Recap: Mining association rules from large datasets

Recap

- **Task 1:** Methods for finding all frequent itemsets efficiently
- **Task 2:** Methods for finding association rules efficiently

Recap

- Frequent itemsets (measure: support)
- Apriori principle
- Apriori algorithm for finding frequent itemsets
  - Prunes really well in practice
  - Makes multiple passes over the dataset

Making a single pass over the data: the AprioriTid algorithm

- The database is **not** used for counting support after the 1st pass!

- Instead information in data structure C_k' is used for counting support in every step

  - C_k' is generated from C_{k-1}'

  - For small values of k, storage requirements for data structures could be larger than the database!

  - For large values of k, storage requirements can be very small

Lecture outline

- **Task 1:** Methods for finding all frequent itemsets efficiently

- **Task 2:** Methods for finding association rules efficiently

Definition: Association Rule

Let **D** be database of transactions

– e.g.:

Transaction ID	Items
2000	A, B, C
1000	A, C
4000	A, D
5000	B, E, F

- Let I be the set of items that appear in the database, e.g., $I=\{A,B,C,D,E,F\}$
- A **rule** is defined by $X \rightarrow Y$, where $X \subset I$, $Y \subset I$, and $X \cap Y = \varnothing$
  - e.g.: $\{B,C\} \rightarrow \{A\}$ is a rule

Definition: Association Rule

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

- **Association Rule**
  - An implication expression of the form **X** $\rightarrow$ **Y**, where **X** and **Y** are non-overlapping itemsets
  - Example:
    {Milk, Diaper} $\rightarrow$ ***{Beer}***

- **Rule Evaluation Metrics**
  - **Support (s)**
    - Fraction of transactions that contain both **X** and **Y**
  - **Confidence (c)**
    - Measures how often items in **Y** appear in transactions that contain **X**

Example:

$$\{\text{Milk}, \text{Diaper}\} \rightarrow \text{Beer}$$

$$s = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{|\text{T}|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{\sigma(\text{Milk}, \text{Diaper})} = \frac{2}{3} = 0.67$$

Example

TID	date	items bought
100	10/10/99	{F,A,D,B}
200	15/10/99	{D,A,C,E,B}
300	19/10/99	{C,A,B,E}
400	20/10/99	{B,A,D}

What is the ***support*** and ***confidence*** of the rule: {B,D} → {A}

- Support:
  - percentage of tuples that contain {A,B,D} = 75%
- Confidence:

$$\frac{\text{number of tuples that contain }\{A, B, D\}}{\text{number of tuples that contain }\{B, D\}} = 100\%$$

Association-rule mining task

- Given a set of transactions **D**, the goal of association rule mining is to find **all** rules having
  - support ≥ ***minsup*** threshold
  - confidence ≥ ***minconf*** threshold

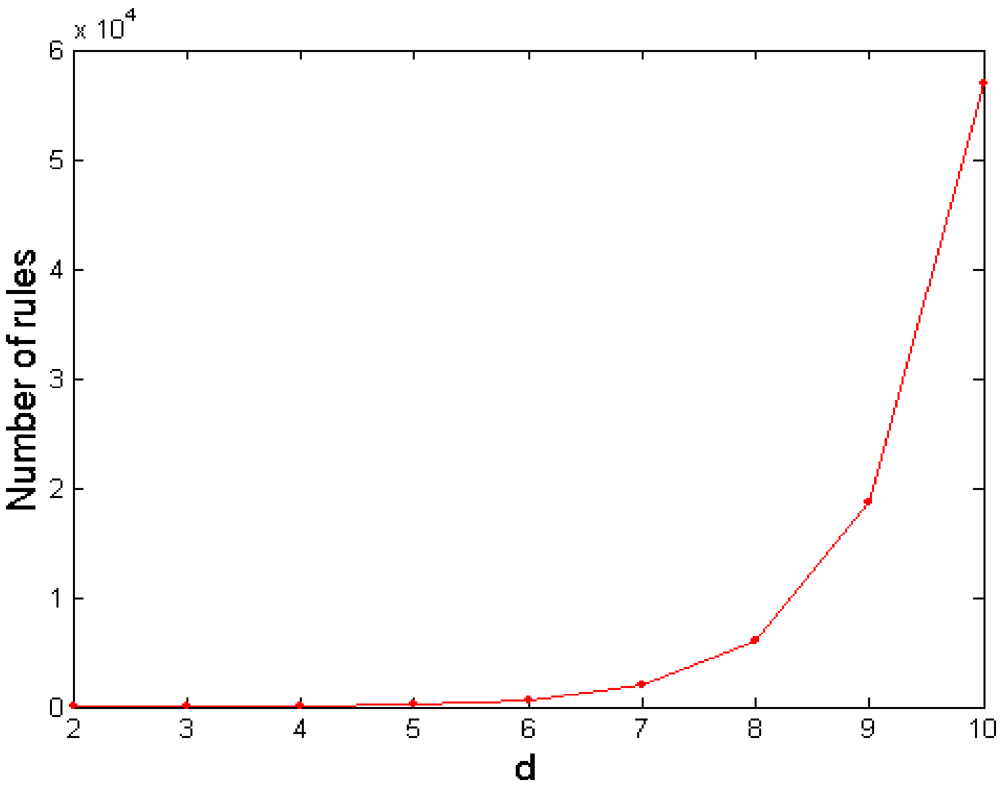
Brute-force algorithm for association-rule mining

- List all possible association rules
- Compute the support and confidence for each rule
- Prune rules that fail the *minsup* and *minconf* thresholds

- $\Rightarrow$ Computationally prohibitive!

How many association rules are there?

- Given **d** unique items in ***I***:
  - Total number of itemsets = **2^d**
  - Total number of possible association rules:



$$R = \sum_{k=1}^{d-1}\left[\binom{d}{k} \times \sum_{j=1}^{d-k}\binom{d-k}{j}\right]$$

$$= 3^d - 2^{d+1} + 1$$

If d=6, R = 602 rules

Mining Association Rules

- Two-step approach:
  - Frequent Itemset Generation
    - Generate all itemsets whose support $\geq$ minsup
  - Rule Generation
    - Generate high confidence rules from each frequent itemset, where each rule is a binary partition of a frequent itemset

Rule Generation – Naive algorithm

- Given a frequent itemset **X**, find all non-empty subsets $y \subset X$ such that $y \rightarrow X - y$ satisfies the minimum confidence requirement

  – If **{A,B,C,D}** is a frequent itemset, candidate rules:

$ABC \rightarrow D$,	$ABD \rightarrow C$,	$ACD \rightarrow B$,	$BCD \rightarrow A$,
$A \rightarrow BCD$,	$B \rightarrow ACD$,	$C \rightarrow ABD$,	$D \rightarrow ABC$
$AB \rightarrow CD$,	$AC \rightarrow BD$,	$AD \rightarrow BC$,	$BC \rightarrow AD$,
$BD \rightarrow AC$,	$CD \rightarrow AB$,		

- If $|X| = k$, then there are $2^k - 2$ candidate association rules (ignoring $X \rightarrow \varnothing$ and $\varnothing \rightarrow X$)

Efficient rule generation

- How to efficiently generate rules from frequent itemsets?
  - In general, confidence does not have an anti-monotone property

    $c(ABC \rightarrow D)$ can be larger or smaller than **$c(AB \rightarrow D)$**

  - ***But confidence of rules generated from the same itemset has an anti-monotone property***
  - Example: $X = \{A,B,C,D\}$:

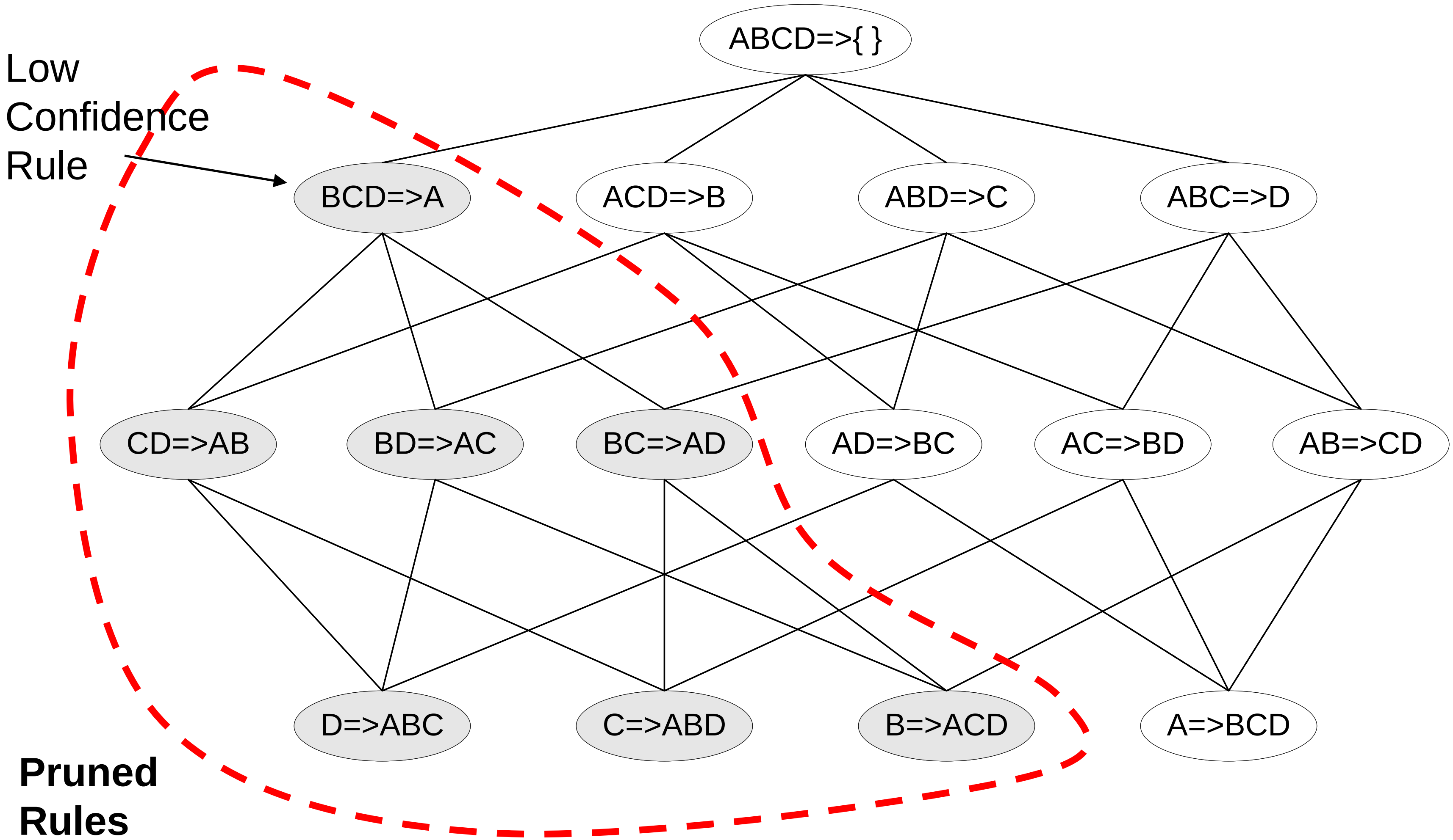
$$c(ABC \rightarrow D) \geq c(AB \rightarrow CD) \geq c(A \rightarrow BCD)$$

  - **Why?**

Confidence is anti-monotone w.r.t. number of items on the RHS of the rule

Rule Generation for Apriori Algorithm

Lattice of rules

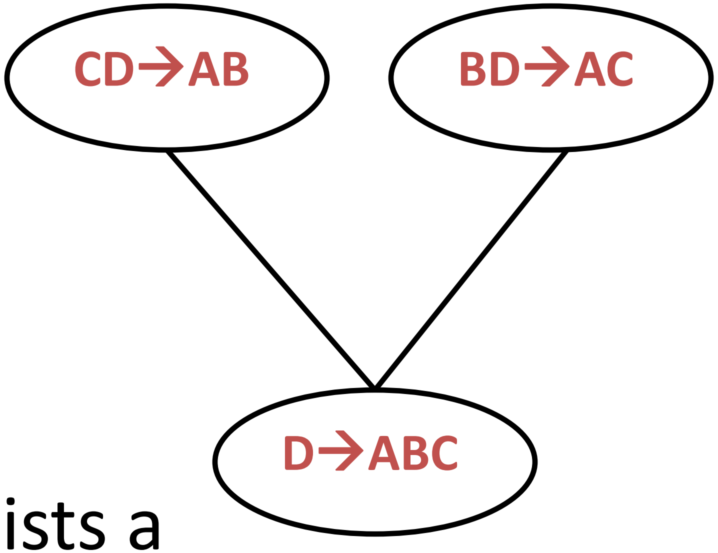


Apriori algorithm for rule generation

- Candidate rule is generated by merging two rules that share the same prefix
  in the rule consequent

- **join(CD→AB,BD—>AC)**
  would produce the candidate rule **D →ABC**

- **Prune** rule **D→ABC** if there exists a subset (e.g., **AD→BC**) that does not have high confidence



Reducing the collection of itemsets: alternative representations and combinatorial problems

Too many frequent itemsets

- If $\{a_1, \ldots, a_{100}\}$ is a frequent itemset, then there are

$$\binom{100}{1} + \binom{100}{2} + \ldots + \binom{100}{100} = 2^{100} - 1$$

1.27*10^{30} frequent sub-patterns!

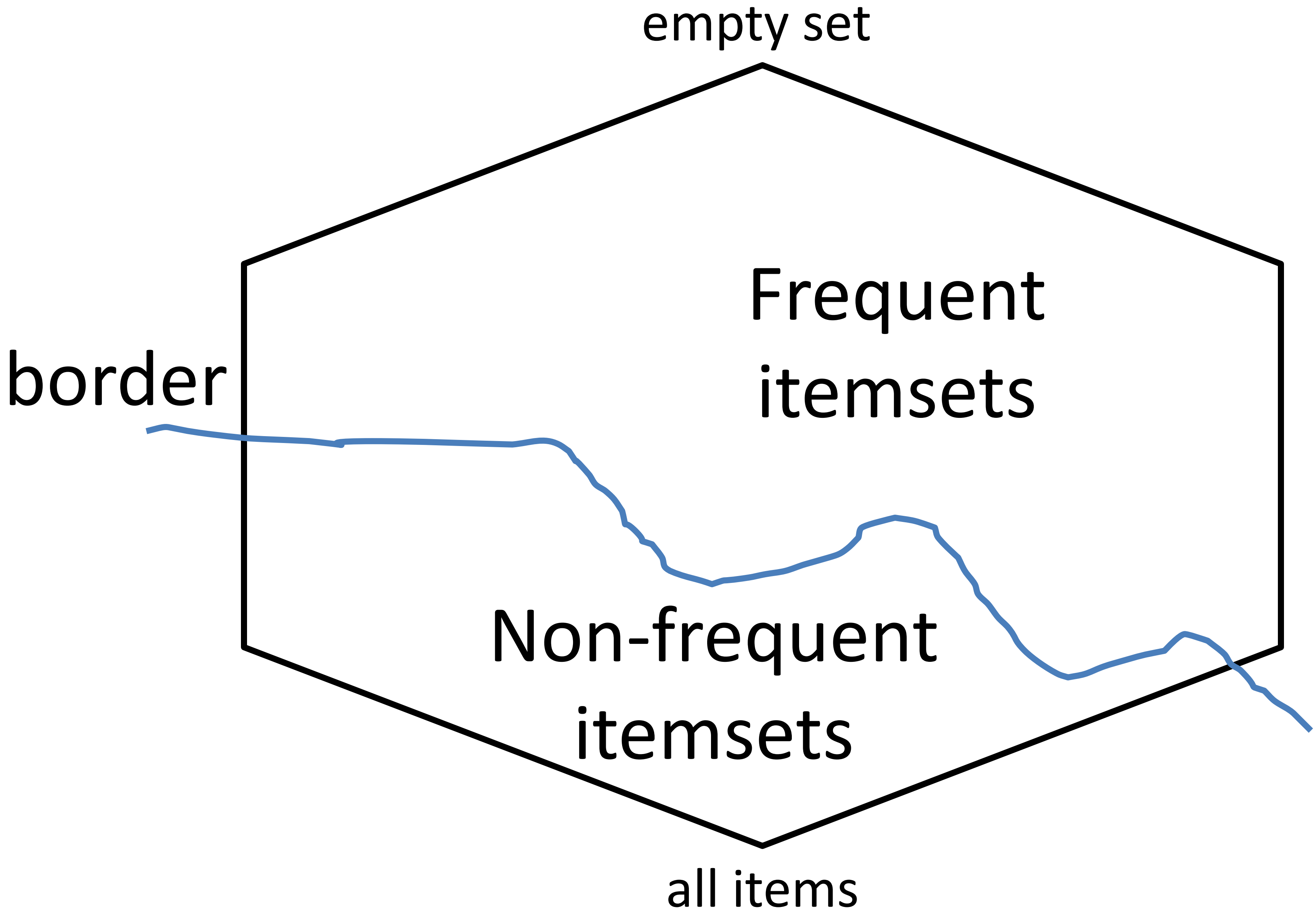
- There should be some more **condensed** way to describe the data

Frequent itemsets maybe too many to be helpful

- If there are many and large frequent itemsets enumerating all of them is costly.

- We may be interested in finding the ***boundary*** frequent patterns.

- **Question:** Is there a good definition of such boundary?


empty set
Frequent
itemsets
border
Non-frequent
itemsets
all items

Borders of frequent itemsets

- Itemset **X** is more ***specific*** than itemset **Y** if **X superset of Y** (notation: **Y<X**). Also, **Y** is more ***general*** than **X** (notation: **X>Y**)

- **The Border:** Let **S** be a collection of frequent itemsets and **P** the lattice of itemsets. The ***border*** **Bd(S)** of **S** consists of all itemsets **X** such that ***all more general itemsets*** than **X** are in **S** and ***no pattern more specific*** than **X** is in **S**.

$$Bd(S) = \left\{ X \in P \,\middle|\, \begin{array}{l} \text{for all } Y \in P \text{ with } Y \prec X \text{ then } Y \in P, \\ \text{and for all } W \in P \text{ with } X \prec W \text{ then } W \notin S \end{array} \right\}$$

Positive and negative border

- **Border**

$$Bd(S) = \left\{ X \in P \middle| \begin{array}{l} \text{for all } Y \in P \text{ with } Y \prec X \text{ then } Y \in S, \\ \text{and for all } W \in P \text{ with } X \prec W \text{ then } W \notin S \end{array} \right\}$$

- **Positive border:** Itemsets in the border that are also frequent (belong in **S**)

$$Bd^{+}(S) = \left\{ X \in S \middle| \text{for all } Y \in P \text{ with } X \prec Y \text{ then } Y \notin S \right\}$$

- **Negative border:** Itemsets in the border that are not frequent (do not belong in **S**)

$$Bd^{-}(S) = \left\{ X \in P \setminus S \middle| \text{for all } Y \in P \text{ with } Y \prec X \text{ then } Y \in S \right\}$$

Examples with borders

- Consider a set of items from the alphabet: **{A,B,C,D,E}** and the collection of frequent sets

$$\mathbf{S} = \{\{A\},\{B\},\{C\},\{E\},\{A,B\},\{A,C\},\{A,E\},\{C,E\},\{A,C,E\}\}$$

- The negative border of collection **S** is

$$\mathbf{Bd^-(S)} = \{\{D\},\{B,C\},\{B,E\}\}$$

- The positive border of collection **S** is

$$\mathbf{Bd^+(S)} = \{\{A,B\},\{A,C,E\}\}$$

Descriptive power of the borders

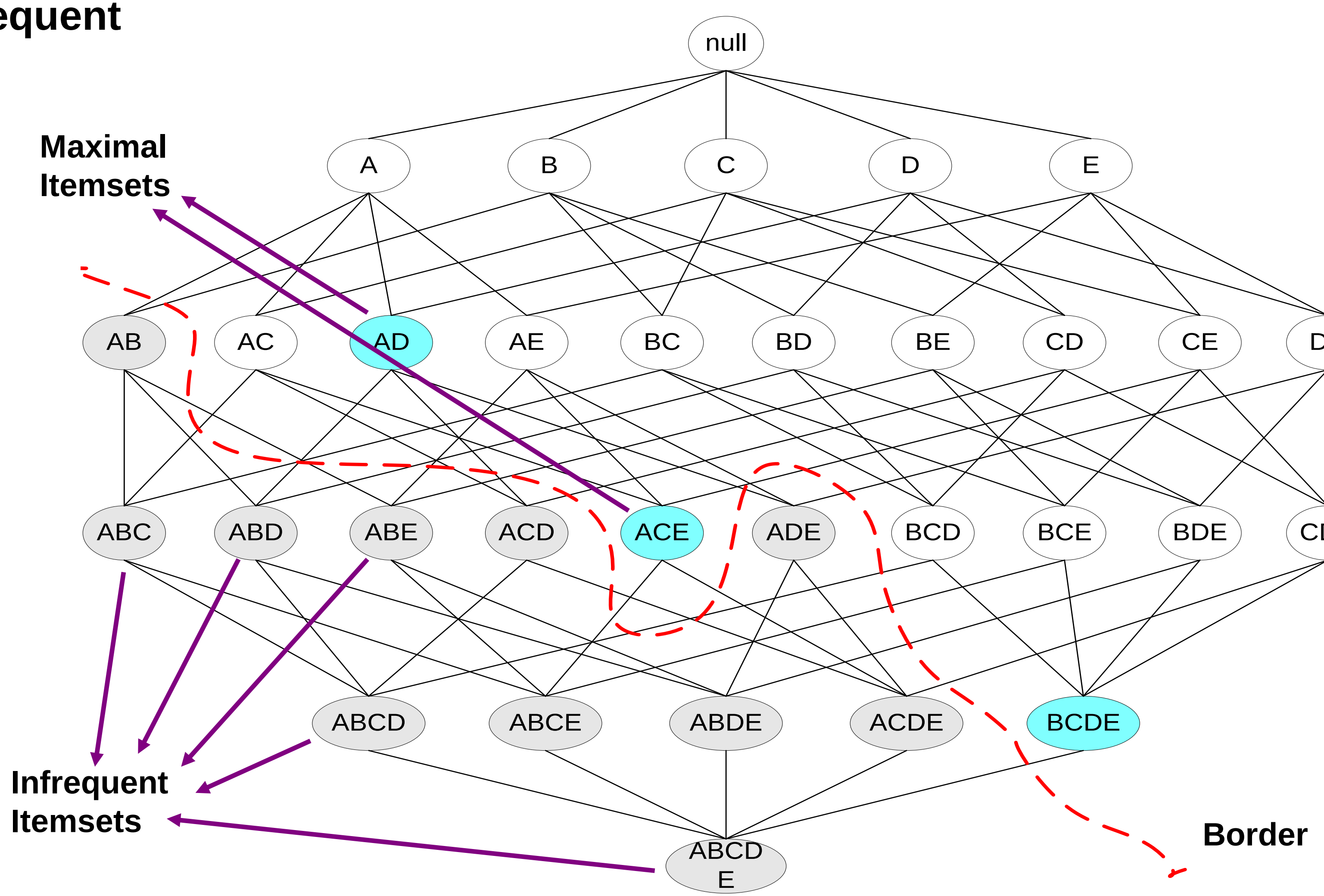
- **Claim:** A collection of frequent sets **S** can be ***fully described*** using only the positive border (**$Bd^+(S)$**) or only the negative border (**$Bd^-(S)$**).

Maximal patterns

Frequent patterns without proper frequent super pattern

Maximal Frequent Itemset

An itemset is maximal frequent if none of its immediate supersets is frequent



Maximal patterns

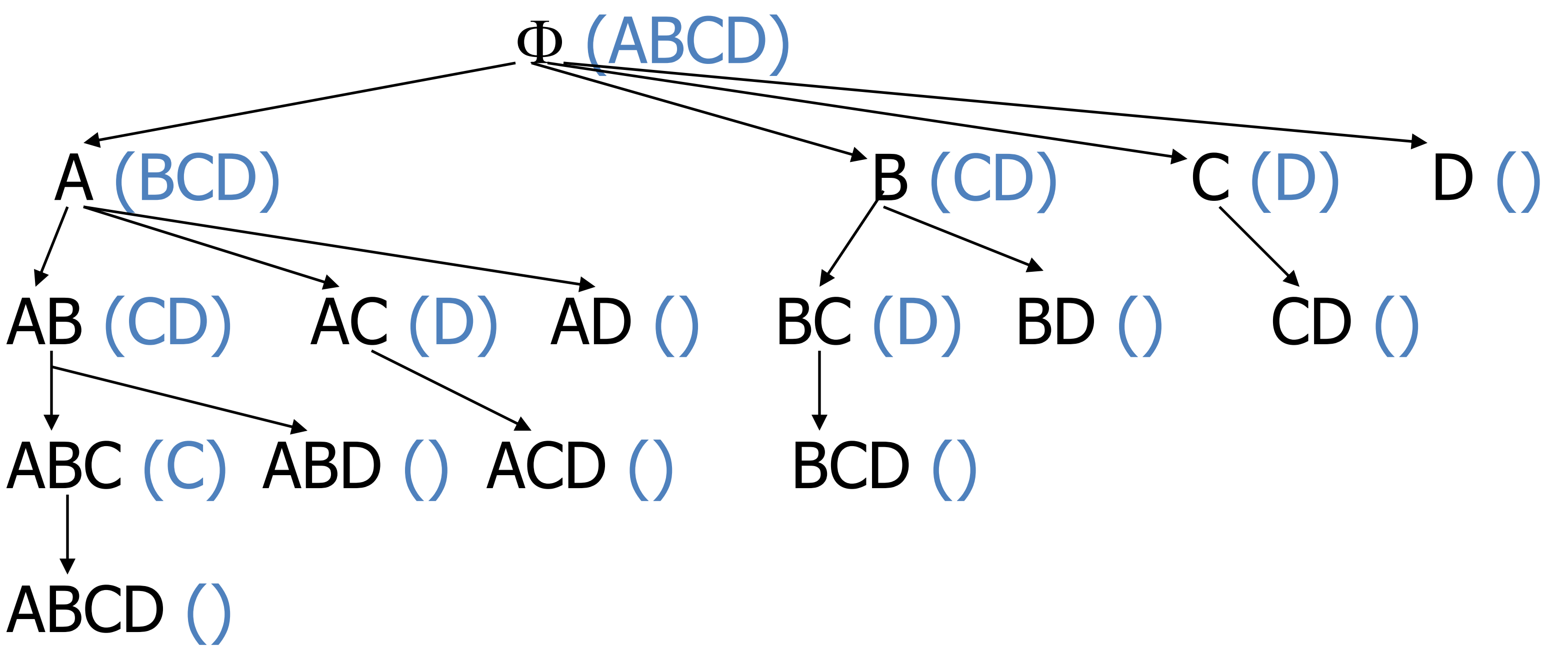
- The set of maximal patterns is the same as the positive border

- Descriptive power of maximal patterns:
  - Knowing the set of all maximal patterns allows us to reconstruct the set of all frequent itemsets!!

  - We can only reconstruct the set not the actual frequencies

MaxMiner: Mining Max-patterns

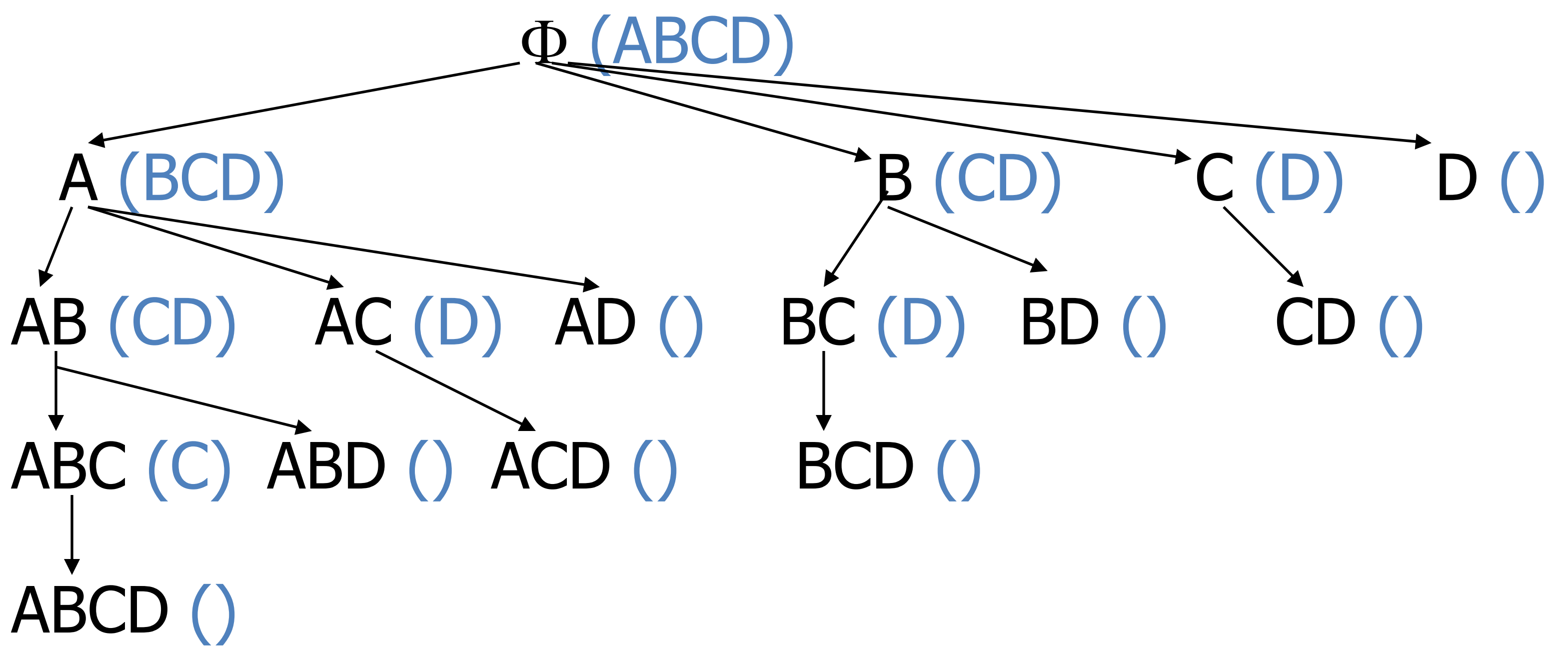
- **Idea:** generate the complete set-enumeration tree one level at a time, while prune if applicable.



Local Pruning Techniques (e.g. at node A)

Check the frequency of **ABCD** and **AB, AC, AD**.

- If **ABCD** is frequent, prune the whole sub-tree.
- If **AC** is NOT frequent, remove **C** from the parenthesis before expanding.

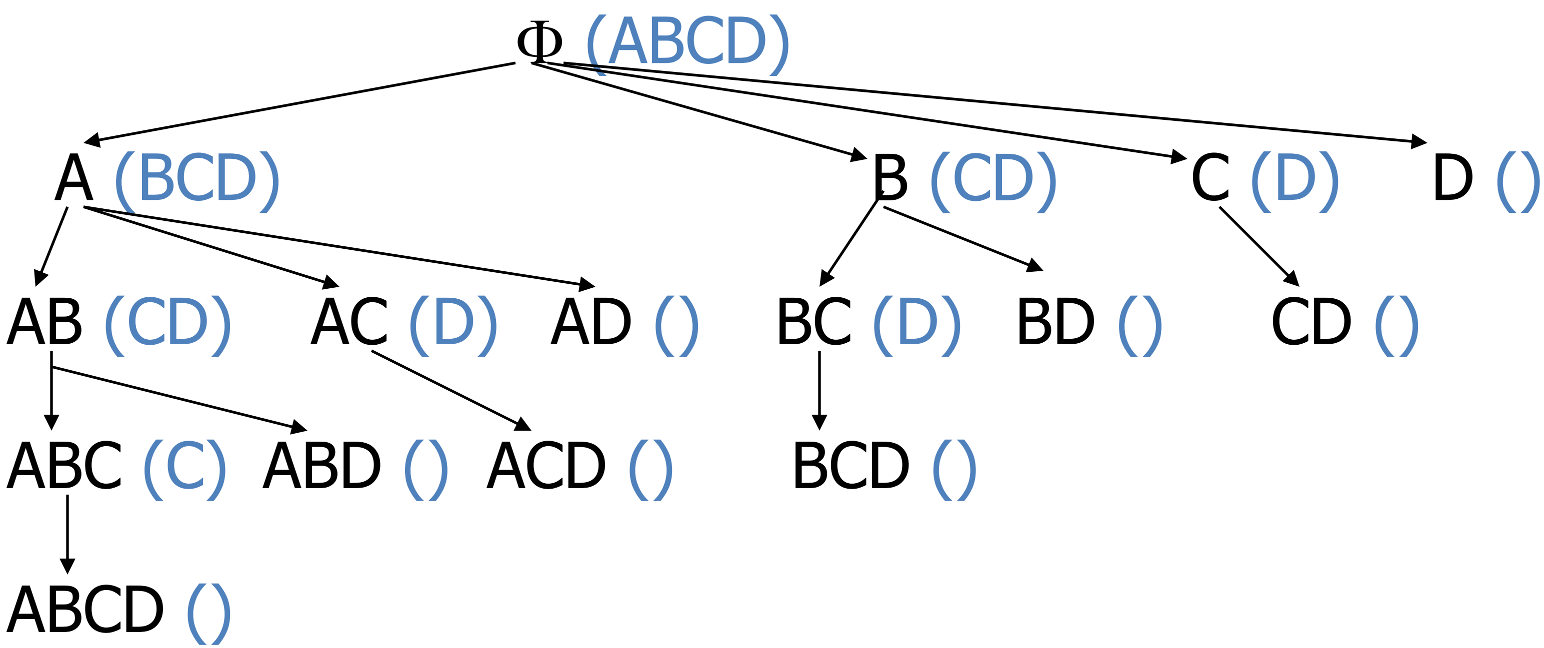


Algorithm MaxMiner

- Initially, generate one node **N=**$\boxed{\Phi\ (ABCD)}$, where **h(N)=Φ** and **t(N)={A,B,C,D}.**
- Consider expanding **N**,
  - If **h(N)∪t(N)** is frequent, do not expand **N**.
  - If for some **i∈t(N), h(N)∪{i}** is NOT frequent, remove **i** from **t(N)** before expanding **N**.
- Apply global pruning techniques...

Global Pruning Technique (across sub-trees)

- When a max pattern is identified **(e.g. ABCD)**, prune all nodes **(e.g. B, C** and **D)** where **h(N)∪t(N)** is a sub-set of it **(e.g. ABCD)**.



Closed patterns

- An itemset is closed if none of its immediate supersets has the same support as the itemset

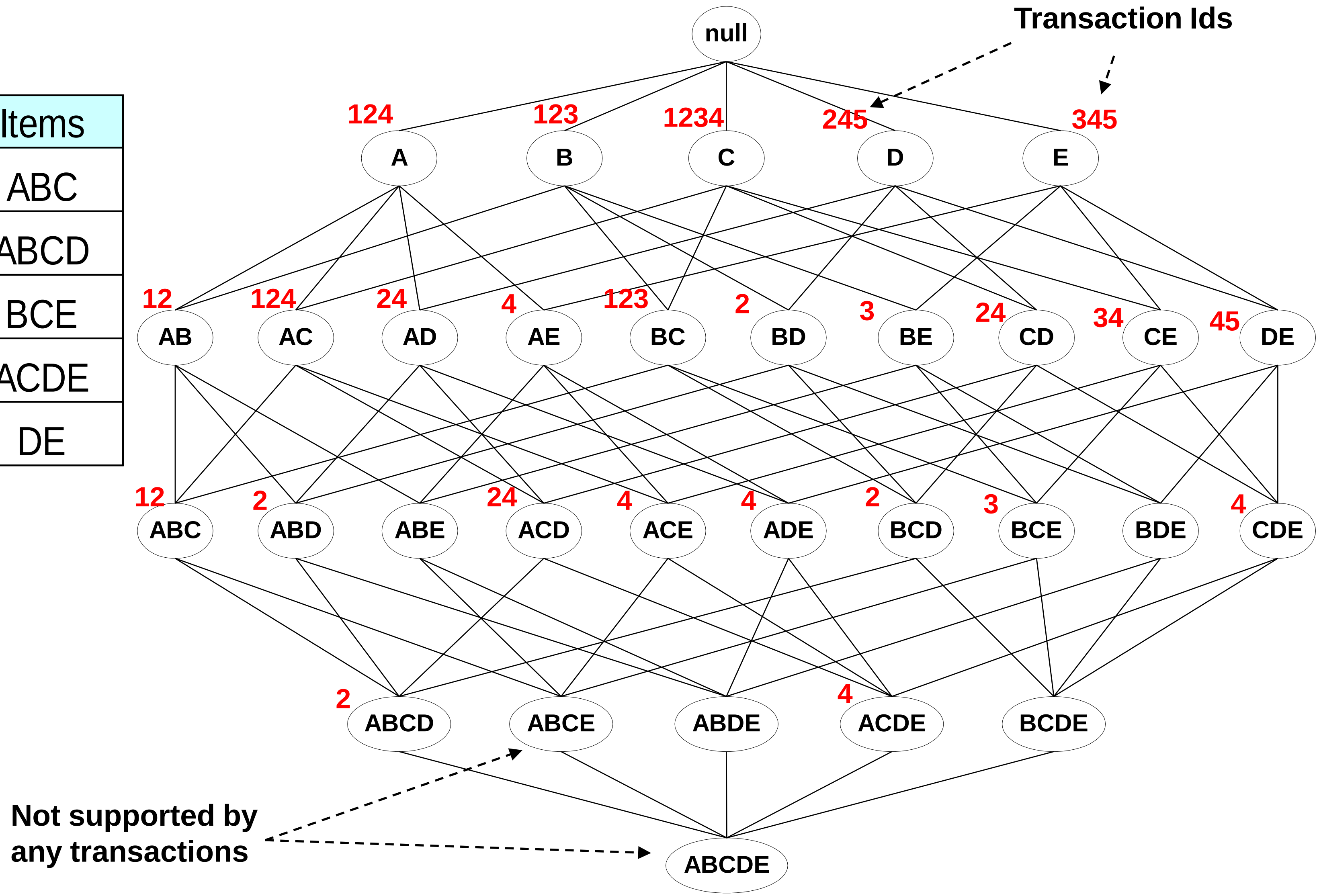
TID	Items
1	{A,B}
2	{B,C,D}
3	{A,B,C,D}
4	{A,B,D}
5	{A,B,C,D}

Itemset	Support
{A}	4
{B}	5
{C}	3
{D}	4
{A,B}	4
{A,C}	2
{A,D}	3
{B,C}	3
{B,D}	4
{C,D}	3

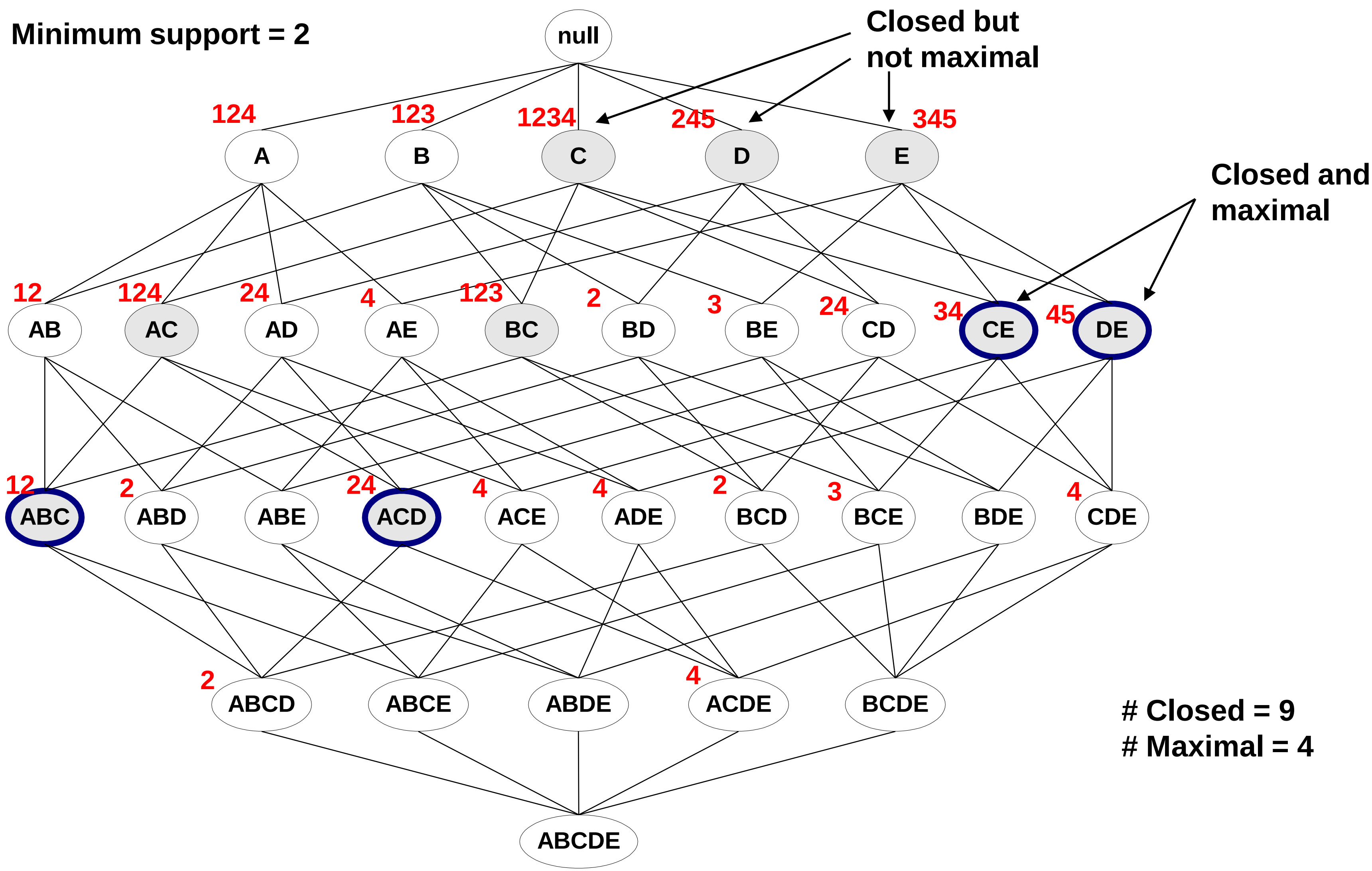
Itemset	Support
{A,B,C}	2
{A,B,D}	3
{A,C,D}	2
{B,C,D}	3
{A,B,C,D}	2

Maximal vs Closed Itemsets

TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE



Maximal vs Closed Frequent Itemsets



Why are closed patterns interesting?

- **s({A,B}) = s(A),** i.e., **conf({A}→{B}) = 1**
- We can infer that for every itemset **X ,**
  s(A U {X}) = s({A,B} U X)
- **No need to count the frequencies of sets X U {A,B} from the database!**
- If there are lots of rules with confidence **1**, then a significant amount of work can be saved
  - Very useful if there are strong correlations between the items and when the transactions in the database are similar

Why closed patterns are interesting?

- Closed patterns and their frequencies alone are sufficient representation for all the frequencies of all frequent patterns

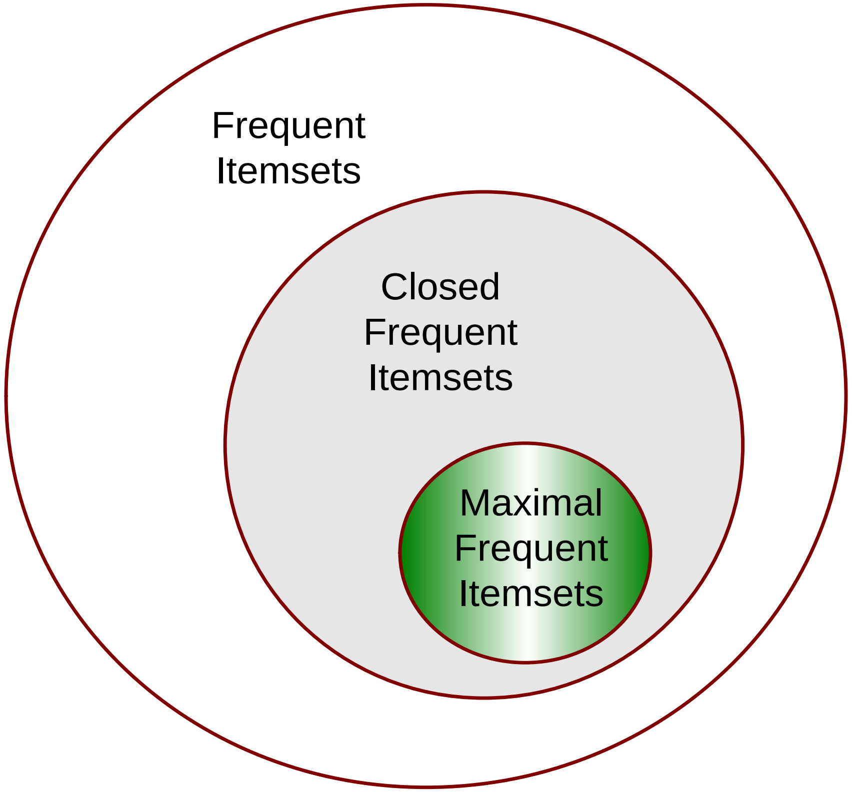
- **Proof:** Assume a frequent itemset **X**:
  - **X** is closed → **s(X)** is known
  - **X** is not closed →

    s(X) = max {s(Y) | Y is closed and X subset of Y}

Maximal vs Closed sets

- Knowing all maximal patterns (and their frequencies) allows us to reconstruct the set of frequent patterns

- Knowing all closed patterns and their frequencies allows us to reconstruct the set of all frequent patterns and their frequencies



A more algorithmic approach to reducing the collection of frequent itemsets

Prototype problems: Covering problems

- Setting:
  - Universe of N elements $U = \{U_1,\ldots,U_N\}$
  - A set of n sets $S = \{s_1,\ldots,s_n\}$
  - Find a collection C of sets in S (C subset of S) such that $\cup_{c \in C} c$ contains many elements from U
- Example:
  - U: set of documents in a collection
  - s_i: set of documents that contain term t_i
  - Find a collection of terms that cover most of the documents

Prototype covering problems

- **Set cover problem:** Find a small collection **C** of sets from **S** such that *all elements in the universe* **U** are covered by some set in **C**
- **Best collection problem:** find a collection **C** of **k** sets from **S** such that the collection covers as many elements from the universe **U** as possible
- Both problems are NP-hard
- Simple approximation algorithms with provable properties are available and very useful in practice

Set-cover problem

- Universe of $\mathbf{N}$ elements $\mathbf{U = \{U_1,\dots,U_N\}}$
- A set of $\mathbf{n}$ sets $\mathbf{S = \{s_1,\dots,s_n\}}$ such that $\mathbf{\cup_i s_i = U}$

- **Question:** Find the smallest number of sets from $\mathbf{S}$ to form collection $\mathbf{C}$ ($\mathbf{C}$ subset of $\mathbf{S}$) such that $\mathbf{\cup_{c \in C} c = U}$

- The set-cover problem is **NP-hard** (what does this mean?)

Trivial algorithm

- Try all subcollections of **S**
- Select the smallest one that covers all the elements in **U**
- The running time of the trivial algorithm is $O(2^{|S|}|U|)$
- This is way too slow

Greedy algorithm for set cover

- Select first the largest-cardinality set **s** from **S**
- Remove the elements from **s** from **U**
- Recompute the sizes of the remaining sets in **S**
- Go back to the first step

As an algorithm

- X = U
- C = {}
- **while** X is not empty **do**
  - For all $s \epsilon S$ let $a_s = |s \text{ intersection } X|$
  - Let s be such that a_s is *maximal*
  - $C = C \cup \{s\}$
  - $X = X \setminus s$

How can this go wrong?

- No global consideration of how good or bad a selected set is going to be

How good is the greedy algorithm?

- Consider a minimization problem
  - In our case we want to minimize the ***cardinality*** of set **C**
- Consider an instance **I**, and cost $a^*(I)$ of the optimal solution
  - $a^*(I)$: is the minimum number of sets in **C** that cover all elements in **U**
- Let **a(I)** be the cost of the approximate solution
  - **a(I)**: is the number of sets in **C** that are picked by the greedy algorithm
- An algorithm for a minimization problem has approximation factor **F** if for all instances **I** we have that

$$a(I) \leq F \times a^*(I)$$

- ***Can we prove any approximation bounds for the greedy algorithm for set cover ?***

How good is the greedy algorithm for set cover?

- ***(Trivial?) Observation***: The greedy algorithm for set cover has approximation factor **$F = s_{max}$**, where **s_{max}** is the set in **S** with the largest cardinality
- **Proof:**
  - $a^*(I) \geq N/|s_{max}|$ or $N \leq |s_{max}| a^*(I)$
  - $a(I) \leq N \leq |s_{max}| a^*(I)$

How good is the greedy algorithm for set cover? A tighter bound

- The greedy algorithm for set cover has approximation factor $\mathbf{F = O(\log |s_{max}|)}$

- **Proof**: (From CLR "Introduction to Algorithms")

Best-collection problem

- Universe of **N** elements $\mathbf{U = \{U_1,\ldots,U_N\}}$
- A set of **n** sets $\mathbf{S = \{s_1,\ldots,s_n\}}$ such that $\mathbf{\cup_i s_i = U}$

- **Question:** Find the a collection **C** consisting of **k** sets from **S** such that $\mathbf{f(C) = |\cup_{c \in C} c|}$ is ***maximized***

- The best-colection problem is NP-hard

- Simple approximation algorithm has approximation factor $\mathbf{F = (e-1)/e}$

Greedy approximation algorithm for the best-collection problem

- $C = \{\}$
- **for every** set s in S and ***not*** in C compute the gain of s:

$$g(s) = f(C \cup \{s\}) - f(C)$$

- Select the set s with the ***maximum*** gain
- $C = C \cup \{s\}$
- **Repeat until** C has k elements

Basic theorem

- The ***greedy*** algorithm for the best-collection problem has approximation factor **$F = (e-1)/e$**

- **C^* : optimal** collection of cardinality **k**
- **C :** collection output by the ***greedy*** algorithm
- **$f(C) \geq (e-1)/e \times f(C^*)$**

Submodular functions and the greedy algorithm

- A function **f** (defined on sets of some universe) is **submodular** if
  - ***for all*** sets **S**, **T** such that **S** is ***subset*** of **T** and **x** ***any*** element in the universe
  - **$f(S \cup \{x\}) - f(S) \geq f(T \cup \{x\}) - f(T)$**

- **Theorem:** For **all maximization** problems where the optimization function is **submodular**, the ***greedy*** algorithm has approximation factor

$$F = (e-1)/e$$

Again: Can you think of a more algorithmic approach to reducing the collection of frequent itemsets

Approximating a collection of frequent patterns

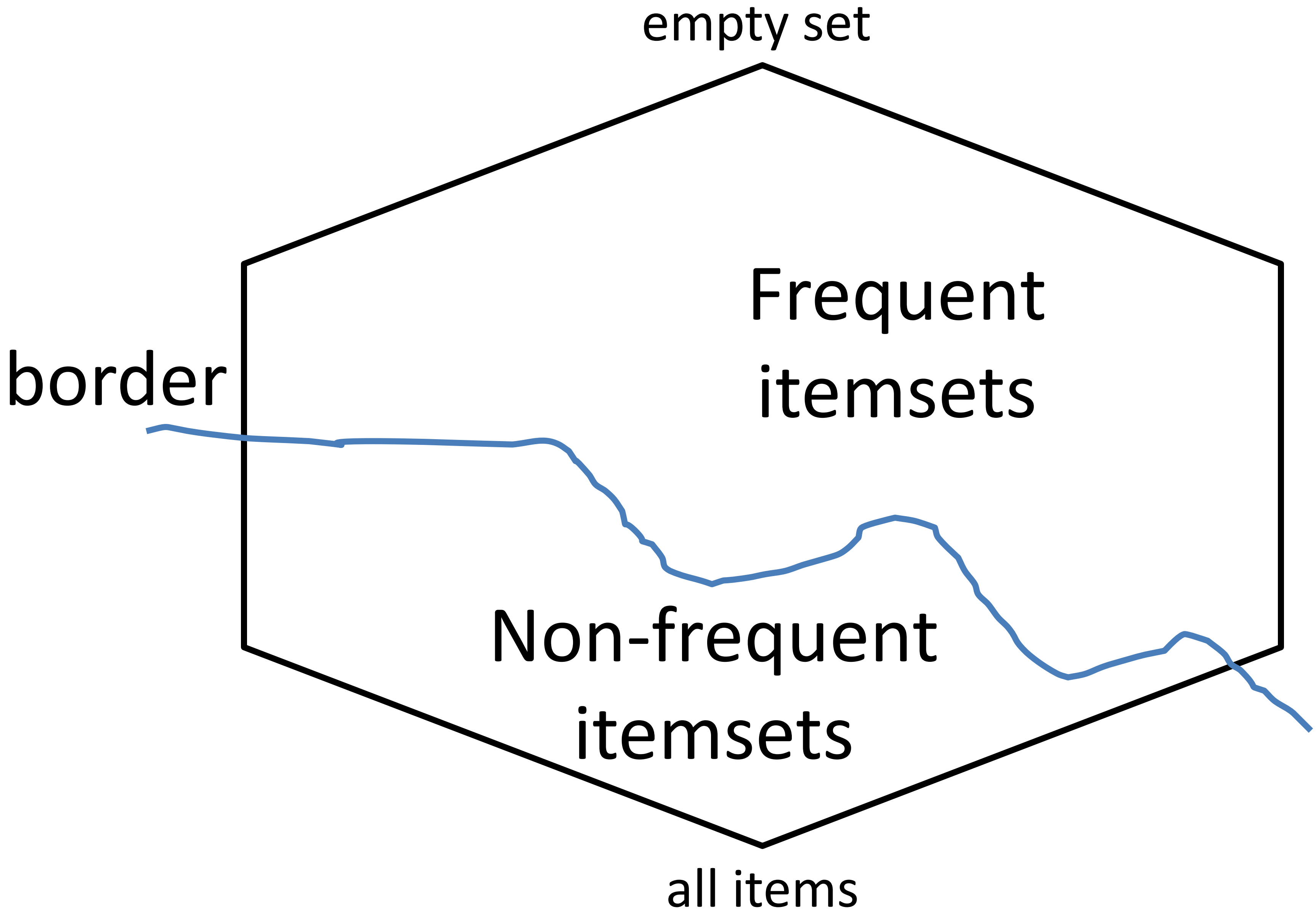
- Assume a collection of frequent patterns **S**

- Each pattern **X ϵ S** is ***described by the patterns that covers***
- **Cov(X) = { Y | Y ϵ S and Y subset of X}**

- **Problem:** Find **k** patterns from **S** to form set **C** such that

$$|\cup_{X \in C} \text{Cov}(X)|$$

is maximized



empty set
Frequent
itemsets
border
Non-frequent
itemsets
all items