Lecture outline

- Classification
- Decision-tree classification

What is classification?

Tid	Home Owner (binary)	Marital Status (categorical)	Annual Income (continuous)	Defaulted Borrower (class)
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Figure 4.6. Training set for predicting borrowers who will default on loan payments.

What is classification?

- **Classification** is the task of ***learning a target function*** **f** that maps attribute set **x** to one of the predefined class labels **y**

	binary	categorical	continuous	class
Tid	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
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Figure 4.6. Training set for predicting borrowers who will default on loan payments.

What is classification?

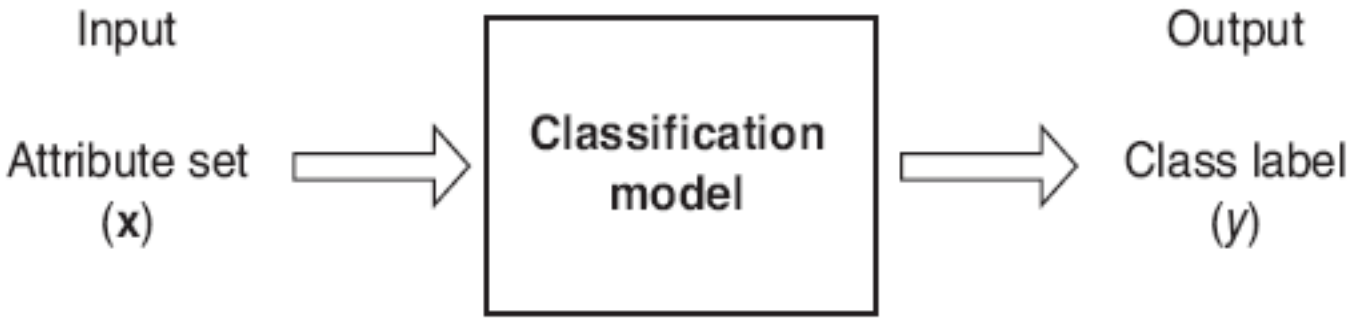


Figure 4.2. Classification as the task of mapping an input attribute set $\mathbf{x}$ into its class label y.

Why classification?

- The target function **f** is known as a ***classification model***
- **Descriptive modeling:** ***Explanatory tool*** to distinguish between objects of different classes (e.g., description of who can pay back his loan)
- **Predictive modeling:** Predict a class of a previously ***unseen*** record

Typical applications

- credit approval
- target marketing
- medical diagnosis
- treatment effectiveness analysis

General approach to classification

- ***Training set*** consists of records with ***known class labels***

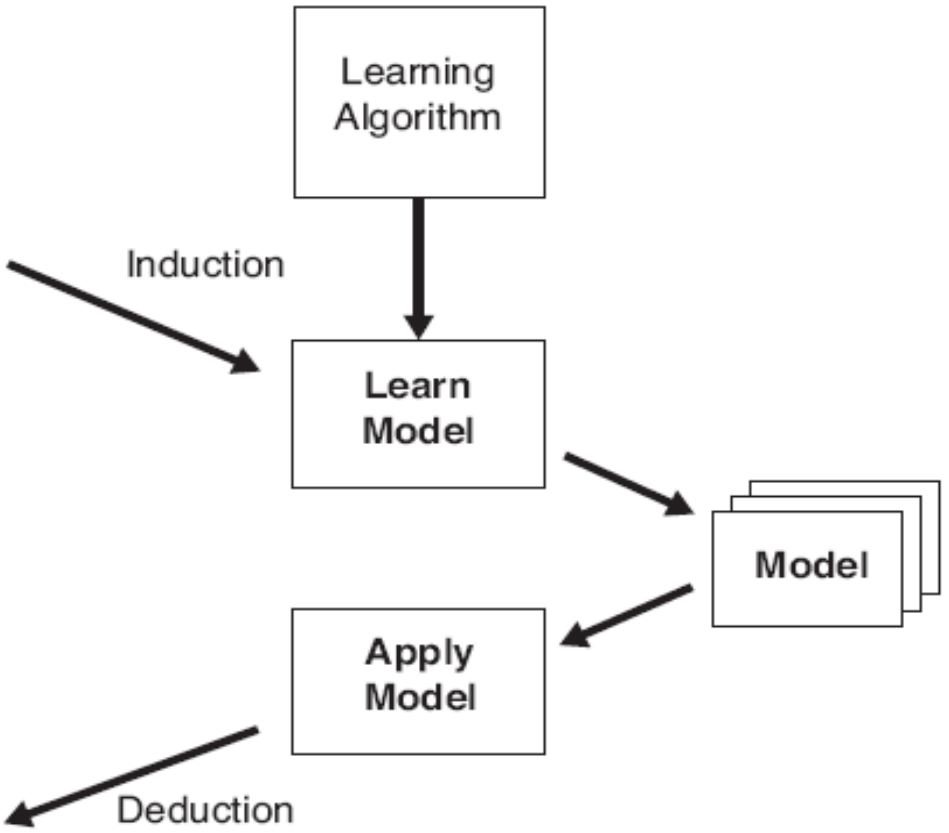
- Training set is used to ***build a classification model***

- The classification model is applied to the ***test set*** that consists of records with ***unknown labels***

General approach to classification

Training Set

Tid	Attrib1	Attrib2	Attrib3	Class
1	Yes	Large	125K	No
2	No	Medium	100K	No
3	No	Small	70K	No
4	Yes	Medium	120K	No
5	No	Large	95K	Yes
6	No	Medium	60K	No
7	Yes	Large	220K	No
8	No	Small	85K	Yes
9	No	Medium	75K	No
10	No	Small	90K	Yes



Test Set

Tid	Attrib1	Attrib2	Attrib3	Class
11	No	Small	55K	?
12	Yes	Medium	80K	?
13	Yes	Large	110K	?
14	No	Small	95K	?
15	No	Large	67K	?

Figure 4.3. General approach for building a classification model.

Evaluation of classification models

- Counts of test records that are correctly (or incorrectly) predicted by the classification model
- **Confusion matrix**

		Predicted Class	
		Class = 1	Class = 0
Actual Class	Class = 1	f_{11}	f_{10}
	Class = 0	f_{01}	f_{00}

$$\text{Accuracy} = \frac{\#\text{ correct predictions}}{\text{total }\#\text{ of predictions}} = \frac{f_{11} + f_{00}}{f_{11} + f_{10} + f_{01} + f_{00}}$$

$$\text{Error rate} = \frac{\#\text{ wrong predictions}}{\text{total }\#\text{ of predictions}} = \frac{f_{10} + f_{01}}{f_{11} + f_{10} + f_{01} + f_{00}}$$

Supervised vs. Unsupervised Learning

- Supervised learning (classification)
  - Supervision: The training data (observations, measurements, etc.) are accompanied by labels indicating the class of the observations
  - New data is classified based on the training set
- Unsupervised learning (clustering)
  - The class labels of training data is unknown
  - Given a set of measurements, observations, etc. with the aim of establishing the existence of classes or clusters in the data

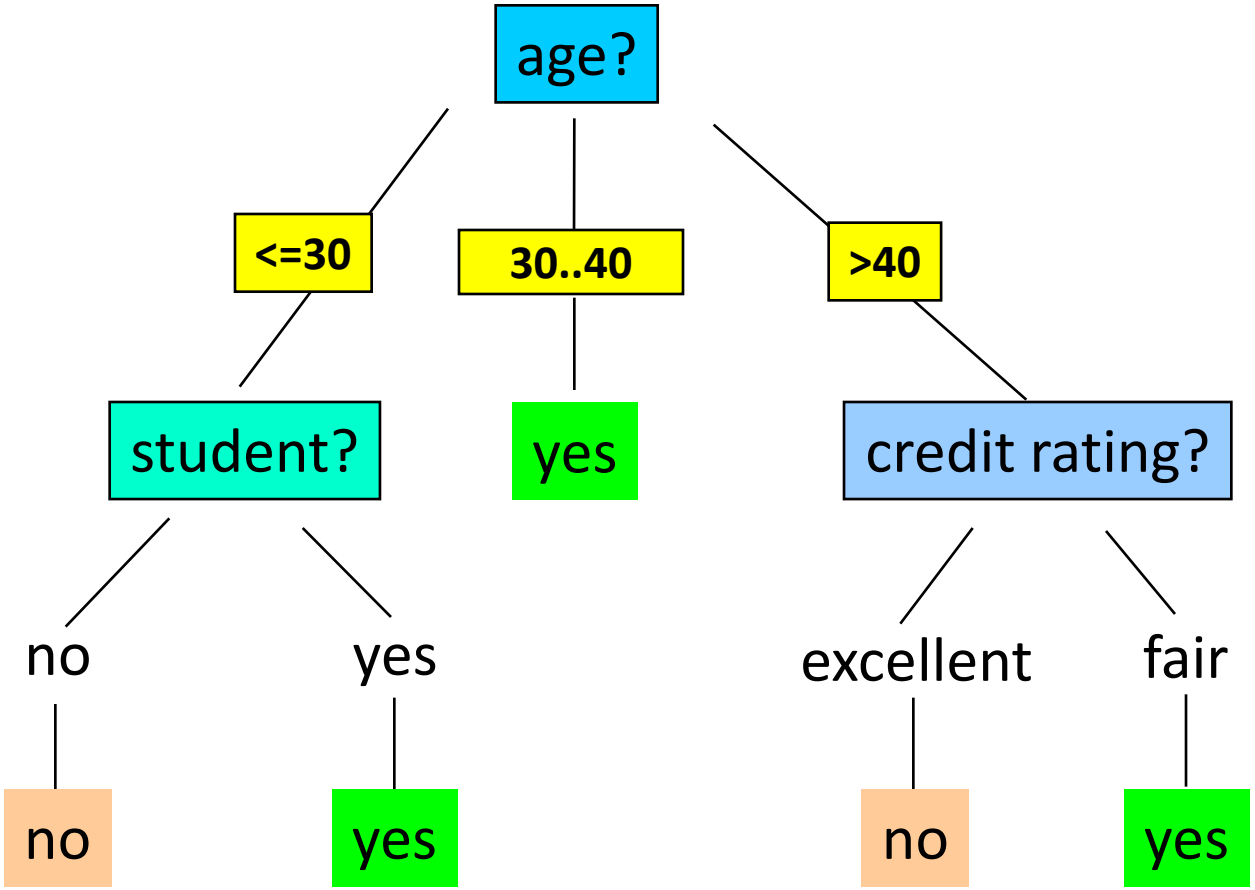
Decision Trees

- Decision tree
  - A flow-chart-like tree structure
  - Internal node denotes a test on an attribute
  - Branch represents an outcome of the test
  - Leaf nodes represent class labels or class distribution
- Decision tree generation consists of two phases
  - **Tree construction**
    - At start, all the training examples are at the root
    - Partition examples recursively based on selected attributes
  - **Tree pruning**
    - Identify and remove branches that reflect noise or outliers
- Use of decision tree: Classifying an unknown sample
  - Test the attribute values of the sample against the decision tree

Training Dataset

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31…40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes
>40	medium	no	excellent	no

Output: A Decision Tree for "*buys_computer*"



Constructing decision trees

- Exponentially many decision trees can be constructed from a given set of attributes

- Finding the most accurate tree is NP-hard

- **In practice: greedy algorithms**
  - Grow a decision tree by making a series of ***locally optimum decisions on which attributes to use*** for partitioning the data

Constructing decision trees: the Hunt's algorithm

- X_t: the set of training records for node **t**
- $y=\{y_1,\ldots,y_c\}$: class labels
- **Step 1:** If all records in X_t belong to the same class y_t, then t is a leaf node labeled as y_t
- **Step 2:** If X_t contains records that belong to more than one class,
  - select ***attribute test condition*** to partition the records into smaller subsets
  - Create a ***child node*** for each outcome of test condition
  - Apply algorithm ***recursively*** for each ***child***

Decision-tree construction (Example)

	binary	categorical	continuous	class
Tid	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
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Figure 4.6. Training set for predicting borrowers who will default on loan payments.

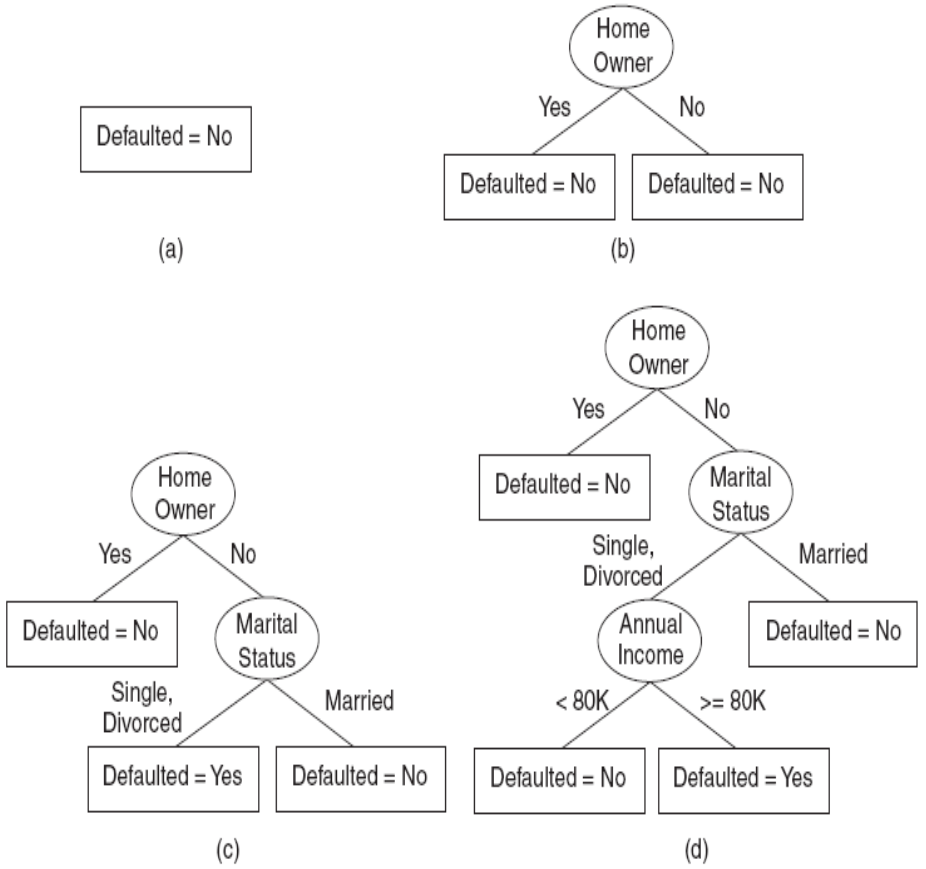


Figure 4.7. Hunt's algorithm for inducing decision trees.

Design issues

- How should the training records be split?
- How should the splitting procedure stop?

Splitting methods

- Binary attributes

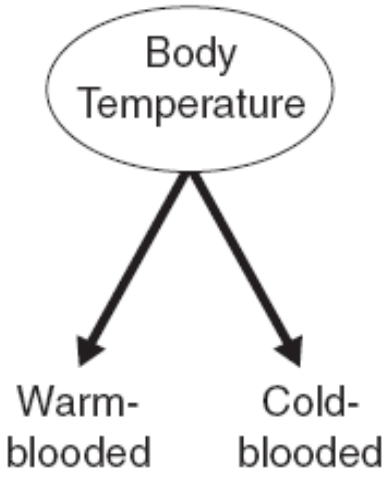


Figure 4.8. Test condition for binary attributes.

Splitting methods

- Nominal attributes

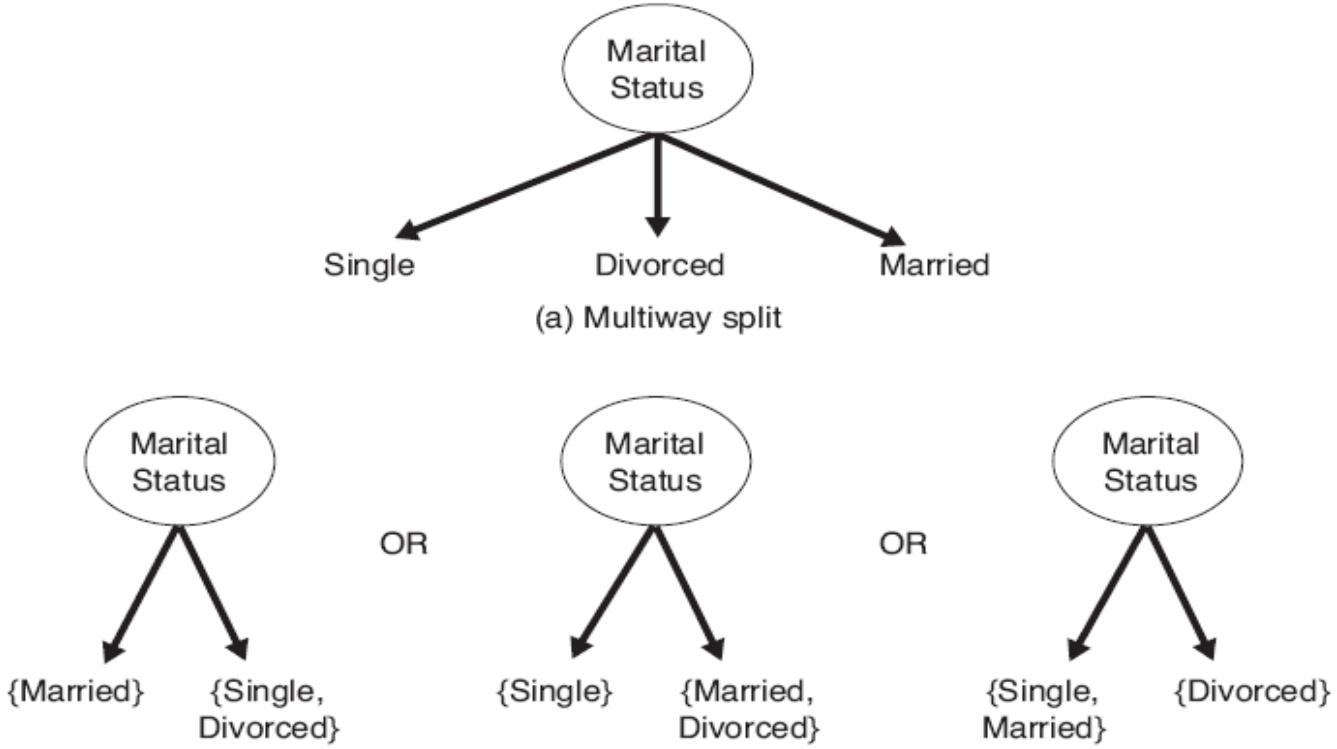


Figure 4.9. Test conditions for nominal attributes.

Splitting methods

- Ordinal attributes

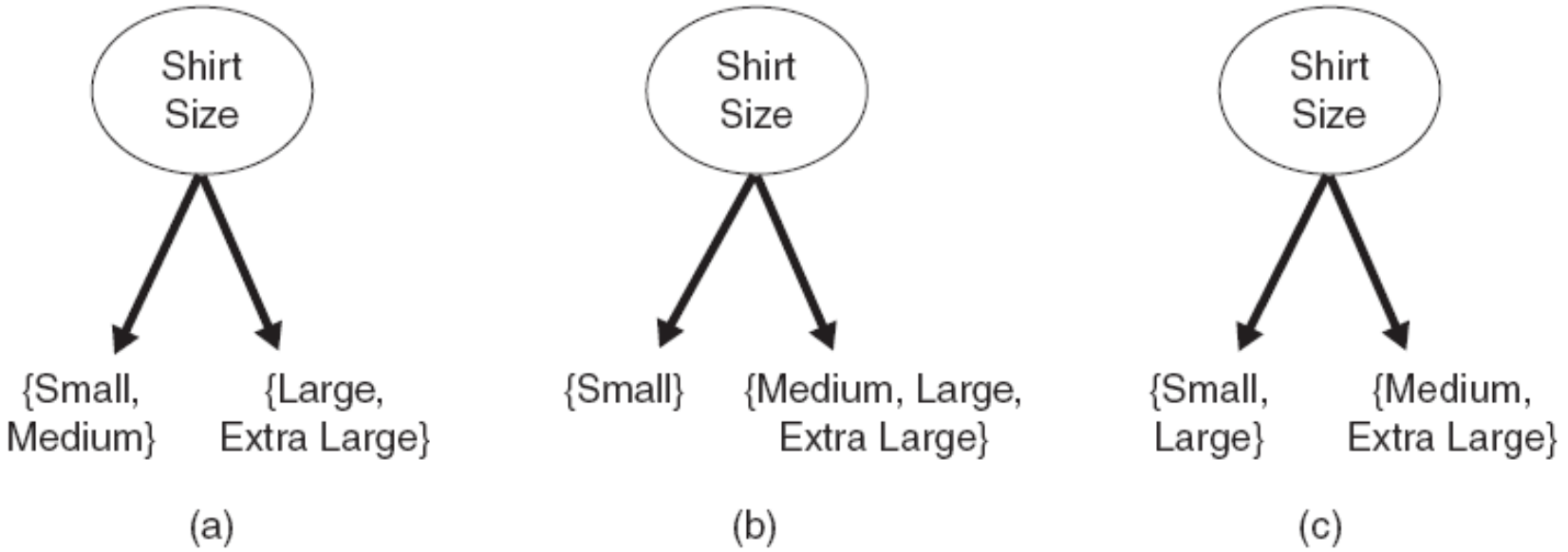


Figure 4.10. Different ways of grouping ordinal attribute values.

Splitting methods

- Continuous attributes

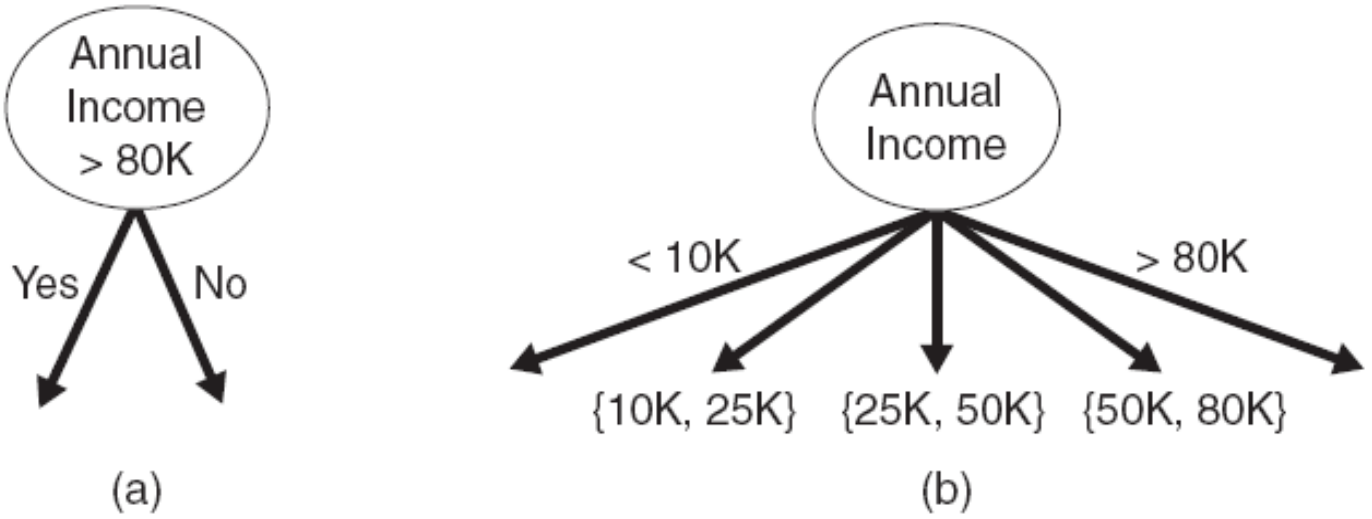


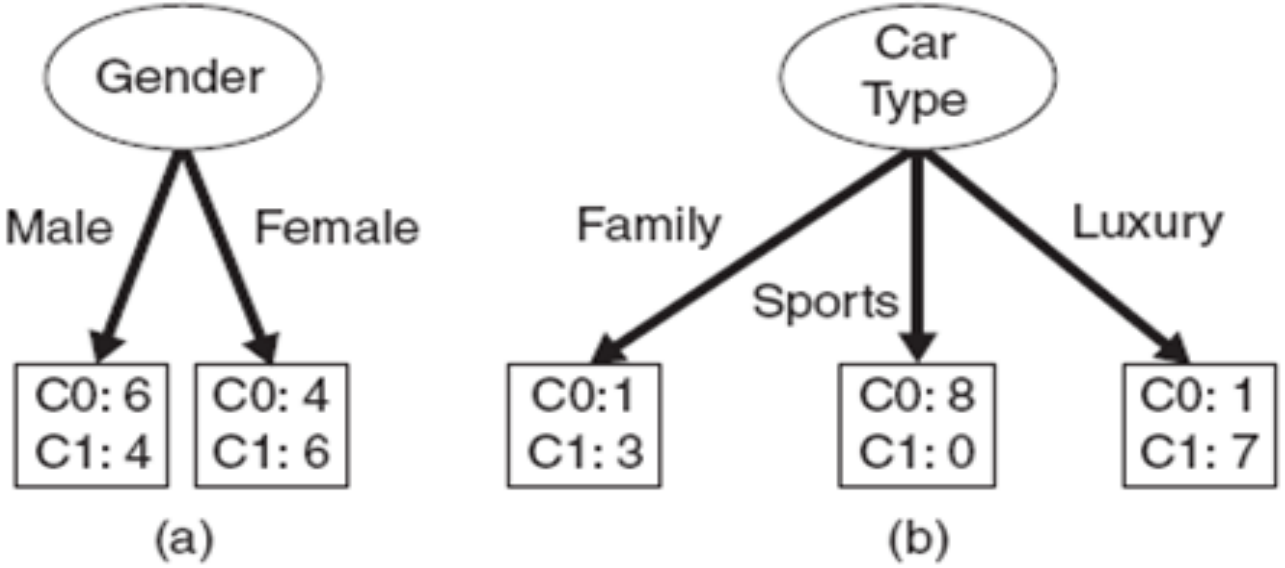
Figure 4.11. Test condition for continuous attributes.

Selecting the best split

- **p(i|t)**: fraction of records belonging to class **i**
- **Best split** is selected based on the degree of **impurity** of the child nodes
  - Class distribution **(0,1)** has **high purity**
  - Class distribution **(0.5,0.5)** has the **smallest purity (highest impurity)**

- **Intuition:** high purity → small value of impurity measures → better split

Selecting the best split



Selecting the best split: Impurity measures

- **p(i|t)**: fraction of records associated with node **t** belonging to class **i**

$$\text{Entropy}(t) = -\sum_{i=1}^{c} p(i \mid t) \log p(i \mid t)$$

$$\text{Gini}(t) = 1 - \sum_{i=1}^{c} p(i \mid t)^2$$

$$\text{Classification error}(t) = 1 - \max_i p(i \mid t)$$

Range of impurity measures

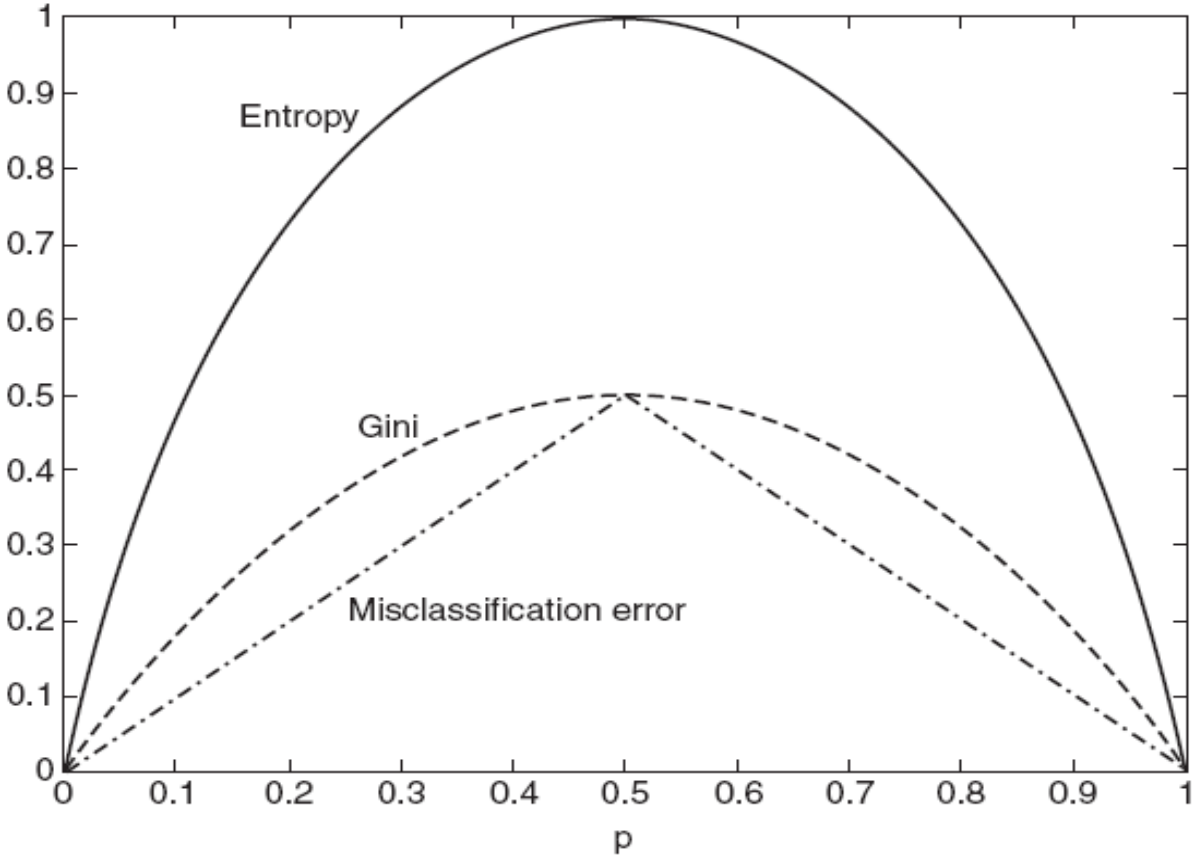


Figure 4.13. Comparison among the impurity measures for binary classification problems.

Impurity measures

- In general the different impurity measures are ***consistent***
- ***Gain of a test condition:*** compare the impurity of the parent node with the impurity of the child nodes

$$\Delta = I(parent) - \sum_{j=1}^{k} \frac{N(v_j)}{N} I(v_j)$$

- Maximizing the gain == minimizing the weighted average impurity measure of children nodes
- If **I() = Entropy(),** then **Δ_{info}** is called **information gain**

Computing gain: example

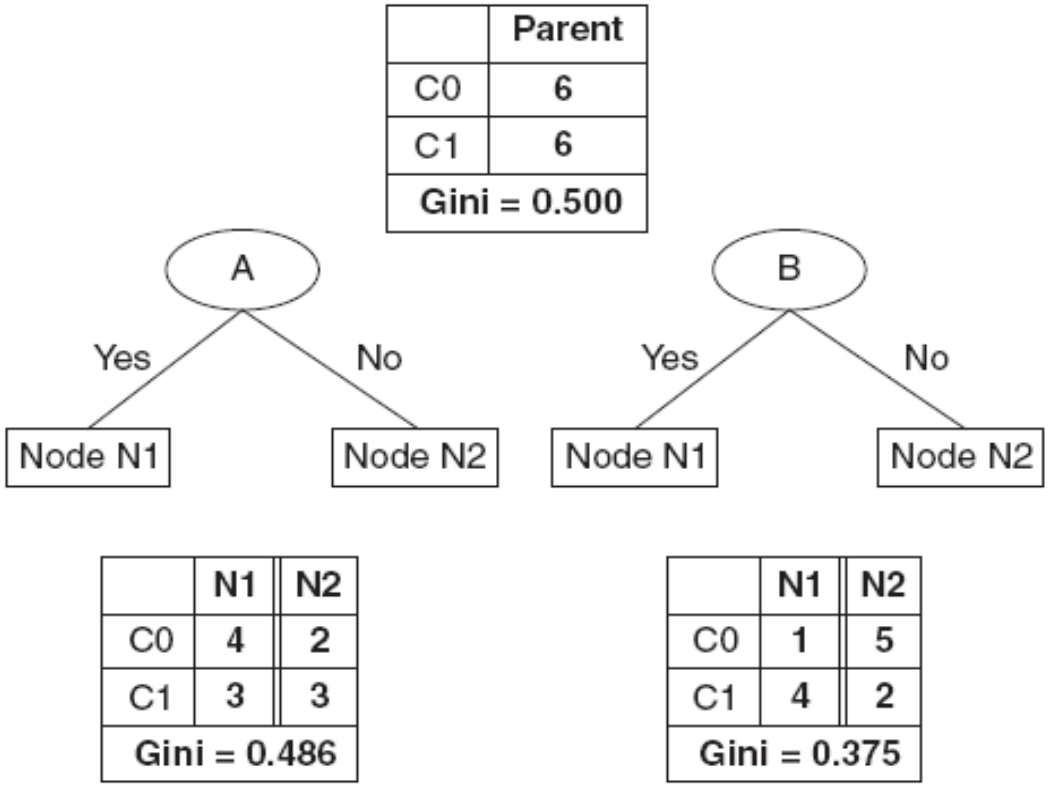


Figure 4.14. Splitting binary attributes.

Is minimizing impurity/ maximizing Δ enough?

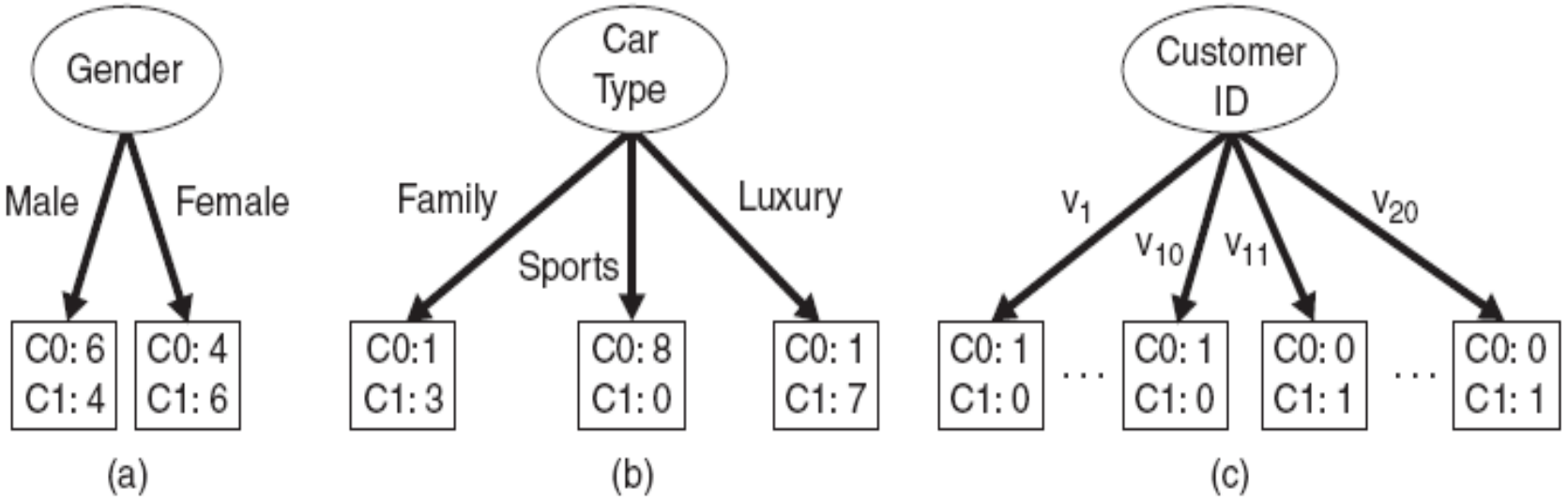


Figure 4.12. Multiway versus binary splits.

Is minimizing impurity/ maximizing Δ enough?

- Impurity measures favor attributes with large number of values

- A test condition with large number of outcomes may not be desirable
  - # of records in each partition is too small to make predictions

Gain ratio

- **Gain ratio = Δ_{info}/Splitinfo**

- **SplitInfo = $-\sum_{i=1\ldots k} p(v_i)\log(p(v_i))$**
- **k:** total number of splits
- If each attribute has the same number of records, **SplitInfo = logk**
- Large number of splits → large **SplitInfo** → small gain ratio

Constructing decision-trees (pseudocode)

GenDecTree(Sample **S**, Features **F**)

1. **If stopping_condition(S,F)** = true **then**
   a. **leaf = createNode()**
   b. **leaf.label= Classify(S)**
   c. **return leaf**
2. **root = createNode()**
3. **root.test_condition = findBestSplit(S,F)**
4. **V = {v| v a** possible outcome of **root.test_condition}**
5. **for *each*** value **vϵV**:
   a. **S_v: = {s | root.test_condition(s)** = **v** and **s ϵ S}**;
   b. **child = TreeGrowth(S_v ,F)** ;
   c. Add **child** as a descent of **root** and label the edge **(root→child) as v**
6. **return root**

Stopping criteria for tree induction

- Stop expanding a node when all the records belong to the same class
- Stop expanding a node when all the records have similar attribute values
- Early termination

Advantages of decision trees

- Inexpensive to construct
- Extremely fast at classifying unknown records
- Easy to interpret for small-sized trees
- Accuracy is comparable to other classification techniques for many simple data sets

Example: C4.5 algorithm

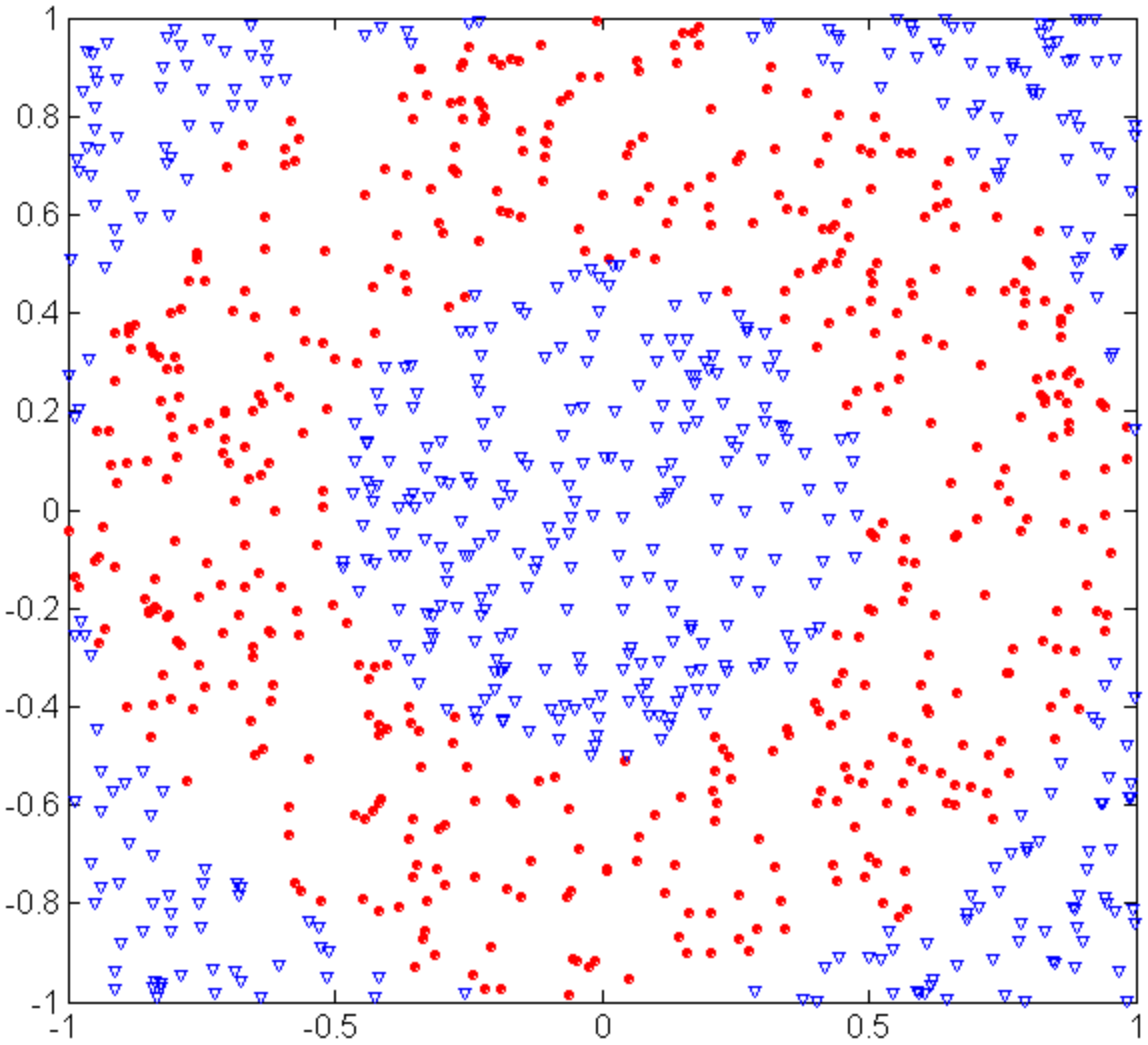
- Simple depth-first construction.
- Uses Information Gain
- Sorts Continuous Attributes at each node.
- Needs entire data to fit in memory.
- Unsuitable for Large Datasets.

- You can download the software from: http://www.cse.unsw.edu.au/~quinlan/c4.5r8.tar.gz

Practical problems with classification

- Unerfitting and overfitting
- Missing values
- Cost of classification

Underfitting and overfitting



triangular data points.

Circular points:

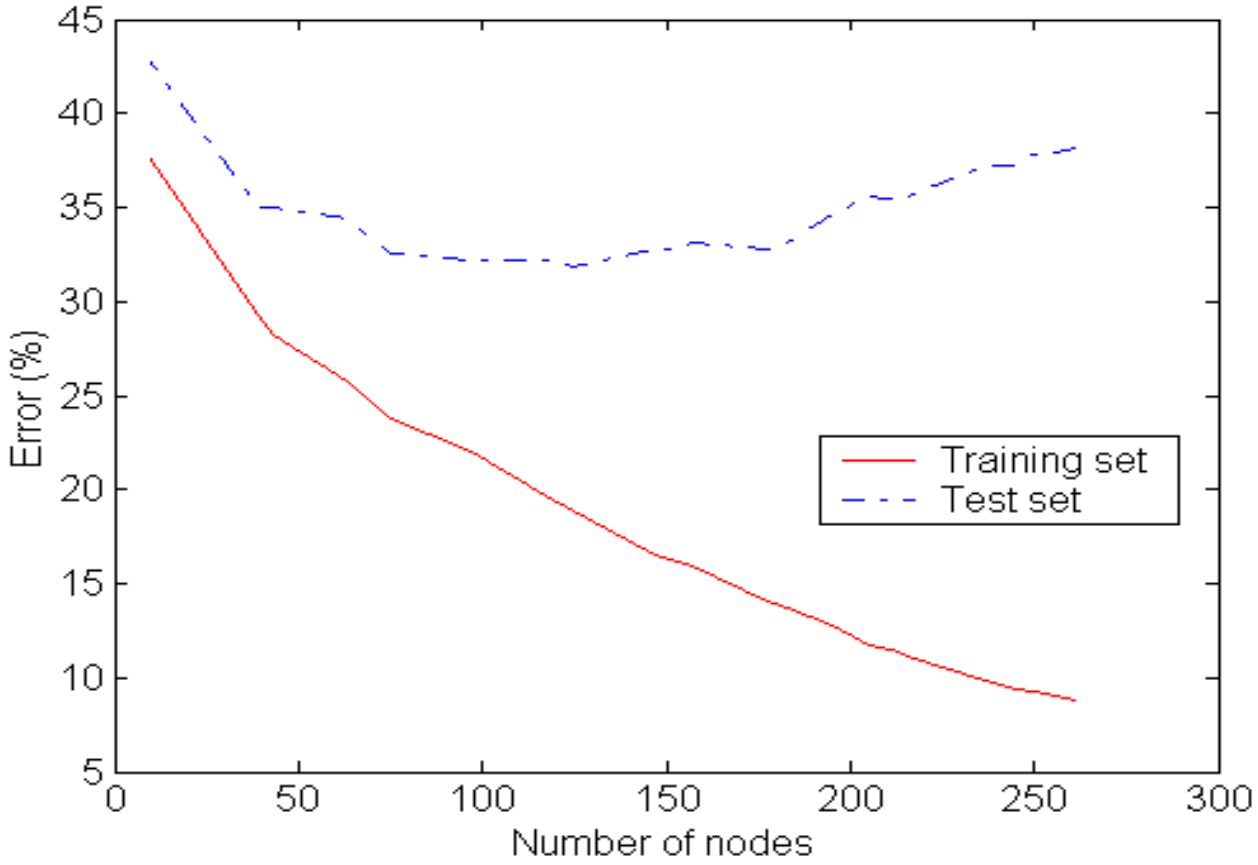
$0.5 \leq \text{sqrt}(x_1^2+x_2^2) \leq 1$

Triangular points:

$\text{sqrt}(x_1^2+x_2^2) > 1$ or

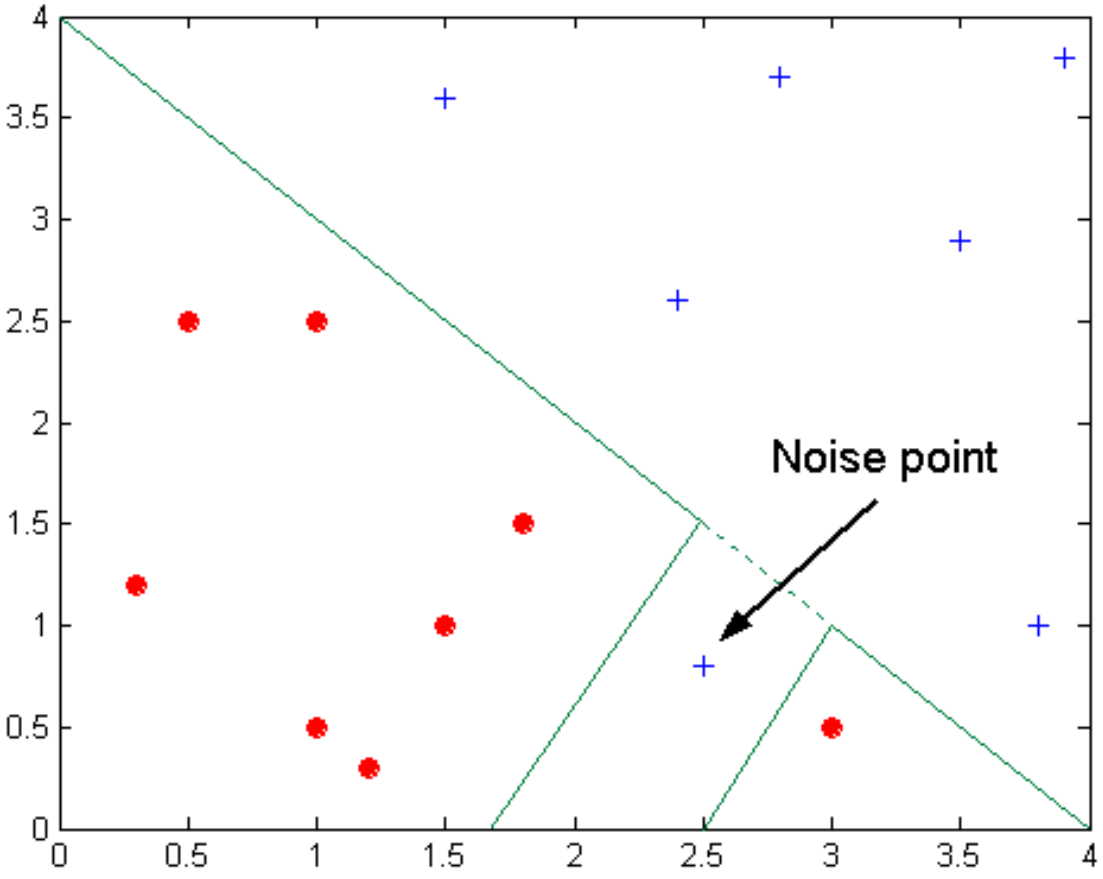
$\text{sqrt}(x_1^2+x_2^2) < 0.5$

Overfitting and underfitting



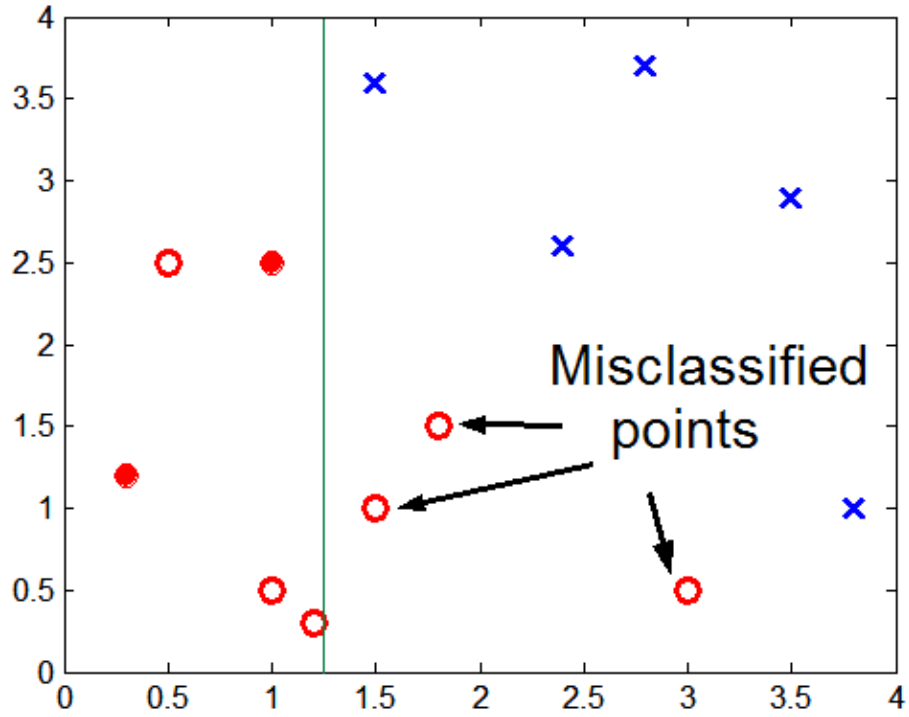
Underfitting: when model is too simple, both training and test errors are large

Overfitting due to noise



Decision boundary is distorted by noise point

Overfitting due to insufficient samples



Lack of data points in the lower half of the diagram makes it difficult to predict correctly the class labels of that region

- Insufficient number of training records in the region causes the decision tree to predict the test examples using other training records that are irrelevant to the classification task

Overfitting: course of action

- Overfitting results in decision trees that are more complex than necessary
- Training error no longer provides a good estimate of how well the tree will perform on previously unseen records
- Need new ways for estimating errors

Methods for estimating the error

- Re-substitution errors: error on training (Σ **e(t)**)
- Generalization errors: error on testing (Σ **e'(t)**)
- Methods for estimating generalization errors:
  - Optimistic approach: **e'(t) = e(t)**
  - Pessimistic approach:
    - For each leaf node: **e'(t) = (e(t)+0.5)**
    - Total errors: **e'(T) = e(T) + N $\times$ 0.5** (N: number of leaf nodes)
    - For a tree with 30 leaf nodes and 10 errors on training (out of 1000 instances):
      Training error = 10/1000 = 1%
      Generalization error = (10 + 30$\times$0.5)/1000 = 2.5%
  - Reduced error pruning (REP):
    - uses ***validation data set*** to estimate generalization error

Addressing overfitting: Occam's razor

- Given two models of similar generalization errors, one should prefer the simpler model over the more complex model

- For complex models, there is a greater chance that it was fitted accidentally by errors in data

- Therefore, one should include model complexity when evaluating a model

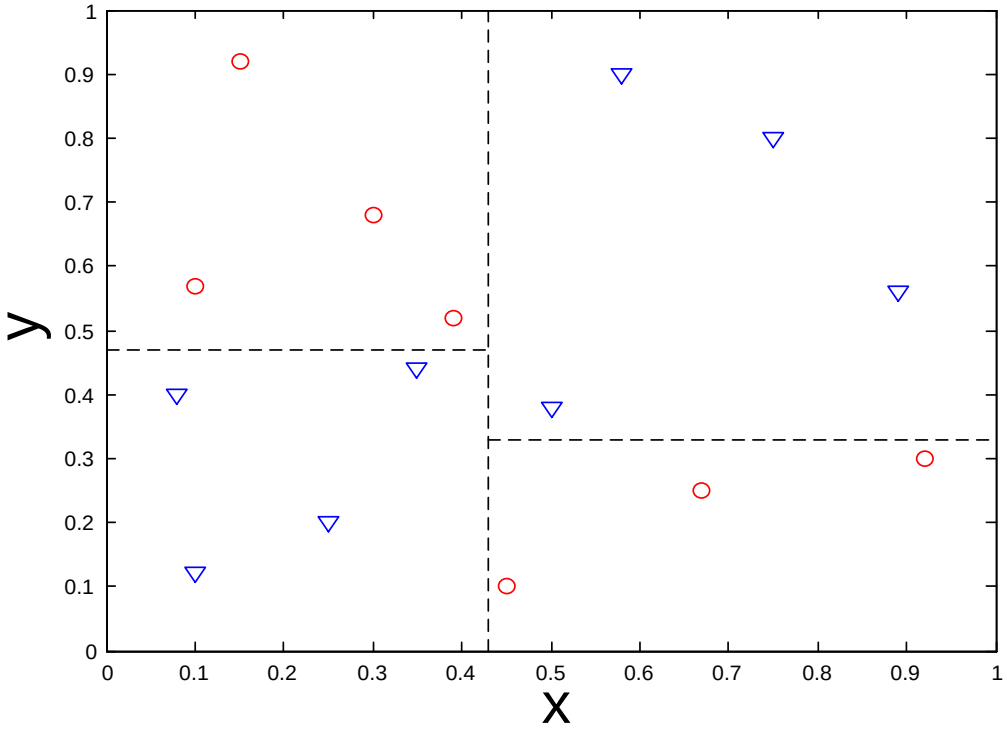
Addressing overfitting: postprunning

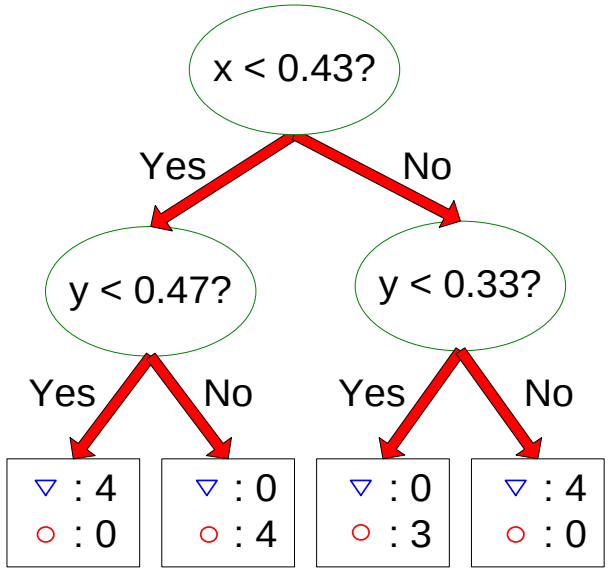
- Grow decision tree to its entirety
- Trim the nodes of the decision tree in a bottom-up fashion
- If generalization error improves after trimming, replace sub-tree by a leaf node.
- Class label of leaf node is determined from majority class of instances in the sub-tree
- Can use MDL for post-pruning

Addressing overfitting: preprunning

- Stop the algorithm before it becomes a fully-grown tree
- Typical stopping conditions for a node:
  - Stop if all instances belong to the same class
  - Stop if all the attribute values are the same
- More restrictive conditions:
  - Stop if number of instances is less than some user-specified threshold
  - Stop if expanding the current node does not improve impurity measures (e.g., Gini or information gain).

Decision boundary for decision trees

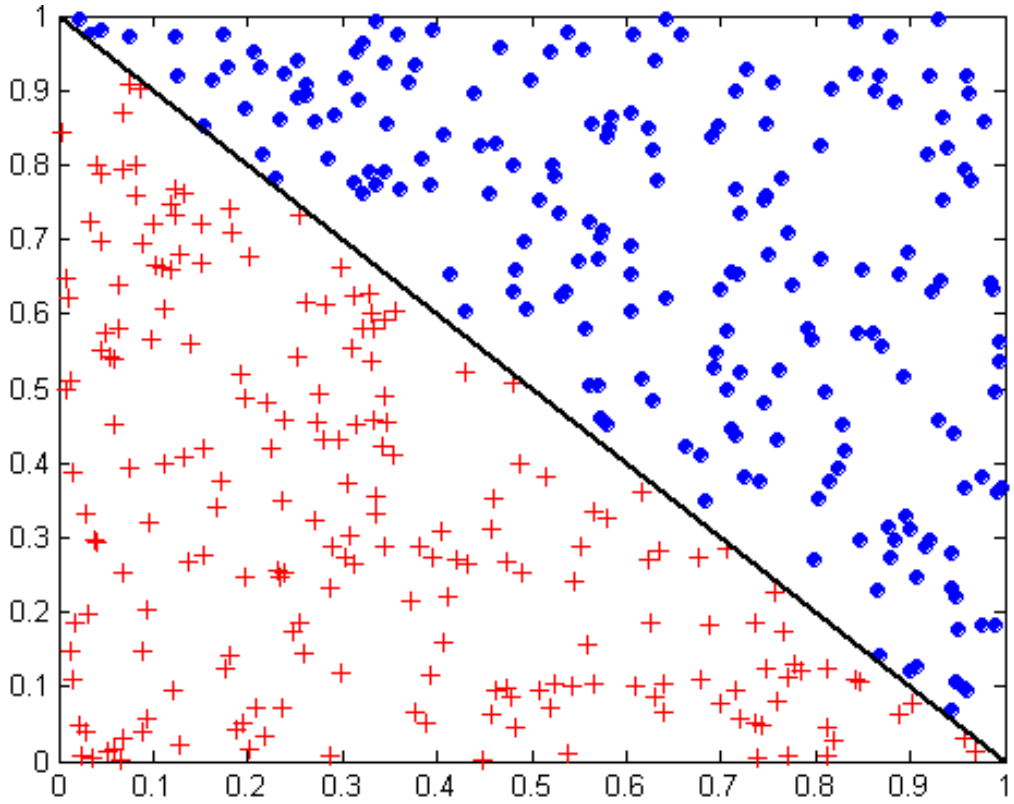


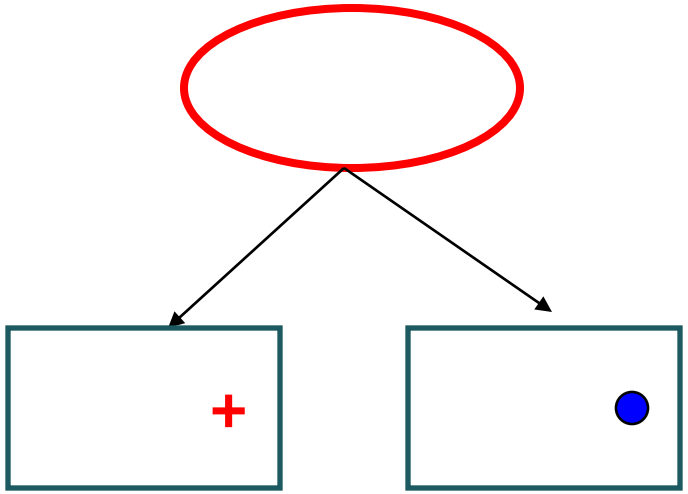


- Border line between two neighboring regions of different classes is known as **decision boundary**

- **Decision boundary in decision trees** is parallel to axes because test condition involves a single attribute at-a-time

Oblique Decision Trees





Not all datasets can be partitioned optimally using test conditions involving single attributes!