## Anonymization and deanonymization of graphs

- Reference:
- Towards identity anonymization in social networks (by Kun Liu and Evimaria Terzi, SIGMOD 2008)


## Growing Privacy Concerns

- Person specific information is being routinely collected.
"Detailed information on an individual's credit, health, and financial status, on characteristic purchasing patterns, and on other personal preferences is routinely recorded and analyzed by a variety of governmental and commercial organizations."
- M. J. Cronin, "e-Privacy?" Hoover Digest, 2000.



## Proliferation of Graph Data



## Linked in

facebook
http://www.touchgraph.com/

## Privacy breaches on graph data

- Identity disclosure
- Identity of individuals associated with nodes is disclosed
- Link disclosure
- Relationships between individuals are disclosed
- Content disclosure
- Attribute data associated with a node is disclosed


## Identity anonymization on graphs

## Identity anonymization on graphs

- Question
- How to share a network in a manner that permits useful analysis without disclosing the identity of the individuals involved?


## Identity anonymization on graphs

- Question
- How to share a network in a manner that permits useful analysis without disclosing the identity of the individuals involved?


## Identity anonymization on graphs

- Question
- How to share a network in a manner that permits useful analysis without disclosing the identity of the individuals involved?
- Observations
- Simply removing the identifying information of the nodes before publishing the actual graph does not guarantee identity anonymization.
L. Backstrom, C. Dwork, and J. Kleinberg, "Wherefore art thou R3579X?: Anonymized social netwoks, hidden patterns, and structural steganography," In WWW 2007.
J. Kleinberg, "Challenges in Social Network Data: Processes, Privacy and Paradoxes, " KDD 2007 Keynote Talk.


## Identity anonymization on graphs

- Question
- How to share a network in a manner that permits useful analysis without disclosing the identity of the individuals involved?
- Observations
- Simply removing the identifying information of the nodes before publishing the actual graph does not guarantee identity anonymization.
L. Backstrom, C. Dwork, and J. Kleinberg, "Wherefore art thou R3579X?: Anonymized social netwoks, hidden patterns, and structural steganography," In WWW 2007.
J. Kleinberg, "Challenges in Social Network Data: Processes, Privacy and Paradoxes, " KDD 2007 Keynote Talk.


## Identity anonymization on graphs

- Question
- How to share a network in a manner that permits useful analysis without disclosing the identity of the individuals involved?
- Observations
- Simply removing the identifying information of the nodes before publishing the actual graph does not guarantee identity anonymization.
L. Backstrom, C. Dwork, and J. Kleinberg, "Wherefore art thou R3579X?: Anonymized social netwoks, hidden patterns, and structural steganography," In WWW 2007.
J. Kleinberg, "Challenges in Social Network Data: Processes, Privacy and Paradoxes, " KDD 2007 Keynote Talk.
- Can we borrow ideas from $k$-anonymity?


## What if you want to prevent the following from happening

- Assume that adversary A knows that B has 327 connections in a social network!
- If the graph is released by removing the identity of the nodes
- A can find all nodes that have degree 327
- If there is only one node with degree 327, A can identify this node as being B.


## Privacy model

[k-degree anonymity] A graph $G(V, E)$ is $k$-degree anonymous if every node in $V$ has the same degree as $k-1$ other nodes in $V$.


## Privacy model

[k-degree anonymity] A graph $G(V, E)$ is $k$-degree anonymous if every node in V has the same degree as $k-1$ other nodes in $V$.


## Privacy model

[k-degree anonymity] A graph $G(V, E)$ is $k$-degree anonymous if every node in V has the same degree as $k-1$ other nodes in $V$.


## Privacy model

[k-degree anonymity] A graph $G(V, E)$ is $k$-degree anonymous if every node in $V$ has the same degree as $k-1$ other nodes in $V$.

[Properties] It prevents the re-identification of individuals by adversaries with a priori knowledge of the degree of certain nodes.

## Outline

- Problem definition
- Algorithms
- Experimental results


## Problem Definition

Given a graph $\boldsymbol{G}(\mathbf{V}, \boldsymbol{E})$ and an integer $\boldsymbol{k}$, modify $\boldsymbol{G}$ via a minimal set of edge addition or deletion operations to construct a new graph $G^{\prime}\left(V^{\prime}, E^{\prime}\right)$ such that

1) $G^{\prime}$ is $k$-degree anonymous;
2) $V^{\prime}=V$;
3) The symmetric difference of $G$ and $G^{\prime}$ is as small as possible

- Symmetric difference between graphs $G(V, E)$ and $G^{\prime}\left(V, E^{\prime}\right)$ :

$$
\operatorname{SymDiff}\left(G^{\prime}, G\right)=\left(E^{\prime} \backslash E\right) \mathrm{U}\left(E \backslash E^{\prime}\right)
$$

## Outline

- Problem definition
- Algorithms
- Experimental results


## GraphAnonymization algorithm

Input: Graph $\mathcal{G}$ with degree sequence $d$, integer $\boldsymbol{k}$ Output: $k$-degree anonymous graph $G^{\prime}$
[Degree Sequence Anonymization]:
Contruct an anonymized degree sequence $d$ ' from the original degree sequence $d$
[Graph Construction]:
[Construct]: Given degree sequence $d^{\prime}$, construct a new graph $G^{0}\left(V, E^{0}\right)$ such that the degree sequence of $G^{0}$ is $d^{6}$ [Transform]: Transform $G^{0}\left(V, E^{0}\right)$ to $G^{\prime}\left(V, E^{\prime}\right)$ so that SymDiff( $G, G)$ is minimized.

## Degree-sequence anonymization

[ $k$-anonymous sequence] A sequence of integers $d$ is $k$-anonymous if every distinct element value in $\boldsymbol{d}$ appears at least $k$ times.

## Degree-sequence anonymization

[k-anonymous sequence] A sequence of integers $\boldsymbol{d}$ is $k$-anonymous if every distinct element value in $\boldsymbol{d}$ appears at least $\boldsymbol{k}$ times.
[100,100, 100, 98, 98,15,15,15]

## Degree-sequence anonymization

[ k -anonymous sequence] A sequence of integers $\boldsymbol{d}$ is $k$-anonymous if every distinct element value in $\boldsymbol{d}$ appears at least $\boldsymbol{k}$ times.
[100,100, 100, 98, 98,15,15,15]
[degree-sequence anonymization] Given degree sequence $\mathbf{d}$, and integer $k$, construct $k$-anonymous sequence $d^{\prime}$ such that $\left\|d^{\prime}-d\right\|$ is minimized

## Degree-sequence anonymization

[ $k$-anonymous sequence] A sequence of integers $\boldsymbol{d}$ is $k$-anonymous if every distinct element value in $\boldsymbol{d}$ appears at least $\boldsymbol{k}$ times.
[100,100, 100, 98, 98,15,15,15]
[degree-sequence anonymization] Given degree sequence $\mathbf{d}$, and integer $k$, construct $k$-anonymous sequence $d^{\prime}$ such that $\left\|d^{\prime}-d\right\|$ is minimized

Increase/decrease of degrees correspond to additions/deletions of edges

## Algorithm for degree-sequence anonymization



## Algorithm for degree-sequence anonymization



## Algorithm for degree-sequence anonymization

```
0 0 0 0
                k=4
    O O O O
```


## DP for degree-sequence anonymization

- $\boldsymbol{d}(1) \geq \boldsymbol{d}(2) \geq \ldots \geq \boldsymbol{d}(i) \geq \ldots \geq \boldsymbol{d}(n)$ : original degree sequence.
- $\boldsymbol{d}^{\prime}(1) \geq \boldsymbol{d}^{\prime}(2) \geq \ldots \geq \boldsymbol{d}^{\prime}(i) \geq \ldots \geq \boldsymbol{d}^{\prime}(n):$ k-anonymized degree sequence.
- $C(i, j)$ : anonymization cost when all nodes $i, i+1, \ldots, j$ are put in the same anonymized group, i.e.,

$$
C(i, j)=\sum_{i=i}^{j}\left(d(i)-d^{*}\right)
$$

- DA(1, $n$ ) : the optimal degree-sequence anonymization cost
- Dynamic Programming with $O\left(n^{2}\right)$

$$
D A(1, i)=\min _{k \leq t \leq i-k}\{D A(1, t)+C(t+1, i)\}
$$

- Dynamic Programming with O(nk)

$$
D A(1, i)=\min _{\max \{\{, i-2 k+1\}\} t \leq i-k}\{D A(1, t)+C(t+1, i)\}
$$

- Dynamic Programming can be done in $\mathrm{O}(\mathrm{n})$ with some additional bookkeeping


## GraphAnonymization algorithm

Input: Graph $G$ with degree sequence $d$, integer $\boldsymbol{k}$ Output: $k$-degree anonymous graph $G^{\prime}$
[Degree Sequence Anonymization]:

- Contruct an anonymized degree sequence d'from the original degree sequence d
[Graph Construction]:
[Construct]: Given degree sequence $d^{\prime}$, construct a new
graph $G^{0}\left(V, E^{0}\right)$ such that the degree sequence of $G^{0}$ is $d^{6}$ [Transform]: Transform $G^{0}\left(V, E^{0}\right)$ to $G^{\prime}\left(V, E^{\prime}\right)$ so that
SymDiff(G',G) is minimized.


## Are all degree sequences realizable?

- A degree sequence $d$ is realizable if there exists a simple undirected graph with nodes having degree sequence d.
- Not all vectors of integers are realizable degree sequences
- $d=\{4,2,2,2,1\}$ ?
- How can we decide?


## Realizability of degree sequences

[Erdös and Gallai] A degree sequence $\boldsymbol{d}$ with $\boldsymbol{d}(1) \geq \boldsymbol{d}(2) \geq \ldots \geq \boldsymbol{d}(i) \geq \ldots \geq$ $\boldsymbol{d}(n)$ and $\Sigma \boldsymbol{d}(i)$ even, is realizable if and only if

$$
\sum_{i=1}^{l} \boldsymbol{d}(i) \leq l(l-1)+\sum_{i=l+1}^{n} \min \{l, \boldsymbol{d}(i)\}, \text { for every } 1 \leq l \leq n-1
$$

## Realizability of degree sequences

[Erdös and Gallai] A degree sequence $\boldsymbol{d}$ with $\boldsymbol{d}(1) \geq \boldsymbol{d}(2) \geq \ldots \geq \boldsymbol{d}(i) \geq \ldots \geq$ $\boldsymbol{d}(n)$ and $\Sigma \boldsymbol{d}(i)$ even, is realizable if and only if

$$
\sum_{i=1}^{l} d(i) \leq l(l-1)+\sum_{i=l+1}^{n} \min \{l, d(i)\}, \text { for every } 1 \leq l \leq n-1
$$

Input: Degree sequence $d^{\prime}$
Output: Graph $G^{0}\left(V, E^{0}\right)$ with degree sequence $d^{\prime}$ or $N O!$
$\rightarrow$ If the degree sequence $d^{\prime}$ is NOT realizable?
-Convert it into a realizable and $\boldsymbol{k}$-anonymous degree sequence

## GraphAnonymization algorithm

Input: Graph $G$ with degree sequence $d$, integer $\boldsymbol{k}$ Output: $k$-degree anonymous graph $G^{\prime}$
[Degree Sequence Anonymization]:
Contruct an anonymized degree sequence d' from the original degree sequence d
[Graph Construction]:
[Construct]: Given degree sequence $d^{\prime \prime}$, construct a new graph $G^{0}\left(V, E^{0}\right)$ such that the degree sequence of $G^{0}$ is $d^{6}$ [Transform]: Transform $G^{0}\left(V, E^{0}\right)$ to $G^{\prime}\left(V, E^{\prime}\right)$ so that SymDiff( $G^{\prime}, G$ ) is minimized.

## Graph-transformation algorithm

- GreedySwap transforms $G^{0}=\left(V, E^{0}\right)$ into $G^{\prime}\left(V, E^{\prime}\right)$ with the same degree

- GreedySwap is a greedy heuristic with several iterations.
- At each step, GreedySwap swaps a pair of edges to make the graph more similar to the original graph G, while leaving the nodes' degrees intact.


## Valid swappable pairs of edges



## Valid swappable pairs of edges



A swap is valid if the resulting graph is simple

## GreedySwap algorithm

Input: A pliable graph $G^{0}\left(V, E^{0}\right)$, fixed graph $G(V, E)$
Output: Graph $G^{\prime}\left(V, E^{\prime}\right)$ with the same degree sequence as $G^{0}\left(V, E^{0}\right)$
$\mathrm{i}=0$
Repeat
find the valid swap in $G^{i}$ that most reduces its symmetric difference with $\boldsymbol{G}$, and form graph $\mathbf{G}^{i+1}$
i++

## Outline

- Problem definition
- Algorithms
- Experimental results


## Experiments

- Datasets: Co-authors, Enron emails, powergrid, ErdosRenyi, small-world and power-law graphs
- Goal: degree-anonymization does not destroy the structure of the graph
- Average path length
- Clustering coefficient
- Exponent of power-law distribution


## Experiments: Clustering coefficient and Avg Path Length

- Co-author dataset
- APL and CC do not change dramatically even for large values of $\boldsymbol{k}$




## Experiments: Edge intersections

Edge intersection achieved by the GreedySwap algorithm for different datasets.

Parenthesis value indicates
the original value of edge intersection

```
Synthetic datasets
Small world graphs* 0.99(0.01)
Random graphs 0.99(0.01)
Power law graphs** 0.93(0.04)
Real datasets
Enron 0.95(0.16)
Powergrid 0.97(0.01)
Co-authors 0.91(0.01)
```


## Experiments: Exponent of power law distributions

| Original | 2.07 |  |
| :--- | :--- | :--- |
| $k=10$ | 2.45 | Co-author dataset |
| $k=15$ | 2.33 |  |
| $k=20$ | 2.28 | Exponent of the power- <br> law distribution as a <br> k=25 |
| $k=50$ | 2.25 |  |
| $k=100$ | 2.05 |  |
| function of $k$ |  |  |

## Conclusions

- Problem and algorithmic aspects of degreeanonymization on graphs.
- Degree-anonymity does not destroy the graph structure in practice


## Inverse anonymization problems

- Given two social networks that share a large portion of their nodes, can you map the nodes of the one network to the other?
- Examples: Twitter and FB. Linkedln and FB etc.


## Questions?

## k-anonvmity on tabular data

[k-Anonymity*] A dataset is $k$-anonymous if every record is indistinguishable from at least ( $k-1$ ) other records.
[Algorithms] Replace specific values with more general, but

* P. Samarati and L. Sweeney, "Generalizing data to provide anonymity when disclosing information," PODS 1998


## k-anonvmity on tabular data

[k-Anonymity*] A dataset is $k$-anonymous if every record is indistinguishable from at least ( $k-1$ ) other records.
[Algorithms] Replace specific values with more general, but

|  | $\mathrm{A}_{1}$ | $\mathrm{A}_{2}$ | $\mathrm{A}_{3}$ | $\mathrm{A}_{4}$ | $\mathrm{A}_{5}$ | $\mathrm{A}_{6}$ | $\mathrm{A}_{7}$ | $\mathrm{A}_{8}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{t}_{1}$ | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| $\mathrm{t}_{2}$ | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| $\mathrm{t}_{3}$ | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 |
| $\mathrm{t}_{4}$ | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 |
| $\mathrm{t}_{5}$ | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| $\mathrm{t}_{6}$ | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |

* P. Samarati and L. Sweeney, "Generalizing data to provide anonymity when disclosing information," PODS 1998


## k-anonvmity on tabular data

[k-Anonymity*] A dataset is $k$-anonymous if every record is indistinguishable from at least ( $k-1$ ) other records.
[Algorithms] Replace specific values with more general, but

|  | $\mathrm{A}_{1}$ | $\mathrm{A}_{2}$ | $\mathrm{A}_{3}$ | $\mathrm{A}_{4}$ | $\mathrm{A}_{5}$ | $\mathrm{A}_{6}$ | $\mathrm{A}_{7}$ | $\mathrm{A}_{8}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{t}_{1}$ | 1 | * | * | 0 | 1 | * | 1 | 0 |
| $\mathrm{t}_{2}$ | 1 | * | * | 0 | 1 | * | 1 | 0 |
| $\mathrm{t}_{3}$ | 1 | * | * | 0 | 1 | * | 1 | 0 |
| $\mathrm{t}_{4}$ | * | 1 | 1 | 1 | * | 0 | 0 | * |
| $\mathrm{t}_{5}$ | * | 1 | 1 | 1 | * | 0 | 0 | * |
| $\mathrm{t}_{6}$ | * | 1 | 1 | 1 | * | 0 | 0 | * |

* P. Samarati and L. Sweeney, "Generalizing data to provide anonymity when disclosing information," PODS 1998


## Anonymizing adjacency matrices

## Anonymizing adjacency matrices

All graphs are unweighted and undirected

## Anonymizing adjacency matrices

Original
Graph

|  | A | B | C | D | E | A (2) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 0 | 1 | 0 | 0 | 1 | $B$ (1) |
| B | 1 | 0 | 0 | 0 | 0 | , |
| C | 0 | 0 | 0 | 1 | 0 |  |
| D | 0 | 0 | 1 | 0 | 0 |  |
| E | 1 | 0 | 0 | 0 | 0 | (1) |

## All graphs are unweighted and undirected

## Anonymizing adjacency matrices



## Experiments: Degree-sequence - an@nymaizati BostoN

- Degree sequences do not change dramatically even for large values of $\boldsymbol{k}$

