Lecture outline

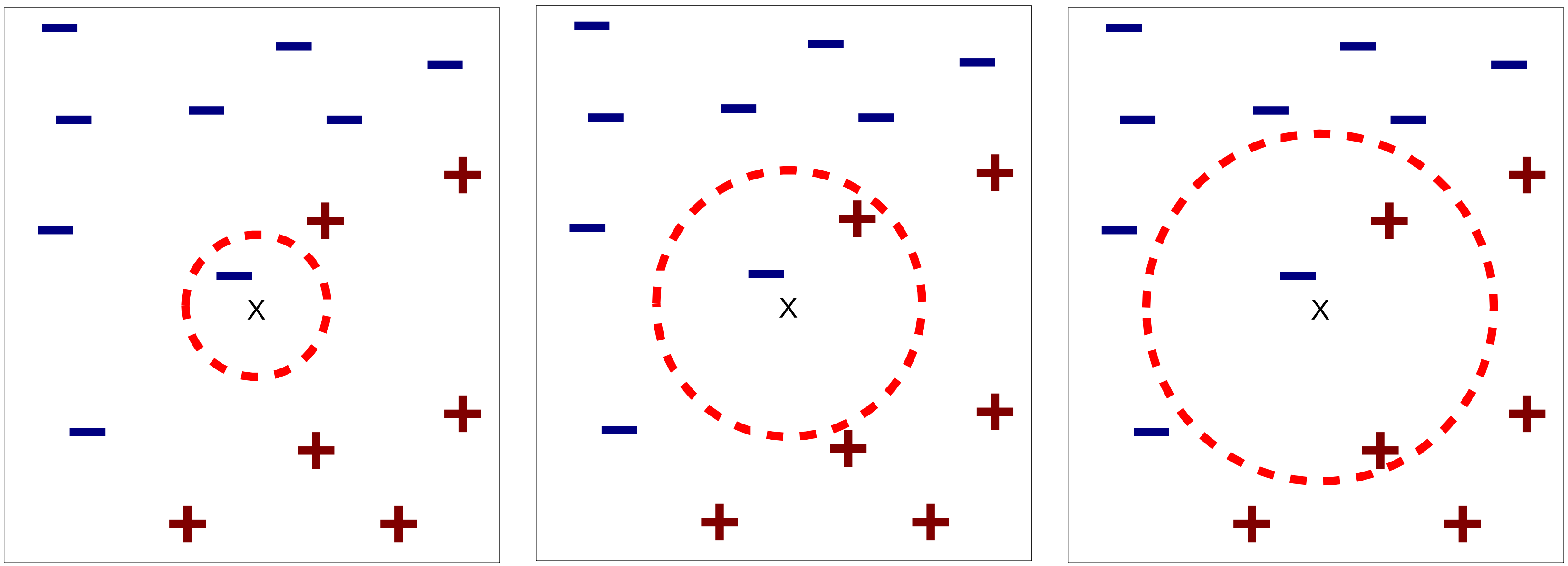
- Classification
- Naïve Bayes classifier
- Nearest-neighbor classifier

Eager vs Lazy learners

- Eager learners: learn the model as soon as the training data becomes available

- Lazy learners: delay model-building until testing data needs to be classified
  - Rote classifier: memorizes the entire training data

k-nearest neighbor classifiers



(a) 1-nearest neighbor (b) 2-nearest neighbor (c) 3-nearest neighbor

k-nearest neighbors of a record **x** are data points that have the **k** smallest distance to **x**

k-nearest neighbor classification

- Given a data record **x** find its **k** closest points
  - Closeness: Euclidean, Hamming, Jaccard distance

- Determine the class of **x** based on the classes in the neighbor list
  - Majority vote
  - Weigh the vote according to distance
    - e.g., weight factor, $w = 1/d^2$
  - Probabilistic voting

Characteristics of nearest-neighbor classifiers

- Instance of ***instance-based*** learning
- No model building (lazy learners)
  - Lazy learners: computational time in classification
  - Eager learners: computational time in model building
- Decision trees try to find global models, k-NN take into account local information
- K-NN classifiers depend a lot on the choice of proximity measure

Bayes Theorem

- **X, Y** random variables
- Joint probability: **Pr(X=x,Y=y)**
- Conditional probability: **Pr(Y=y | X=x)**
- Relationship between joint and conditional probability distributions

$$\Pr(X,Y) = \Pr(X \mid Y) \times \Pr(Y) = \Pr(Y \mid X) \times \Pr(X)$$

- **Bayes Theorem**:

$$\Pr(Y \mid X) = \frac{\Pr(X \mid Y)\Pr(Y)}{\Pr(X)}$$

Bayes Theorem for Classification

- **X**: attribute set
- ***Y***: class variable
- ***Y*** depends on **X** in a ***non-determininstic*** way
- We can capture this dependence using

$\Pr(Y|\mathbf{X})$: Posterior probability

vs

$\Pr(Y)$: Prior probability

Building the Classifier

- Training phase:
  - Learning the posterior probabilities **Pr(Y|X)** for every combination of **X** and **Y** based on training data

- Test phase:
  - For test record **X’**, compute the class **Y’** that *maximizes the posterior probability* **Pr(Y’|X’)**

Bayes Classification: Example

	binary	categorical	continuous	class
Tid	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Figure 4.6. Training set for predicting borrowers who will default on loan payments.

X'=(Home Owner=No, Marital Status=Married, AnnualIncome=120K)

Compute: **Pr(Yes|X')**, **Pr(No|X')** pick No or Yes with max Prob.

How can we compute these probabilities??

Computing posterior probabilities

- Bayes Theorem

$$\Pr(Y \mid X) = \frac{\Pr(X \mid Y)\Pr(Y)}{\Pr(X)}$$

- **P(X)** is constant and can be ignored
- **P(Y):** estimated from training data; compute the fraction of training records in each class
- **P(X|Y)**?

Naïve Bayes Classifier

$$\Pr(X \mid Y = y) = \prod_{i=1}^{d} \Pr(X_i \mid Y = y)$$

- Attribute set **X = {X_1,…,X_d}** consists of **d** attributes

- Conditional independence:
  - **X** conditionally independent of **Y**, given **X**: **Pr(X|Y,Z) = Pr(X|Z)**
  - **Pr(X,Y|Z) = Pr(X|Z)xPr(Y|Z)**

Naïve Bayes Classifier

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$$\Pr(Y \mid X) = \frac{\Pr(Y) \prod_{i=1}^{d} \Pr(X_i \mid Y)}{\Pr(X)}$$

Conditional probabilities for categorical attributes

- Categorical attribute X_i
- $Pr(X_i = x_i | Y = y)$: fraction of training instances in class y that take value x_i on the i-th attribute

$Pr(\text{homeOwner} = \text{yes} | \text{No}) = 3/7$

$Pr(\text{MaritalStatus} = \text{Single} | \text{Yes}) = 2/3$

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Figure 4.6. Training set for predicting borrowers who will default on loan payments.

Estimating conditional probabilities for continuous attributes?

- Discretization?

- How can we discretize?

Naïve Bayes Classifier: Example

- **X' = (HomeOwner = No, MaritalStatus = Married, Income=120K)**
- Need to compute **Pr(Y|X')** or **Pr(Y)xPr(X'|Y)**
- But **Pr(X'|Y)** is
  - **Y = No**:
    - Pr(HO=No|No)xPr(MS=Married|No)xPr(Inc=120K|No) = 4/7x4/7x0.0072 = 0.0024
  - **Y=Yes**:
    - Pr(HO=No|Yes)xPr(MS=Married|Yes)xPr(Inc=120K|Yes) = 1x0x1.2x10^{-9} = 0

Naïve Bayes Classifier: Example

- **X' = (HomeOwner = No, MaritalStatus = Married, Income=120K)**
- Need to compute **Pr(Y|X')** or **Pr(Y)xPr(X'|Y)**
- But **Pr(X'|Y = Yes)** is **0**?
- Correction process:

$$\Pr(X_i = x_i \mid Y = y_j) = \frac{n_c + mp}{n + m}$$

n_c: number of training examples from class $\mathbf{y_j}$ that take value $\mathbf{x_i}$
n: total number of instances from class $\mathbf{y_j}$
m: equivalent sample size (balance between prior and posterior)
p: user-specified parameter (prior probability)

Characteristics of Naïve Bayes Classifier

- Robust to isolated noise points
  - noise points are averaged out
- Handles missing values
  - Ignoring missing-value examples
- Robust to irrelevant attributes
  - If $\mathbf{X_i}$ is irrelevant, $\mathbf{P(X_i|Y)}$ becomes almost uniform
- Correlated attributes degrade the performance of NB classifier