Lecture outline

- Classification
- Naïve Bayes classifier
-

Bayes Theorem

- **X, Y** random variables
- Joint probability: **Pr(X=x,Y=y)**
- Conditional probability: **Pr(Y=y | X=x)**
- Relationship between joint and conditional probability distributions

$$\Pr(X,Y) = \Pr(X \mid Y) \times \Pr(Y) = \Pr(Y \mid X) \times \Pr(X)$$

- **Bayes Theorem**:

$$\Pr(Y \mid X) = \frac{\Pr(X \mid Y)\Pr(Y)}{\Pr(X)}$$

Bayes Theorem for Classification

- **X**: attribute set
- **Y**: class variable
- **Y** depends on **X** in a **non-determininstic** way
- We can capture this dependence using

Pr(Y|X) : Posterior probability

vs

Pr(Y): Prior probability

Building the Classifier

- Training phase:
  - Learning the posterior probabilities **Pr(Y|X)** for every combination of **X** and **Y** based on training data

- Test phase:
  - For test record **X'**, compute the class **Y'** that maximizes the posterior probability **Pr(Y'|X')**

Bayes Classification: Example

	binary	categorical	continuous	class
Tid	**Home Owner**	**Marital Status**	**Annual Income**	**Defaulted Borrower**
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Figure 4.6. Training set for predicting borrowers who will default on loan payments.

X'=(Home Owner=No, Marital Status=Married, AnnualIncome=120K)

Compute: **Pr(Yes|X'), Pr(No|X')** pick No or Yes with max Prob.

How can we compute these probabilities??

Computing posterior probabilities

- Bayes Theorem

$$\Pr(Y \mid X) = \frac{\Pr(X \mid Y)\Pr(Y)}{\Pr(X)}$$

- **P(X)** is constant and can be ignored
- **P(Y):** estimated from training data; compute the fraction of training records in each class
- **P(X|Y)**?

Naïve Bayes Classifier

$$\Pr(X \mid Y = y) = \prod_{i=1}^{d} \Pr(X_i \mid Y = y)$$

- Attribute set **X = {X$_1$,…,X$_d$}** consists of **d** attributes
- Conditional independence:
  - **X** conditionally independent of **Y**, given **X**: **Pr(X|Y,Z) = Pr(X|Z)**
  - **Pr(X,Y|Z) = Pr(X|Z)xPr(Y|Z)**

Naïve Bayes Classifier

$$\Pr(X|Y=y) = \prod_{i=1}^{d} \Pr(X_i|Y=y)$$

- Attribute set $\mathbf{X = \{X_1,\ldots,X_d\}}$ consists of $\mathbf{d}$ attributes

$$\Pr(X|Y) = \frac{\Pr(Y)\prod_{i=1}^{d}\Pr(X_i|Y)}{\Pr(X)}$$

Conditional probabilities for categorical attributes

- Categorical attribute X_i
- Pr(Xi = xi|Y=y): fraction of training instances in class y that take value x_i on the i-th attribute

Pr(homeOwner = yes|No) = 3/7

Pr(MaritalStatus = Single| Yes) = 2/3

Tid	Home Owner (binary)	Marital Status (categorical)	Annual Income (continuous)	Defaulted Borrower (class)
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
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Figure 4.6. Training set for predicting borrowers who will default on loan payments.

Estimating conditional probabilities for continuous attributes?

- Discretization?

- How can we discretize?

Naïve Bayes Classifier: Example

- **X' = (HomeOwner = No, MaritalStatus = Married, Income=120K)**
- Need to compute **Pr(Y|X')** or **Pr(Y)xPr(X'|Y)**
- But **Pr(X'|Y)** is
  - **Y = No**:
    - Pr(HO=No|No)xPr(MS=Married|No) xPr(Inc=120K|No) = 4/7x4/7x0.0072 = 0.0024
  - **Y=Yes**:
    - Pr(HO=No|Yes)xPr(MS=Married|Yes) xPr(Inc=120K|Yes) = 1x0x1.2x10^{-9} = 0

Naïve Bayes Classifier: Example

- **X' = (HomeOwner = No, MaritalStatus = Married, Income=120K)**
- Need to compute **Pr(Y|X')** or **Pr(Y)xPr(X'|Y)**
- But **Pr(X'|Y = Yes)** is **0**?
- Correction process:

$$\Pr(X_i = x_i \mid Y = y_j) = \frac{n_c + mp}{n + m}$$

n_c: number of training examples from class **y_j** that take value **x_i**
n: total number of instances from class **y_j**
m: equivalent sample size (balance between prior and posterior)
p: user-specified parameter (prior probability)

Characteristics of Naïve Bayes Classifier

- Robust to isolated noise points
  - noise points are averaged out
- Handles missing values
  - Ignoring missing-value examples
- Robust to irrelevant attributes
  - If X_i is irrelevant, $P(X_i|Y)$ becomes almost uniform
- Correlated attributes degrade the performance of NB classifier