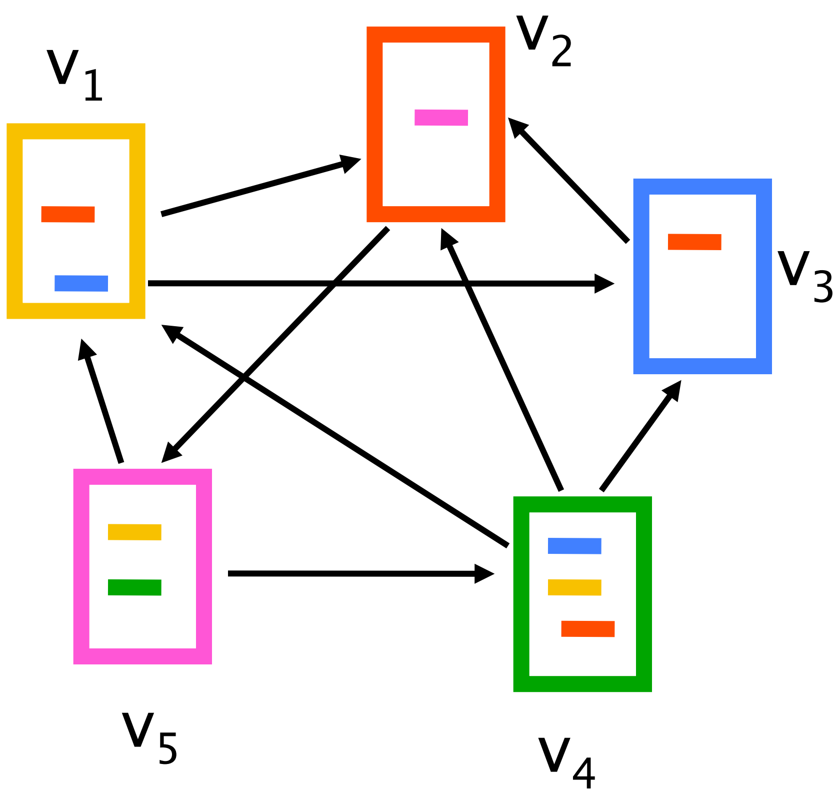
Random walks

- Random walks on graphs correspond to Markov Chains
  - The set of states S is the set of nodes of the graph G
  - The transition probability matrix is the probability that we follow an edge from one node to another

An example

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 1/2 \end{bmatrix}$$



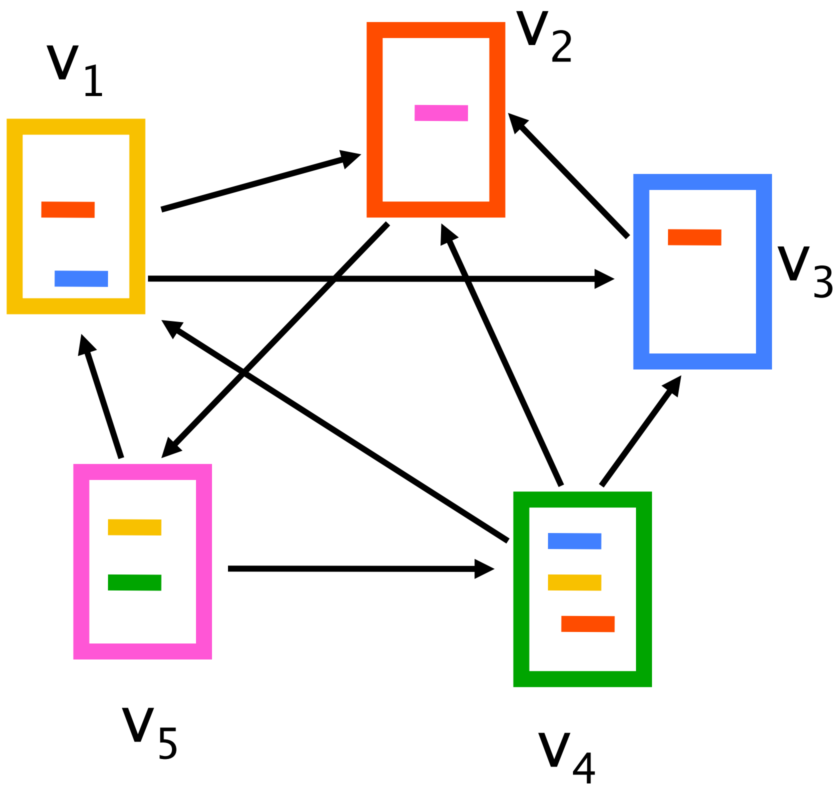
State probability vector

- The vector $q^t = (q^t{}_1, q^t{}_2, \ldots, q^t{}_n)$ that stores the probability of being at state i at time t
  - $q^0{}_i$ = the probability of starting from state i

$$q^t = q^{t-1} P$$

An example

$$P = \begin{bmatrix} 0 & 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2 & 0 \end{bmatrix}$$



$q^{t+1}_1 = 1/3\ q^t_4 + 1/2\ q^t_5$

$q^{t+1}_2 = 1/2\ q^t_1 + q^t_3 + 1/3\ q^t_4$

$q^{t+1}_3 = 1/2\ q^t_1 + 1/3\ q^t_4$

$q^{t+1}_4 = 1/2\ q^t_5$

$q^{t+1}_5 = q^t_2$

Stationary distribution

- A stationary distribution for a MC with transition matrix P, is a probability distribution π, such that $\pi = \pi P$

- A MC has a unique stationary distribution if
  - it is irreducible
    - the underlying graph is strongly connected
  - it is aperiodic
    - for random walks, the underlying graph is not bipartite
- The probability π_i is the fraction of times that we visited state i as $t \rightarrow \infty$
- The stationary distribution is an eigenvector of matrix P
  - the principal left eigenvector of P – stochastic matrices have maximum eigenvalue 1

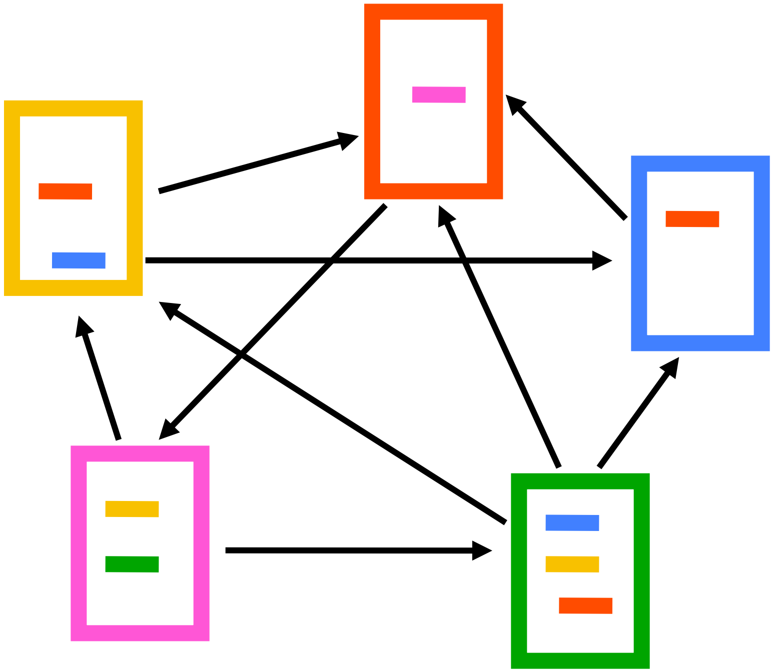
Computing the stationary distribution

- The Power Method
  - Initialize to some distribution q^0
  - Iteratively compute $q^t = q^{t-1}P$
  - After enough iterations $q^t \approx \pi$
  - Power method because it computes $q^t = q^0P^t$
- Why does it converge?
  - follows from the fact that any vector can be written as a linear combination of the eigenvectors
    - $q^0 = v_1 + c_2v_2 + \ldots c_nv_n$
- Rate of convergence
  - determined by $\lambda_2{}^t$

The PageRank random walk

- Vanilla random walk
  - make the adjacency matrix stochastic and run a random walk

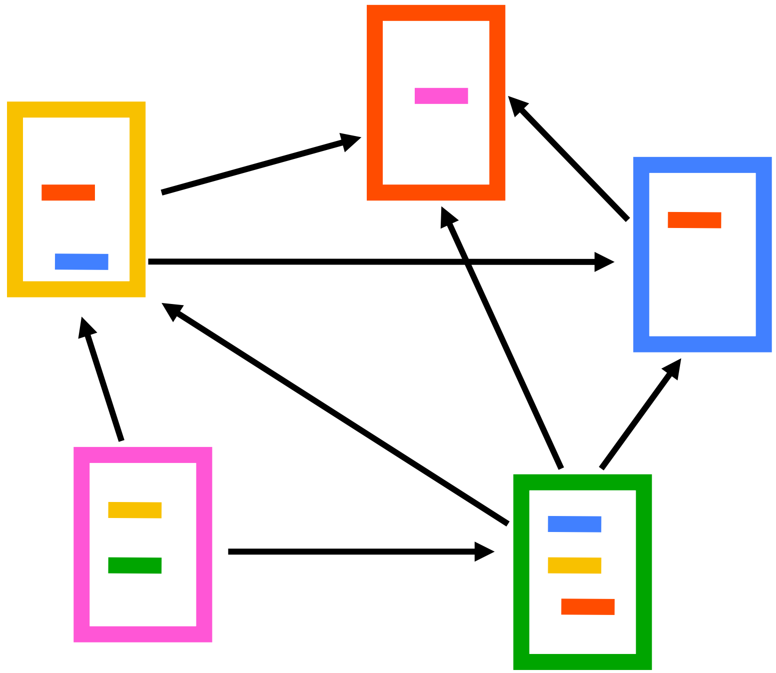
$$P = \begin{bmatrix} 0 & 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2 & 0 \end{bmatrix}$$



The PageRank random walk

- What about sink nodes?
  - what happens when the random walk moves to a node without any outgoing inks?

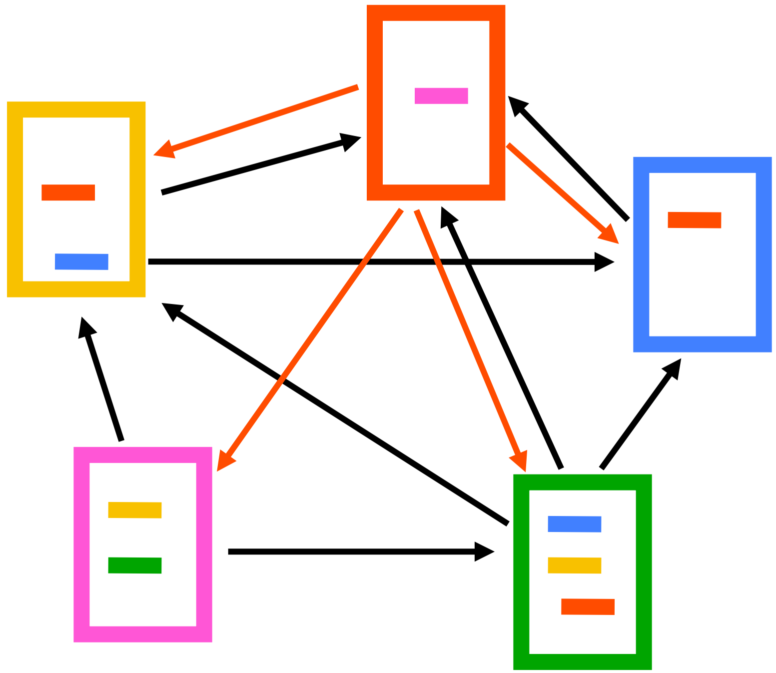
$$P = \begin{bmatrix} 0 & 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2 & 0 \end{bmatrix}$$



The PageRank random walk

- Replace these row vectors with a vector v
  - typically, the uniform vector

$$P' = \begin{bmatrix} 0 & 1/2 & 1/2 & 0 & 0 \\ \boxed{1/5 \quad 1/5 \quad 1/5 \quad 1/5 \quad 1/5} \\ 0 & 1 & 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2 & 0 \end{bmatrix}$$



$$P' = P + dv^T \qquad d = \begin{cases} 1 & \text{if i is sink} \\ 0 & \text{otherwise} \end{cases}$$

The PageRank random walk

- How do we guarantee irreducibility?
  - add a random jump to vector v with prob α
    - typically, to a uniform vector

$$P'' = \alpha \begin{bmatrix} 0 & 1/2 & 1/2 & 0 & 0 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \\ 0 & 1 & 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 1/2 \end{bmatrix} + (1-\alpha) \begin{bmatrix} 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \end{bmatrix}$$

$P'' = \alpha P' + (1-\alpha) u v^T$, where u is the vector of all 1s

Effects of random jump

- Guarantees irreducibility
- Motivated by the concept of random surfer
- Offers additional flexibility
  - personalization
  - anti-spam
- Controls the rate of convergence
  - the second eigenvalue of matrix P'' is α

A PageRank algorithm

- Performing vanilla power method is now too expensive – the matrix is not sparse

$$q^0 = v$$
t = 1
repeat
$$q^t = (P'')^T q^{t-1}$$
$$\delta = ||q^t - q^{t-1}||$$
t = t +1
until $\delta < \varepsilon$

Efficient computation of
$$q^t = (P'')^T q^{t-1}$$

$$q^t = aP'^T q^{t-1}$$
$$\beta = ||q^{t-1}||_1 - ||q^t||_1$$
$$q^t = q^t + \beta v$$

Random walks on undirected graphs

- In the stationary distribution of a random walk on an undirected graph, the probability of being at node i is proportional to the (weighted) degree of the vertex

- Random walks on undirected graphs are not "interesting"

Research on PageRank

- Specialized PageRank
  - personalization [BP98]
    - instead of picking a node uniformly at random favor specific nodes that are related to the user
  - topic sensitive PageRank [H02]
    - compute many PageRank vectors, one for each topic
    - estimate relevance of query with each topic
    - produce final PageRank as a weighted combination
- Updating PageRank [Chien et al 2002]
- Fast computation of PageRank
  - numerical analysis tricks
  - node aggregation techniques
  - dealing with the "Web frontier"