Covering problems



Prototype problems: Covering problems

- Setting:
 - Universe of N elements $U = \{U_1, ..., U_N\}$
 - A set of n sets $S = \{s_1, \dots, s_n\}$
 - Find a collection C of sets in S (C subset of S) such that U_{c∈C}c contains many elements from U
- Example:
 - U: set of documents in a collection
 - s_i: set of documents that contain term t_i
 - Find a collection of terms that cover most of the documents



Prototype covering problems

- Set cover problem: Find a small collection C of sets from S such that all elements in the universe U are covered by some set in C
- Best collection problem: find a collection C of k sets from S such that the collection covers as many elements from the universe U as possible
- Both problems are NP-hard
- Simple approximation algorithms with provable properties are available and very useful in practice



Set-cover problem

- Universe of N elements $U = \{U_1, ..., U_N\}$
- A set of **n** sets $S = \{s_1, ..., s_n\}$ such that $U_i s_i = U$
- Question: Find the smallest number of sets from S to form collection C (C subset of S) such that U_{c∈C}C=U
- The set-cover problem is NP-hard (what does this mean?)





Try all subcollections of S



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- Try all subcollections of S
- Select the smallest one that covers all the elements in $\ensuremath{\textbf{U}}$



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- Try all subcollections of S
- Select the smallest one that covers all the elements in $\ensuremath{\textbf{U}}$
- The running time of the trivial algorithm is O(2^{|S|}|U|)
- This is way too slow



Greedy algorithm for set cover

- Select first the largest-cardinality set s from S
- Remove the elements from s from U
- Recompute the sizes of the remaining sets in S
- Go back to the first step



As an algorithm

- X = U
- **C** = {}
- while X is not empty do
 - For all scS let a_s=|s intersection X|
 - Let **s** be such that a_s is maximal
 - C = C U {s}
 - $X = X \setminus s$



How can this go wrong?

 No global consideration of how good or bad a selected set is going to be



How good is the greedy algorithm?



How good is the greedy algorithm?

- Consider a minimization problem
 - In our case we want to minimize the cardinality of set C
- Consider an instance I, and cost a*(I) of the optimal solution
 - a*(I): is the minimum number of sets in C that cover all elements in U
- Let **a(I)** be the cost of the approximate solution
 - **a(I)**: is the number of sets in **C** that are picked by the greedy algorithm
- An algorithm for a minimization problem has approximation factor F if for all instances I we have that

 $a(I) \leq F \times a^*(I)$

 Can we prove any approximation bounds for the greedy algorithm for set cover ?



How good is the greedy algorithm for set cover?



How good is the greedy algorithm for set cover?

 (Trivial?) Observation: The greedy algorithm for set cover has approximation factor F = s_{max}, where s_{max} is the set in S with the largest cardinality



How good is the greedy algorithm for set cover?

- (Trivial?) Observation: The greedy algorithm for set cover has approximation factor F = s_{max}, where s_{max} is the set in S with the largest cardinality
- Proof:
 - $a^*(I) \ge N/|s_{max}|$ or $N \le |s_{max}|a^*(I)$
 - $a(I) \le N \le |s_{max}|a^*(I)$



How good is the greedy algorithm for set cover? A tighter bound

- The greedy algorithm for set cover has approximation factor $F = O(\log |s_{max}|)$
- Proof: (From CLR "Introduction to Algorithms")



Best-collection problem

- Universe of N elements $U = \{U_1, ..., U_N\}$
- A set of **n** sets $S = \{s_1, \dots, s_n\}$ such that $U_i s_i = U$
- Question: Find the a collection C consisting of k sets from S such that f (C) = |U_{c∈C}c| is maximized
- The best-colection problem is NP-hard
- Simple approximation algorithm has approximation factor F = (e-1)/e



Greedy approximation algorithm for the best-collection problem

• C = {}

 for every set s in S and not in C compute the gain of s:

 $g(s) = f(C \cup \{s\}) - f(C)$

- Select the set s with the maximum gain
- C = C U {s}
- Repeat until C has k elements



Basic theorem

- The **greedy** algorithm for the best-collection problem has approximation factor F = (e-1)/e
- C* : optimal collection of cardinality k
- C : collection output by the greedy algorithm
- $f(C) \ge (e-1)/e \times f(C^*)$







- Finding team of experts in a social network
 - Figure 1. Lappas, K. Liu, E. Terzi KDD 2009]



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- Finding team of experts in a social network
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Expertise location in social networks: "How do I find an effective team of people that collectively can perform a given task"



Setting

- Experts (defining the set V, with |V|=n):
 - Every expert i is associated with a set of skills X_i
- Tasks
 - Every task T is associated with a set of skills (T) required for performing the task
- A social network of experts (G=(V,E))
 - Edges between experts indicate ability to work well together

	Team Formation
Experts' skills	Known
Participation of experts in teams	Unknown
Network structure	Known





Given a task and a set of experts organized in a network find the subset of experts that can effectively perform the task



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- Task: set of required skills
- Expert: has a set of skills
- Network: represents strength of relationships



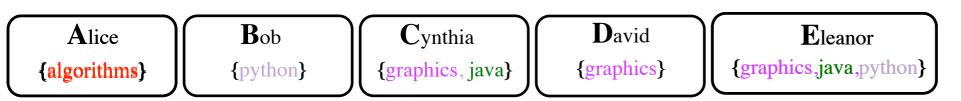
Expertise networks

- Collaboration networks (e.g., DBLP graph, coauthor networks)
- Organizational structure of companies
- LinkedIn
- Geographical (map) of experts



What makes a team effective for a task?

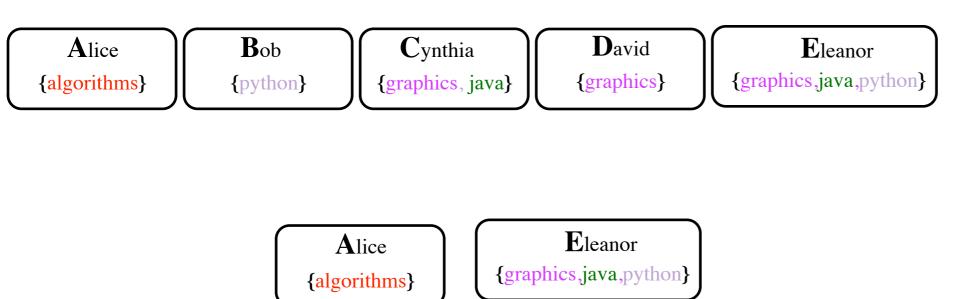
T = {algorithms, java, graphics, python}





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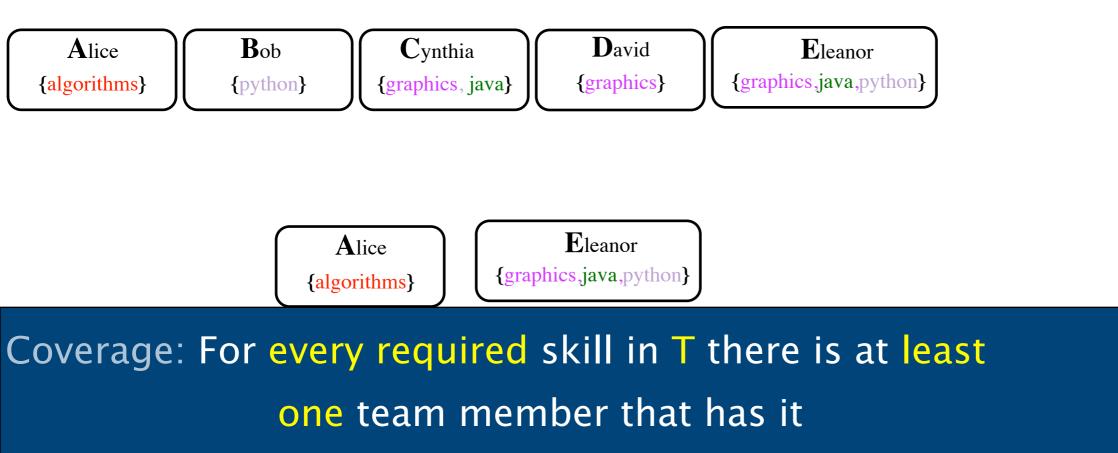
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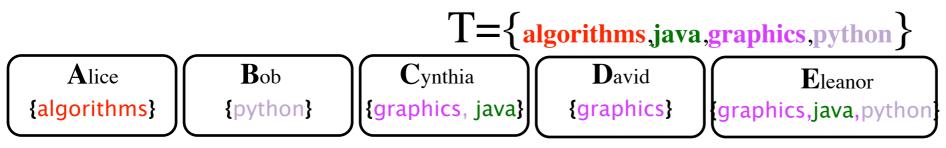
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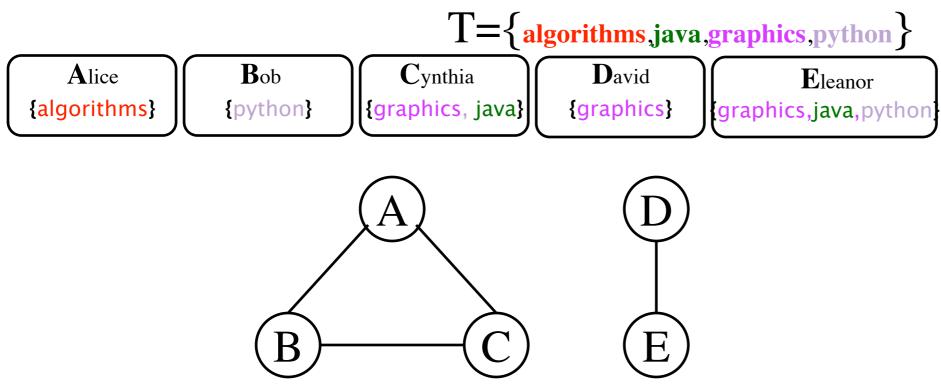


Given a task and a set of individuals, find the subset (team) of individuals that can perform the given task.

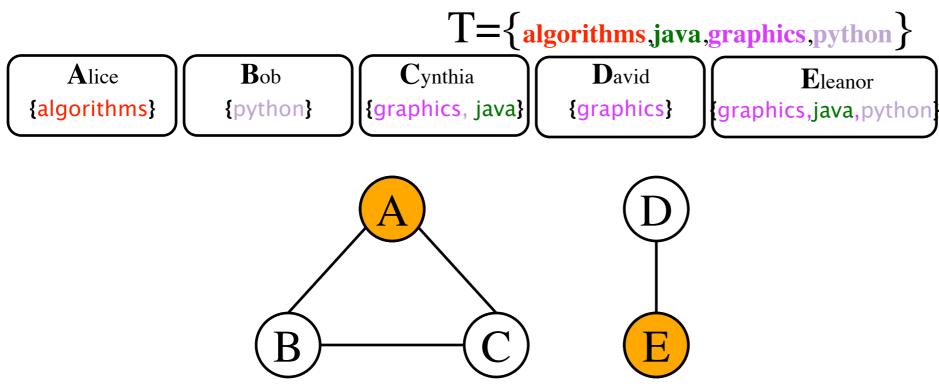




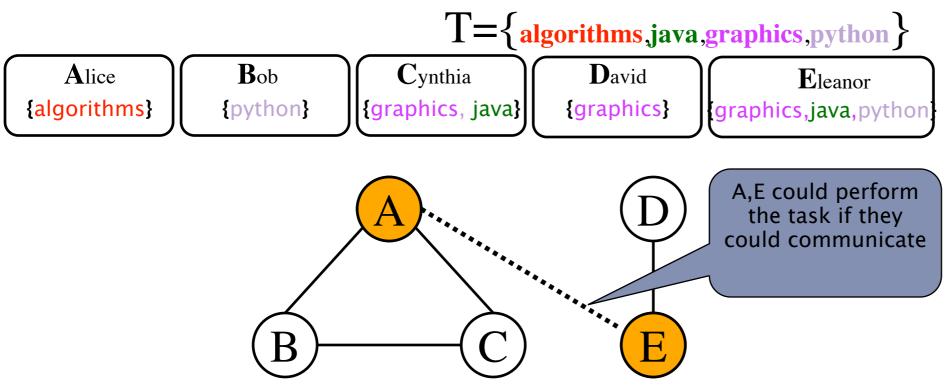




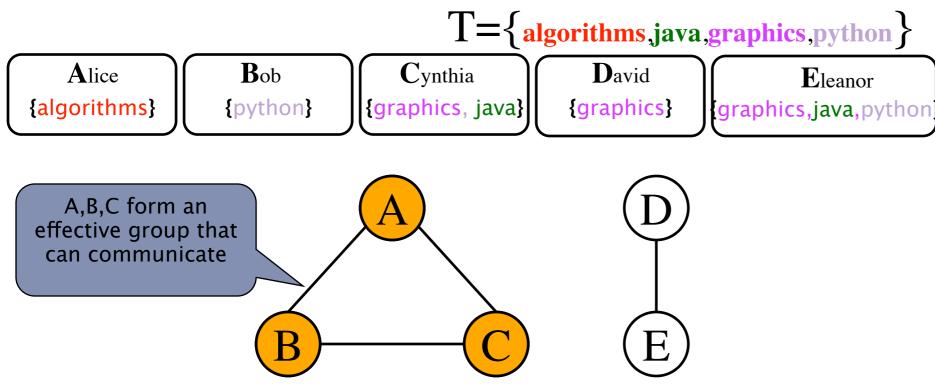




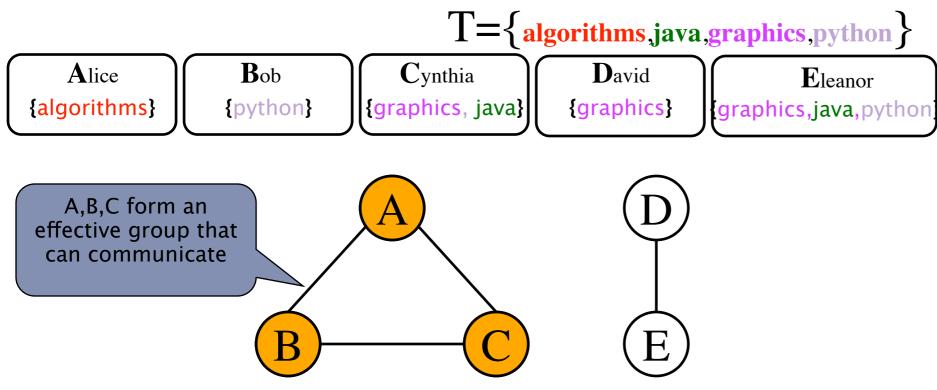












Communication: the members of the team must be able to efficiently communicate and work together





Given a task and a social network of individuals, find the subset (team) of individuals that can effectively perform the given task.



Given a task and a social network of individuals, find the subset (team) of individuals that can effectively perform the given task.



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Given a task and a social network of individuals, find the subset (team) of individuals that can effectively perform the given task.

Thesis: Good teams are teams that have the necessary skills and can also communicate effectively



Diameter of the subgraph defined by the group members



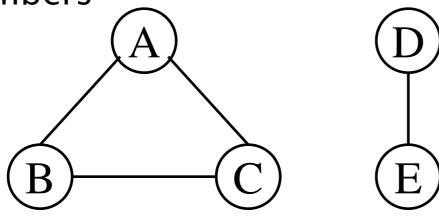
The longest shortest path between any two nodes in the subgraph

Diameter of the subgraph defined by the group members



The longest shortest path between any two nodes in the subgraph

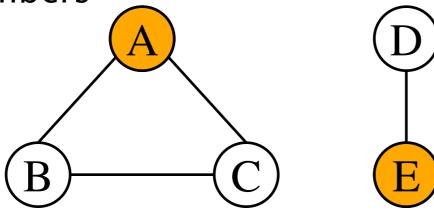
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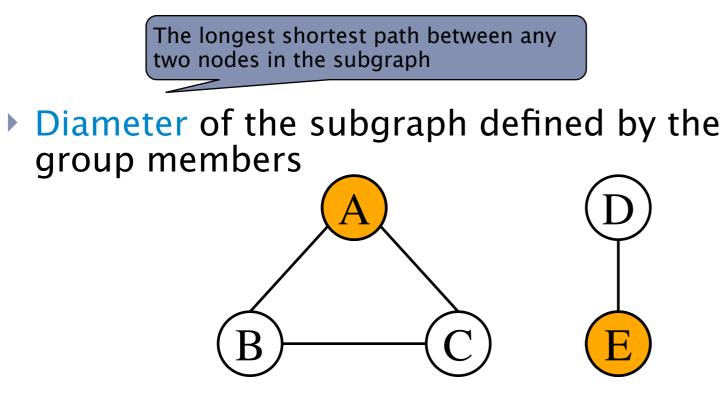


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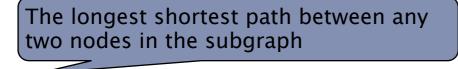




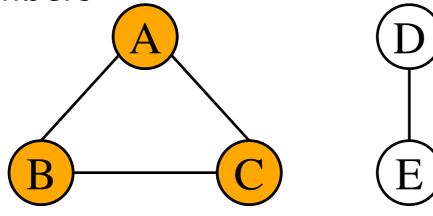


diameter = infty

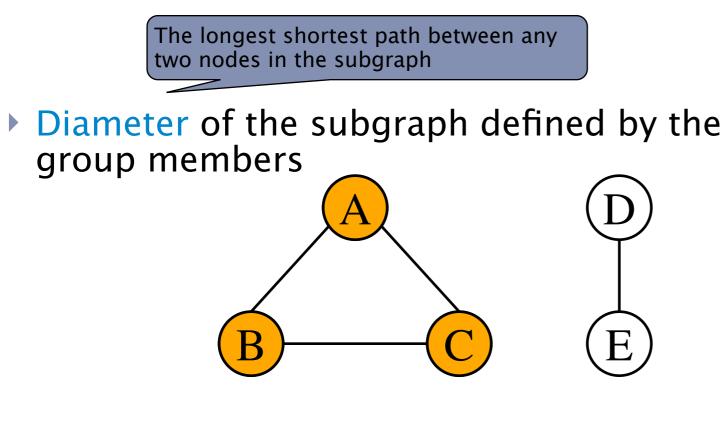




Diameter of the subgraph defined by the group members







diameter
$$= 1$$



MST (Minimum spanning tree) of the subgraph defined by the group members



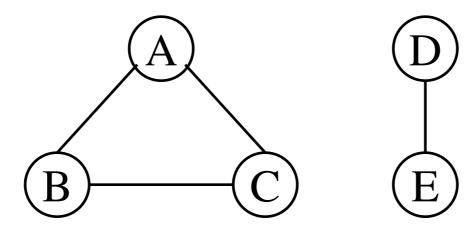
The total weight of the edges of a tree that spans all the team nodes

 MST (Minimum spanning tree) of the subgraph defined by the group members



The total weight of the edges of a tree that spans all the team nodes

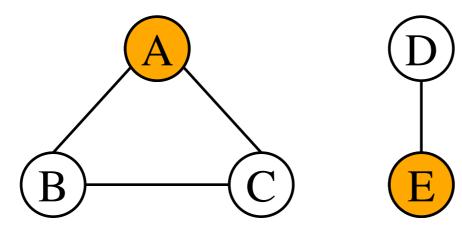
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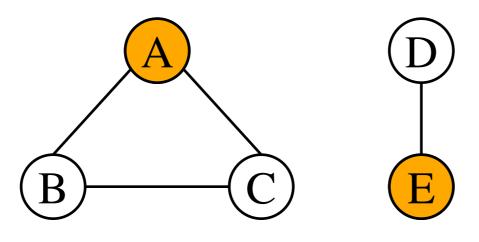
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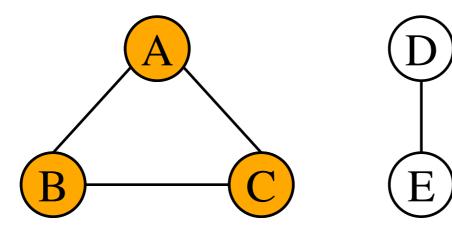


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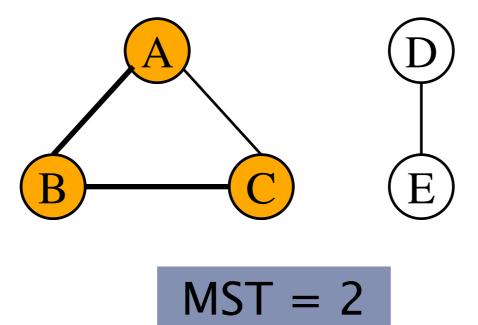
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The total weight of the edges of a tree that spans all the team nodes

 MST (Minimum spanning tree) of the subgraph defined by the group members







Given a task and a social network G of experts, find the subset (team) of experts that can perform the given task and they define a subgraph in G with the minimum diameter.

Problem is NP-hard



Algorithms for minimizing the diameter : RarestFirst

- Find Rarest skill α_{rare} required for a task
- S_{rare} group of people that have α_{rare}
- Evaluate star graphs, centered at individuals from S_{rare}
- Report cheapest star

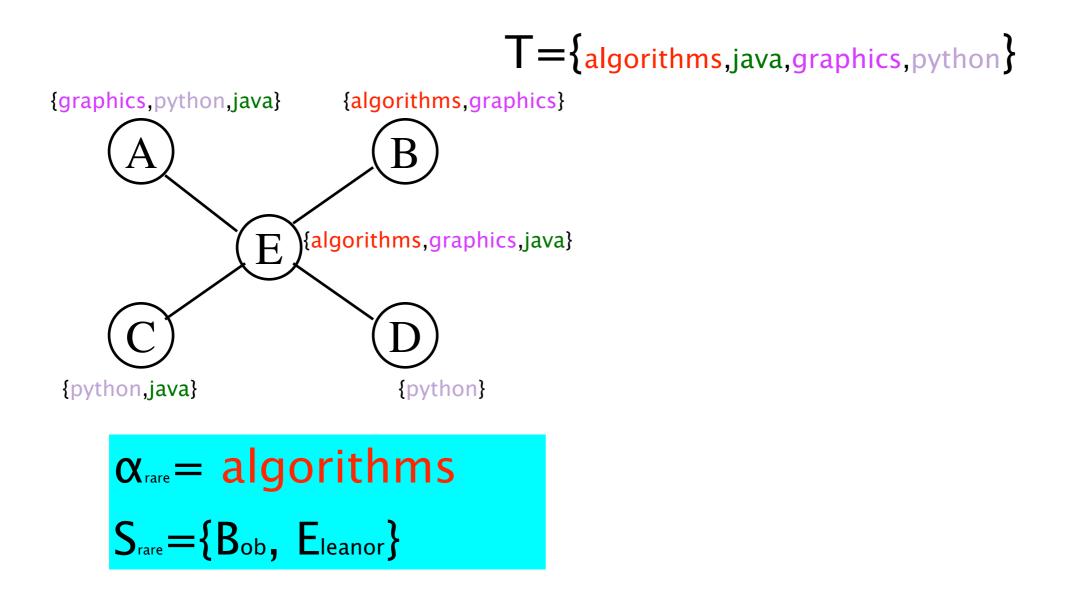


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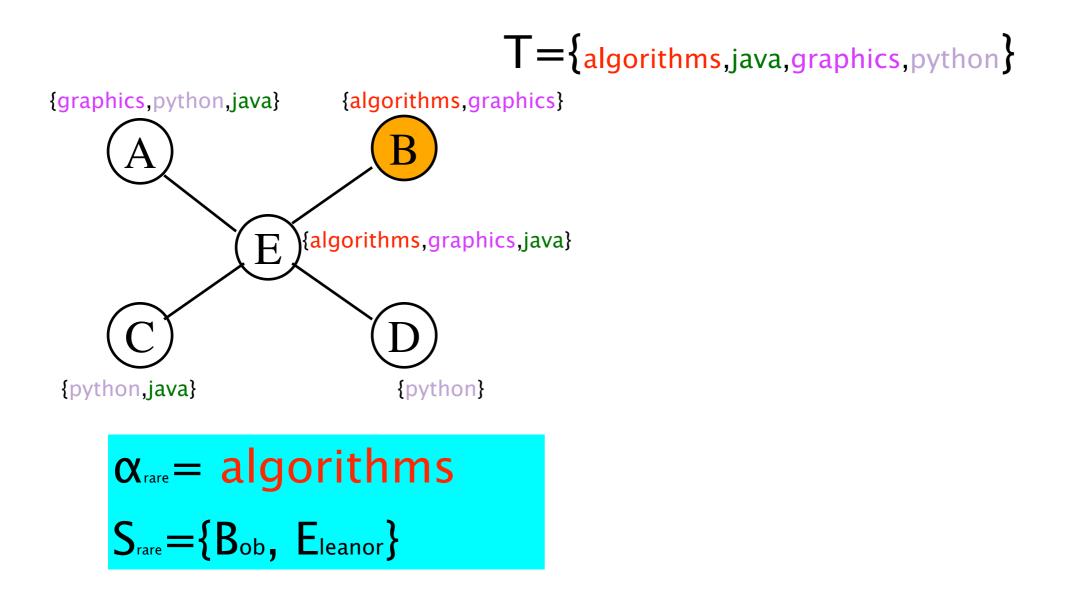
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- Report cheapest star

Running time: Quadratic to the number of nodes

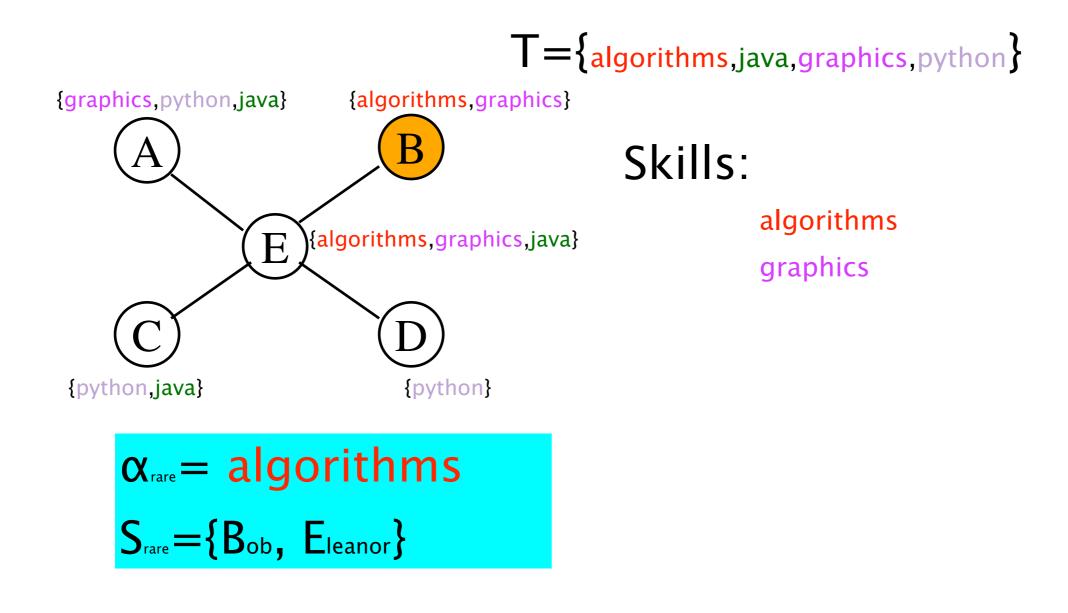




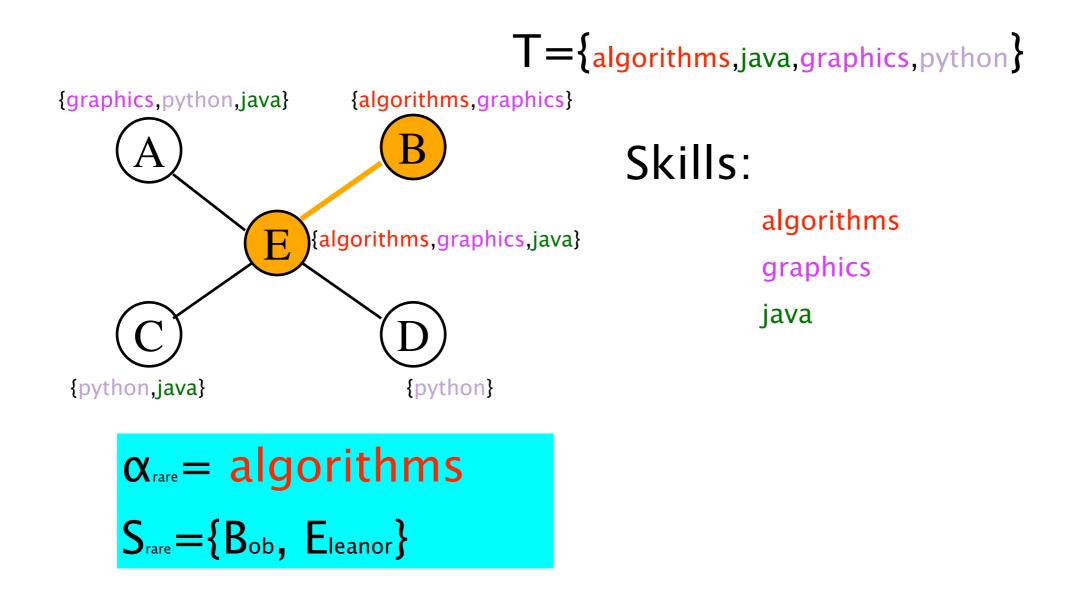




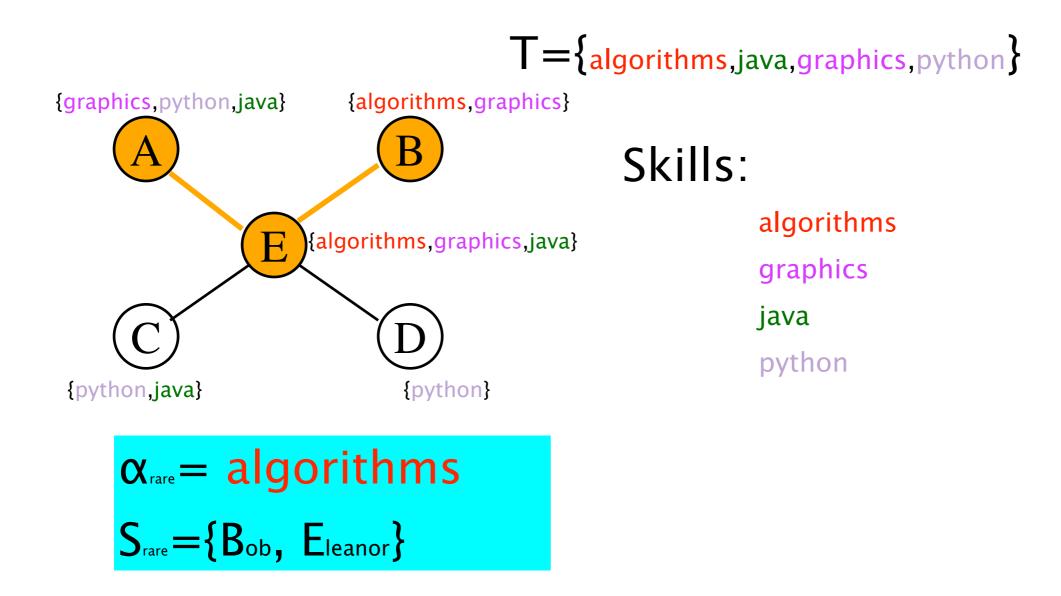




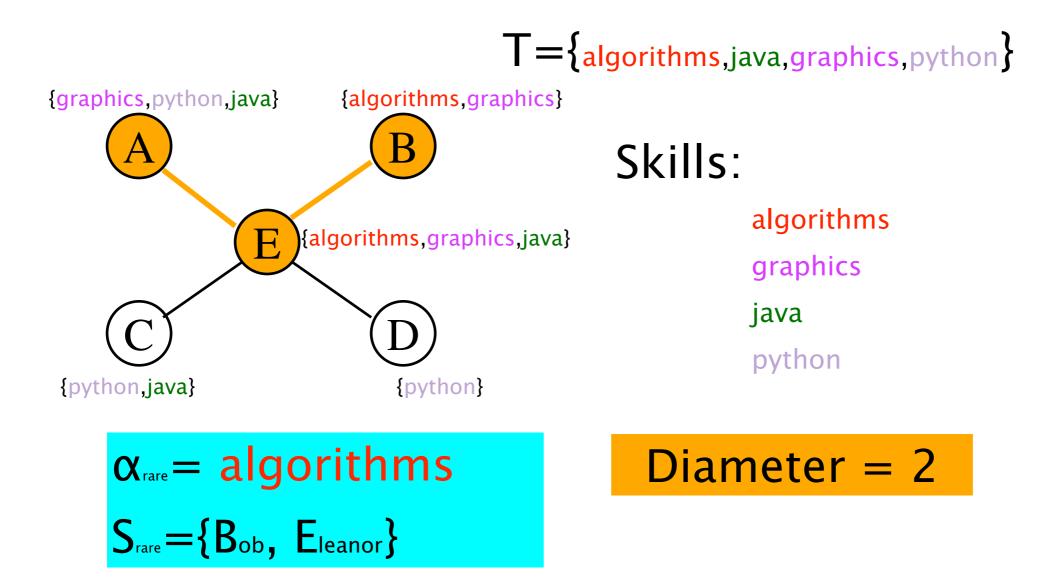




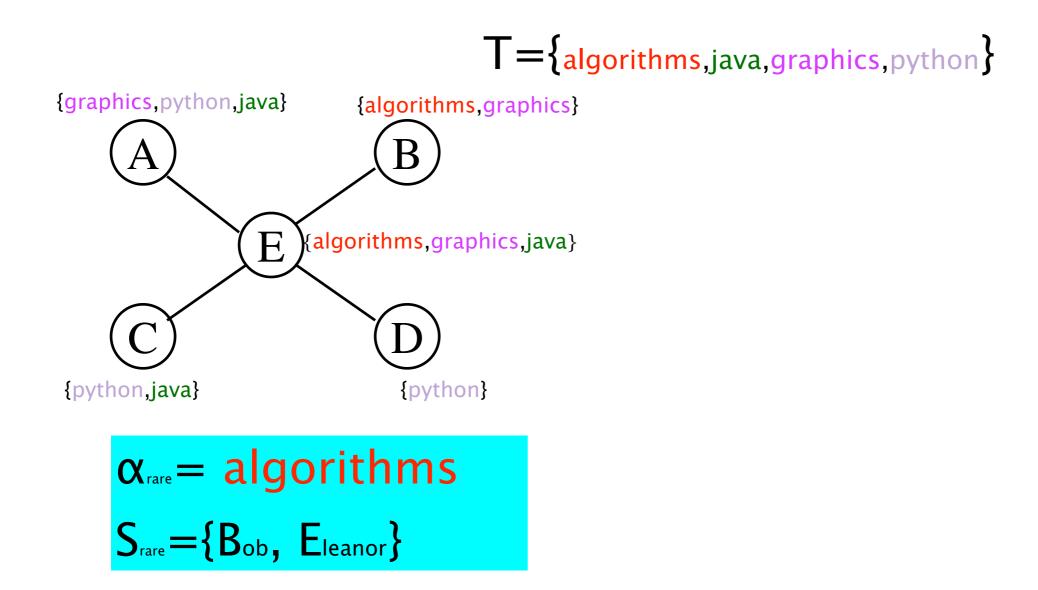




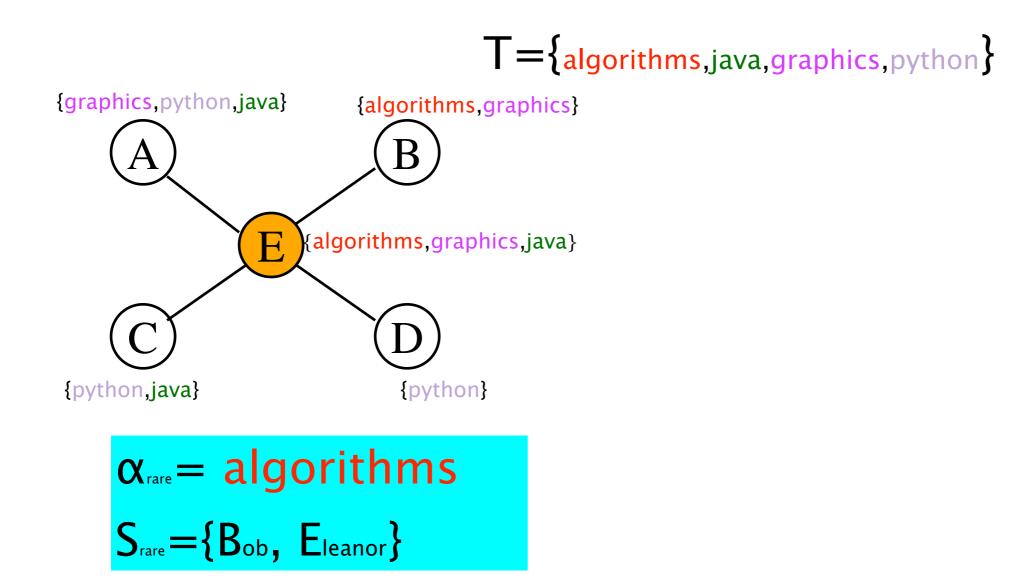




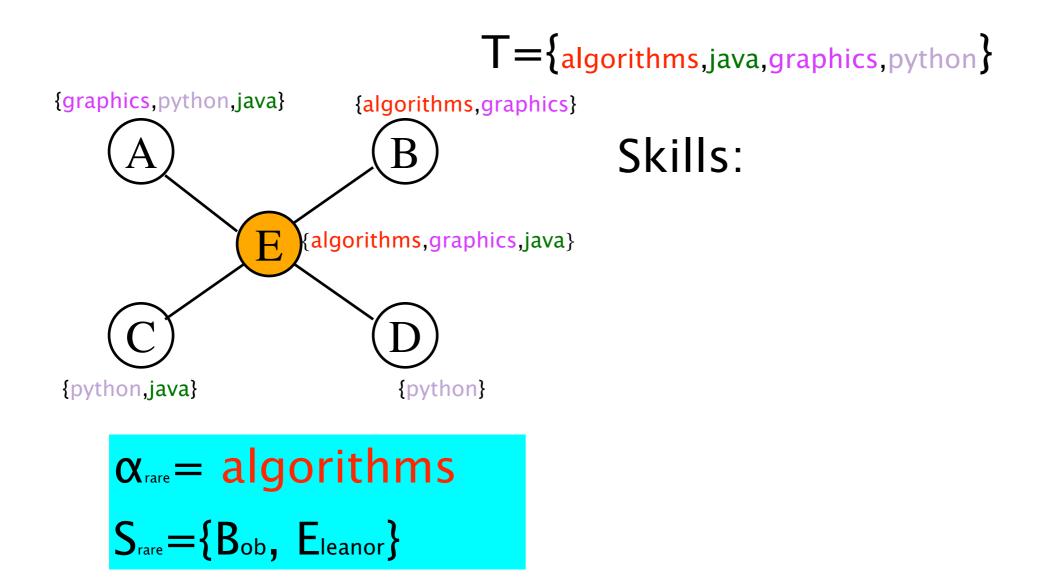




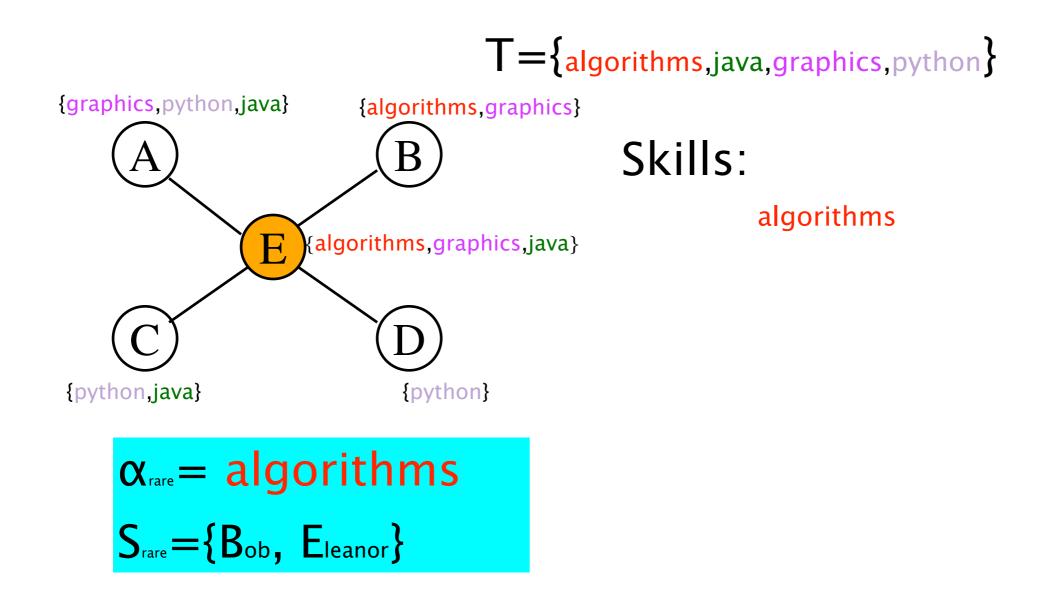




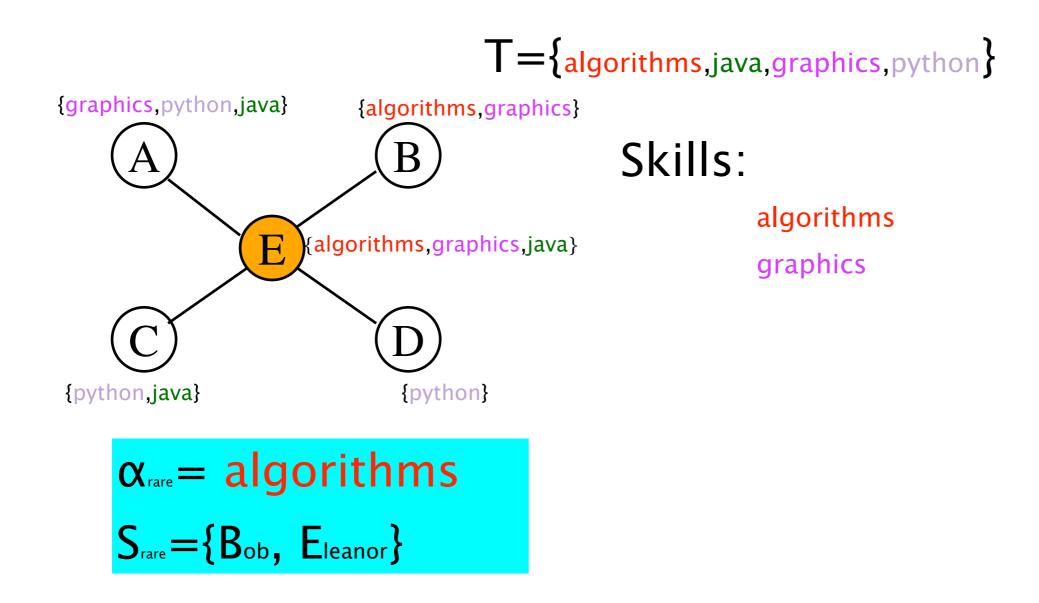




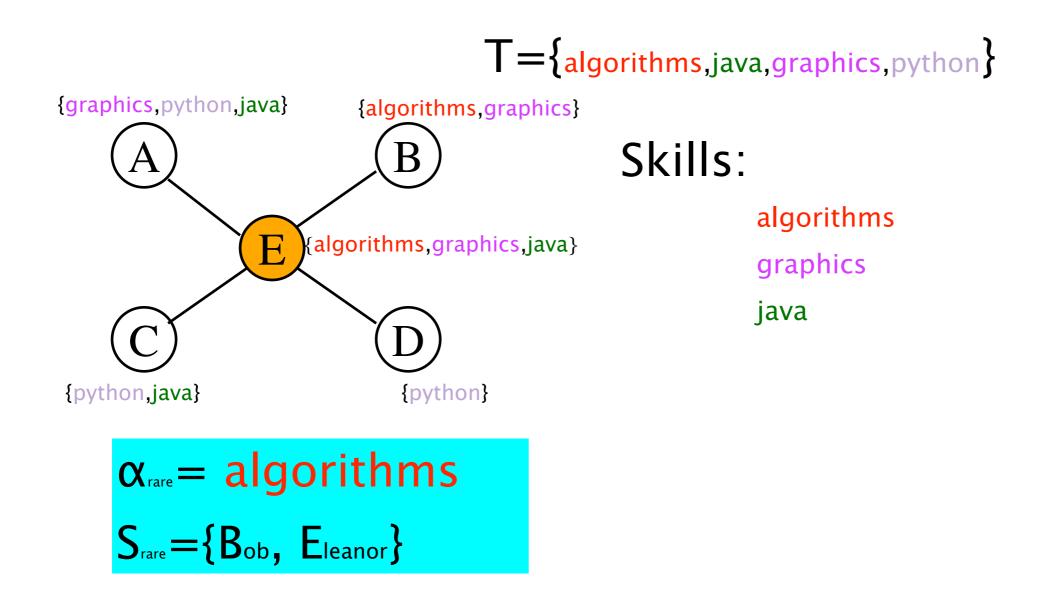




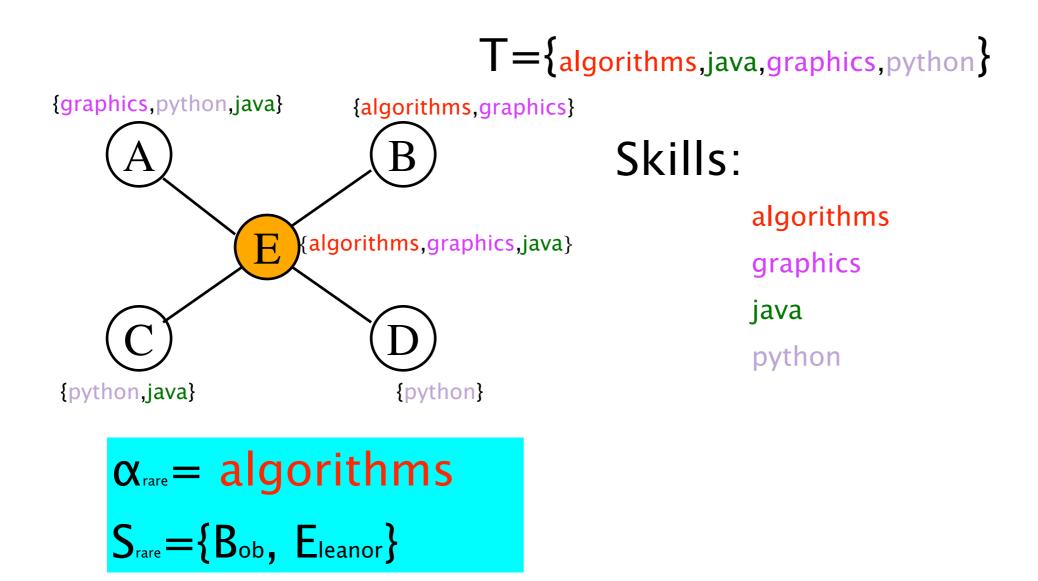




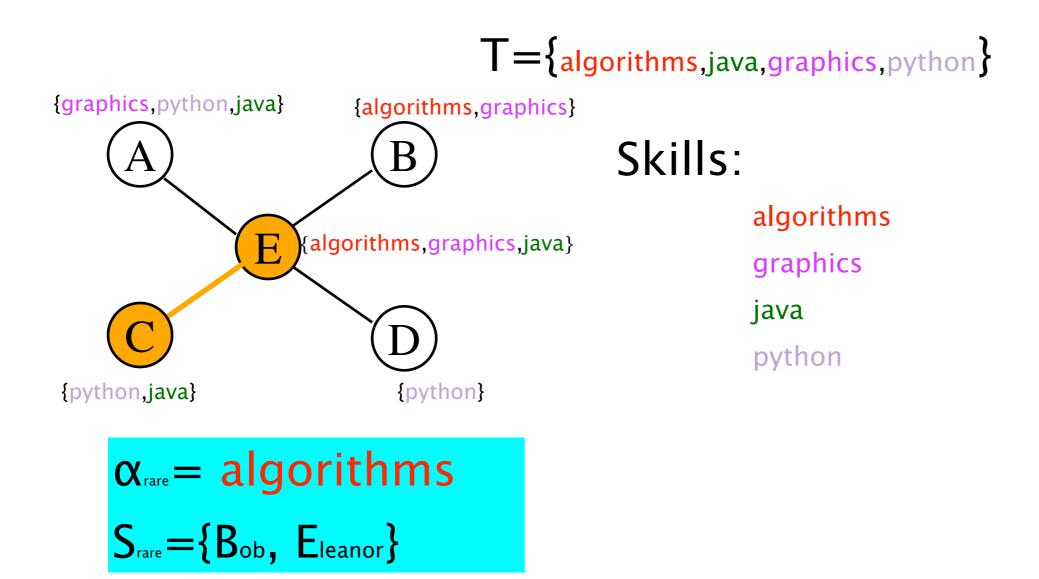




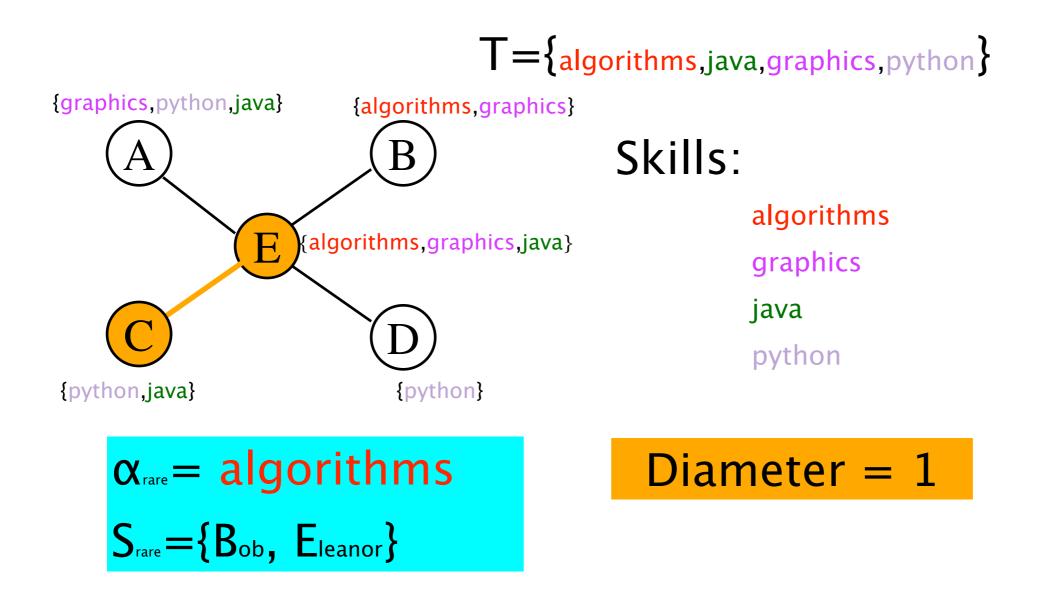




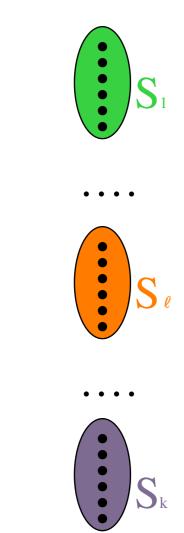








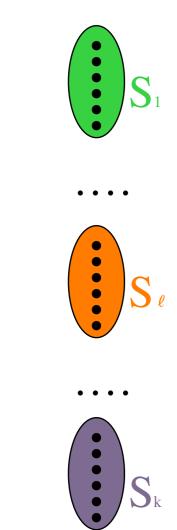






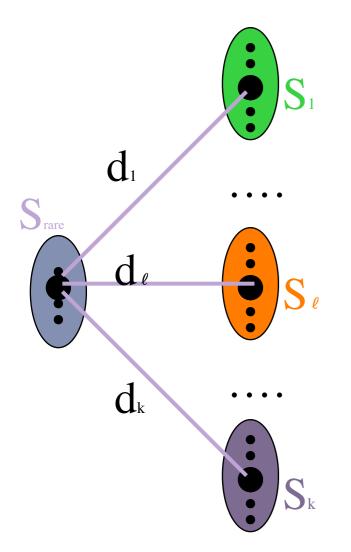
Srare

•

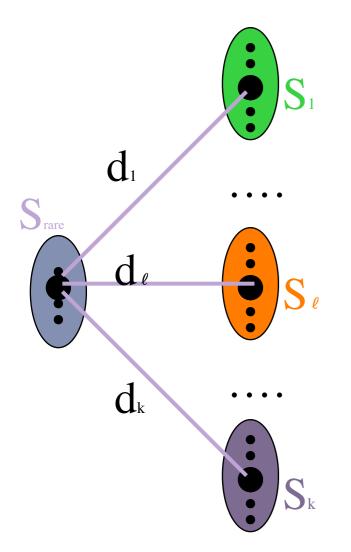




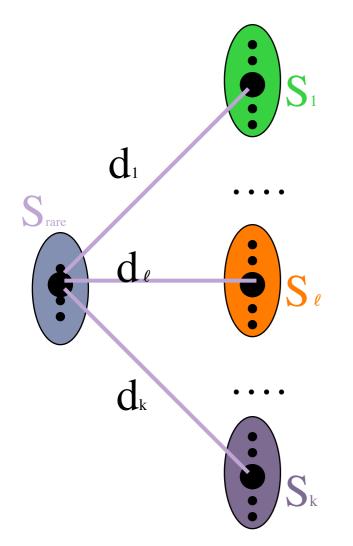
Srare





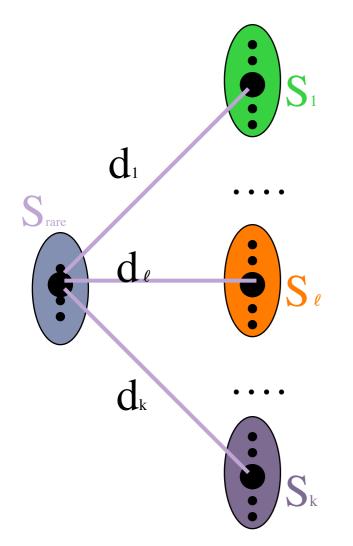






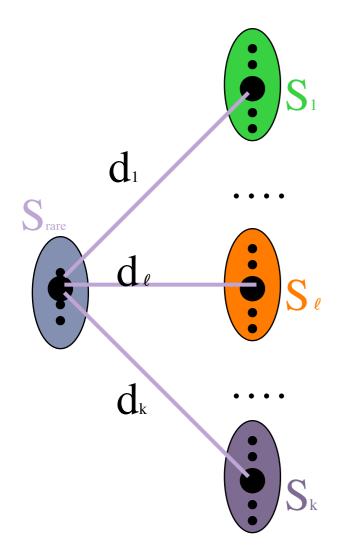
$$D = \max \{ d_{\ell}, d_k, d_{\ell k} \}$$





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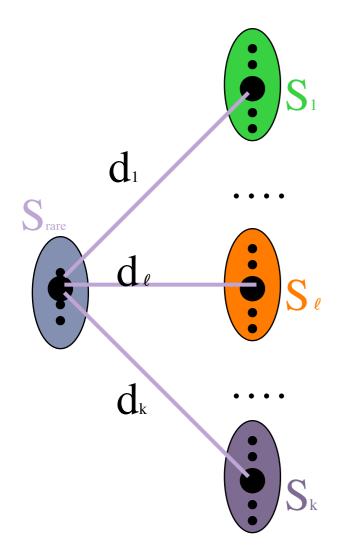




$$D = \max \{ d_{\ell}, d_k, d_{\ell k} \}$$

Fact: OPT $\geq d_{\ell}$

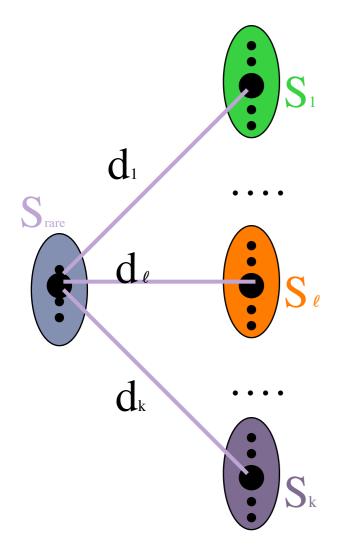




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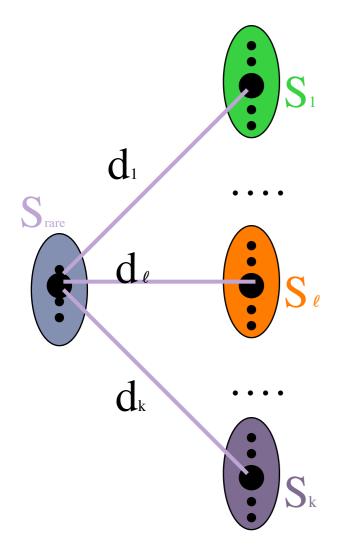


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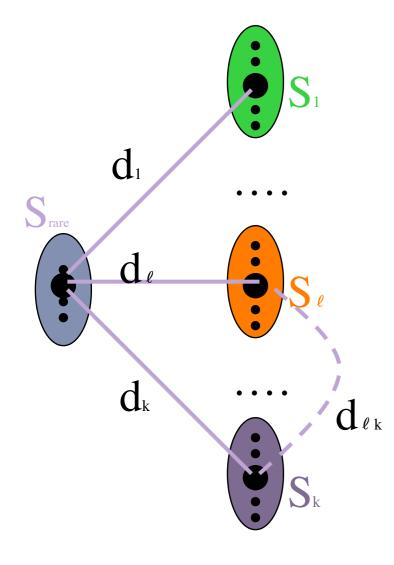


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$$D = \max \{ d_{\ell}, d_k, d_{\ell k} \}$$

Fact: OPT $\geq d_{\ell}$

Fact: OPT $\geq d_k$

$$D \leq d_{\ell k} \leq d_{\ell} + d_{k} \leq 2*OPT$$



Problem definition – v.1.2



Problem definition – v.1.2

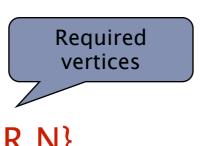
Given a task and a social network G of experts, find the subset (team) of experts that can perform the given task and they define a subgraph in G with the minimum MST cost.

Problem is NP-hard



The **SteinerTree** problem

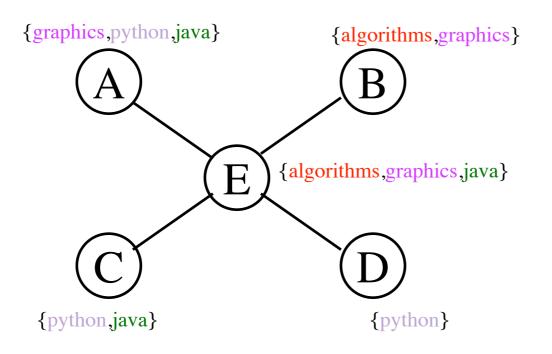
► Graph G=(V,E)



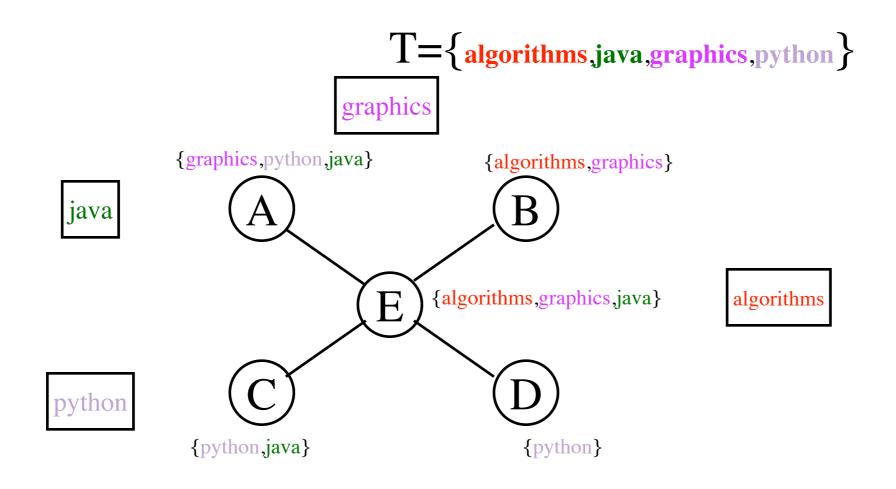
- Partition of V into V = {R,N}
- Find G' subgraph of G such that G' contains all the required vertices (R) and MST(G') is minimized



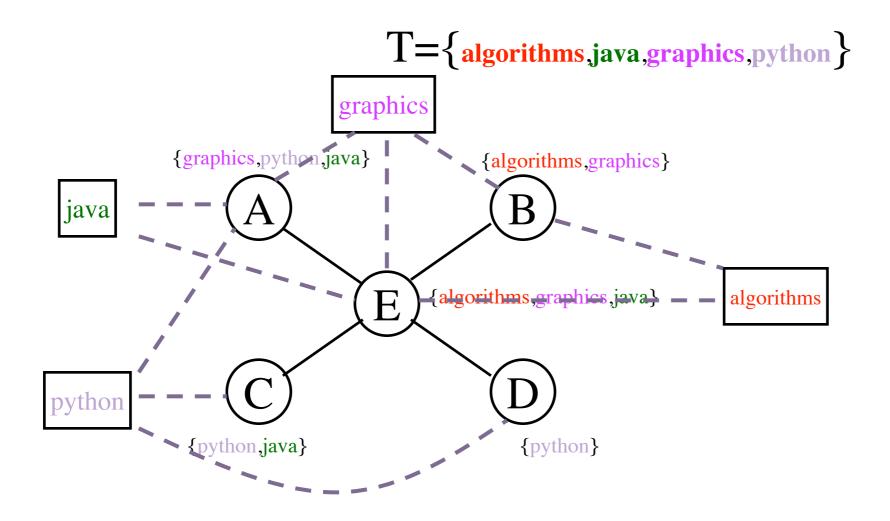
 $T = \{algorithms, java, graphics, python\}$



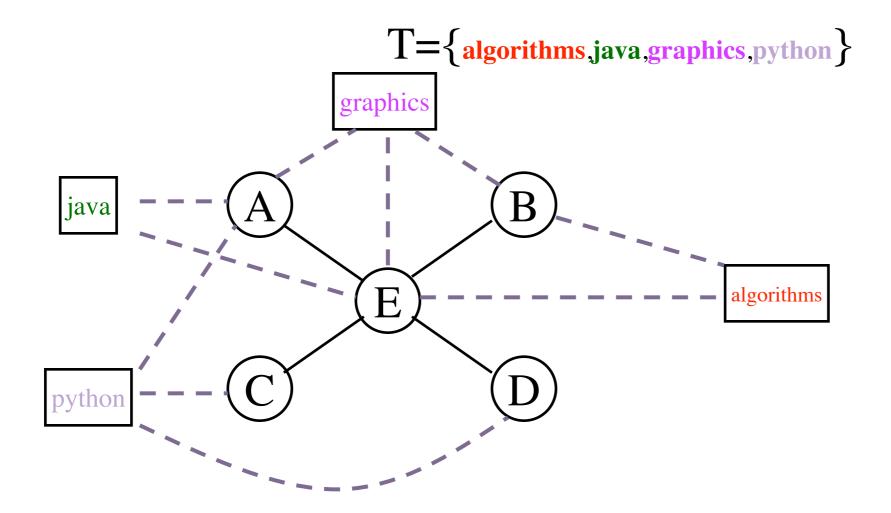




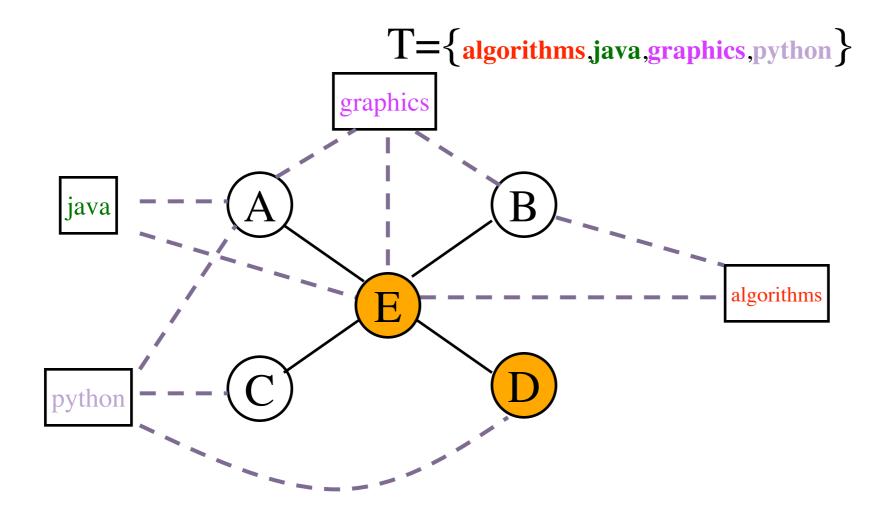




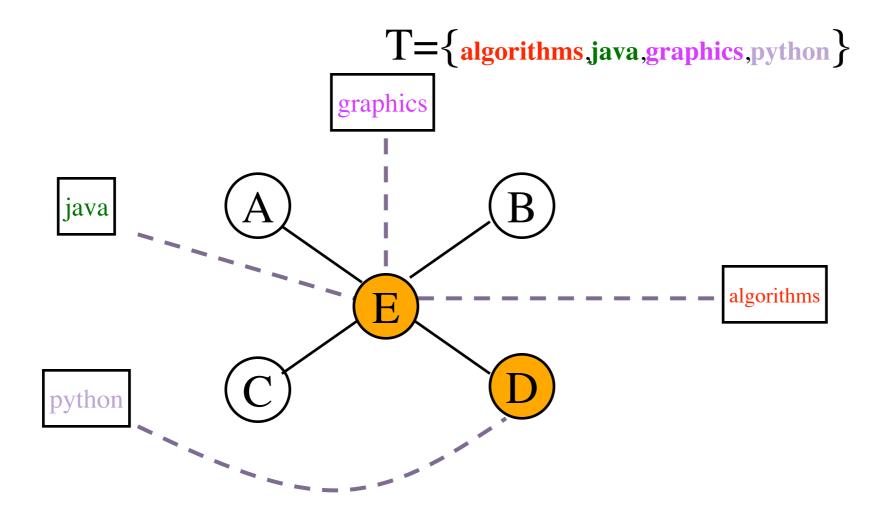




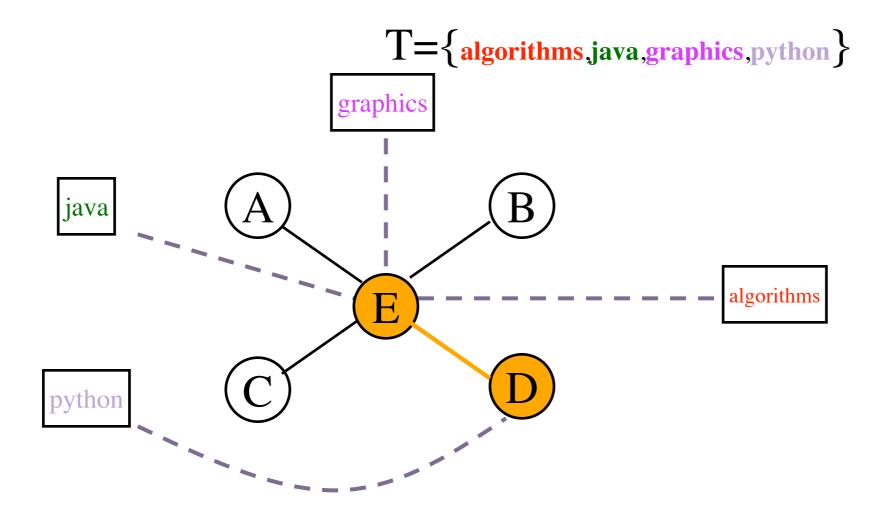




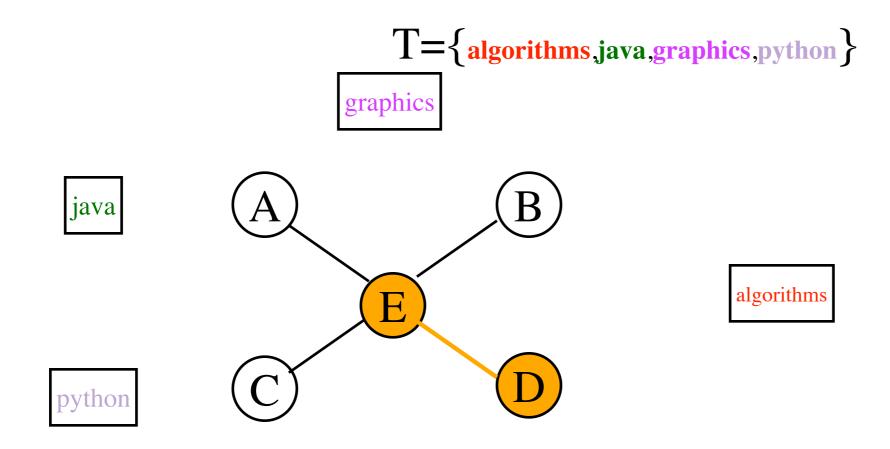




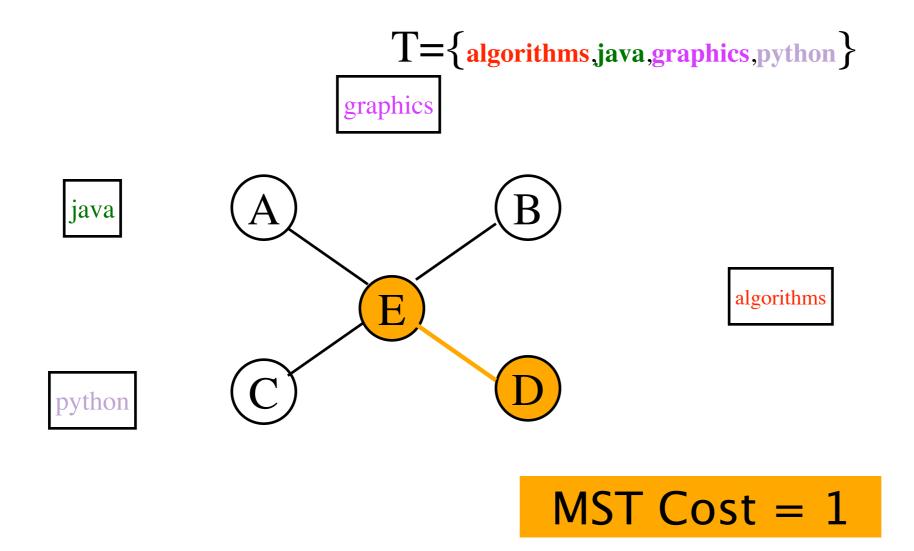














Other ways of exploiting the **SteinerTree** problem

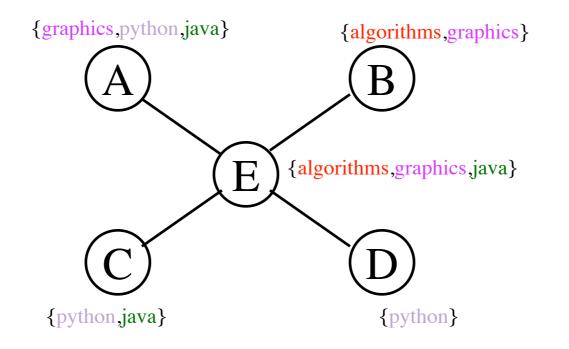
- Graph G(V,E)

 Required vertices

 Partition of V into V = {R,N}
- Find G' subgraph of G such that G' contains all the required vertices (R) and MST(G') is minimized

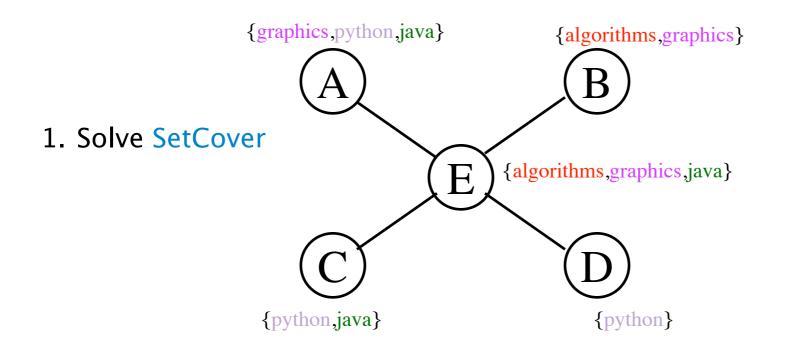


 $T = \{algorithms, java, graphics, python\}$



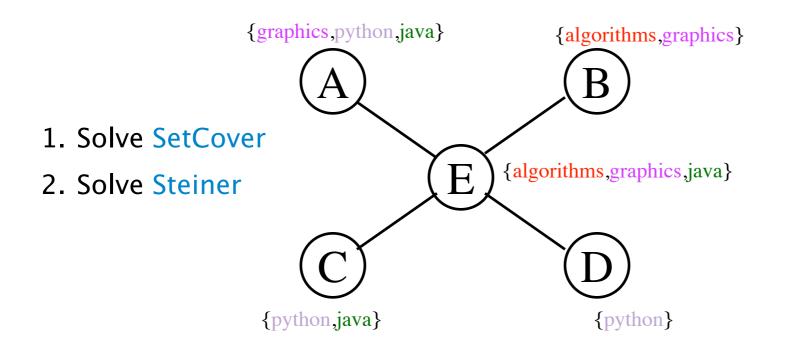


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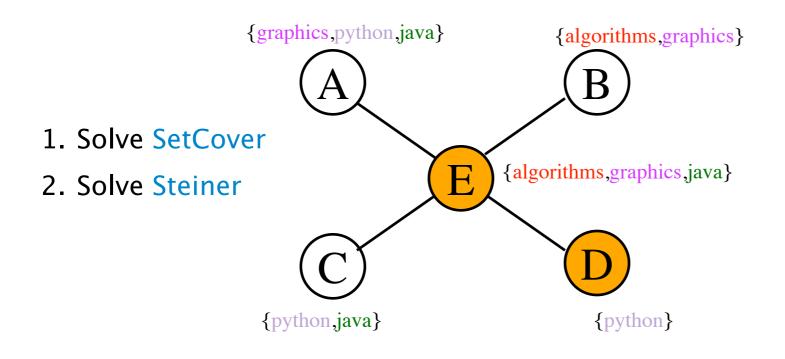


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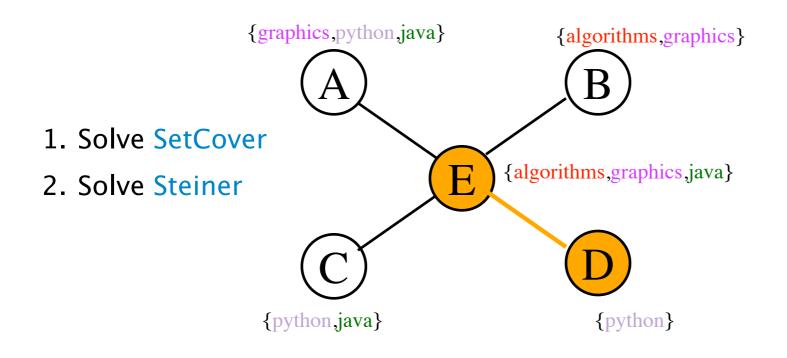


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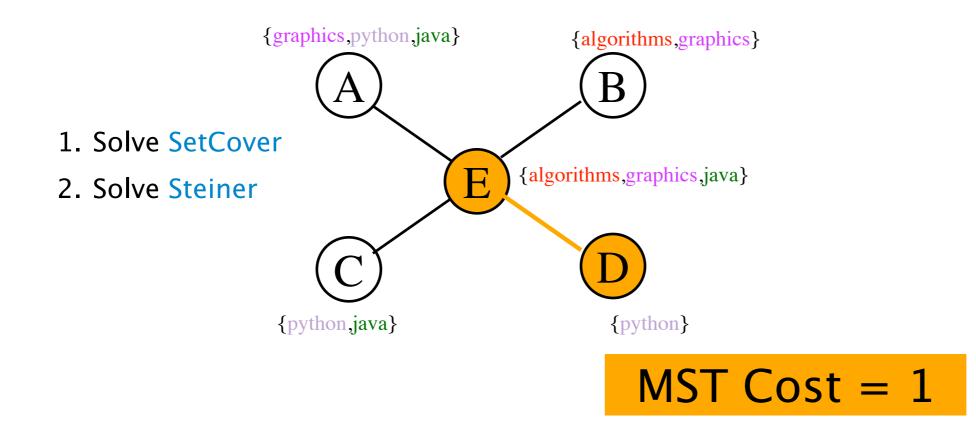


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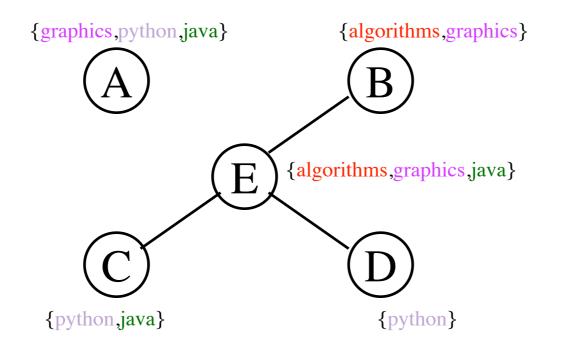
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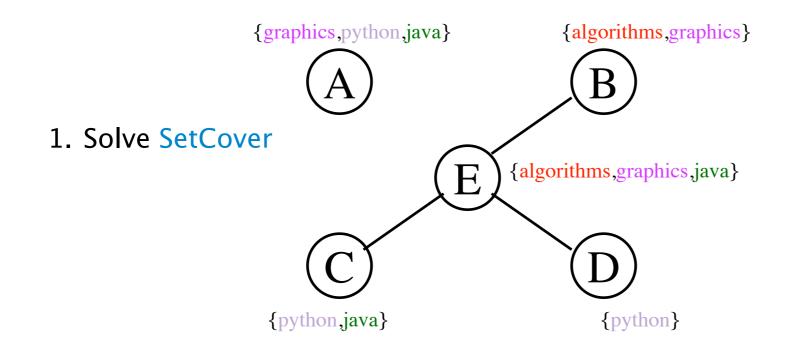


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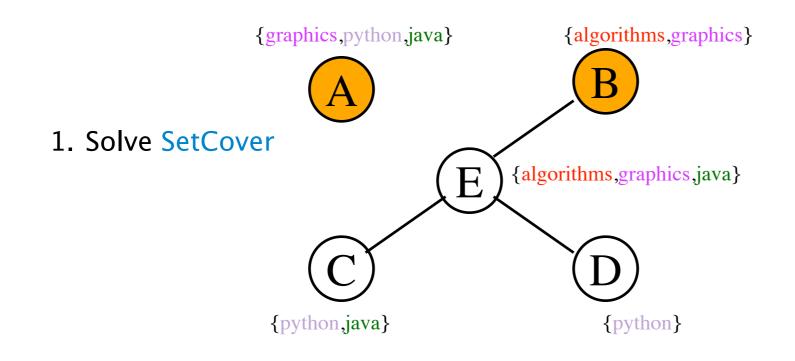


 $T = \{algorithms, java, graphics, python\}$



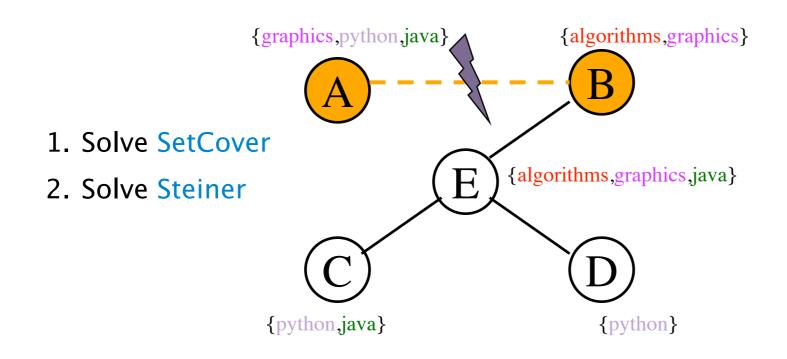


 $T = \{algorithms, java, graphics, python\}$



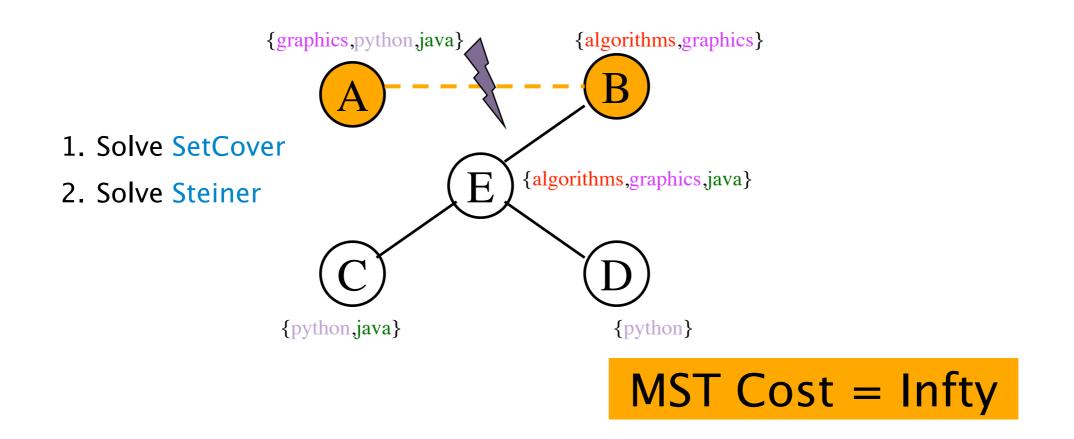


 $T = \{algorithms, java, graphics, python\}$



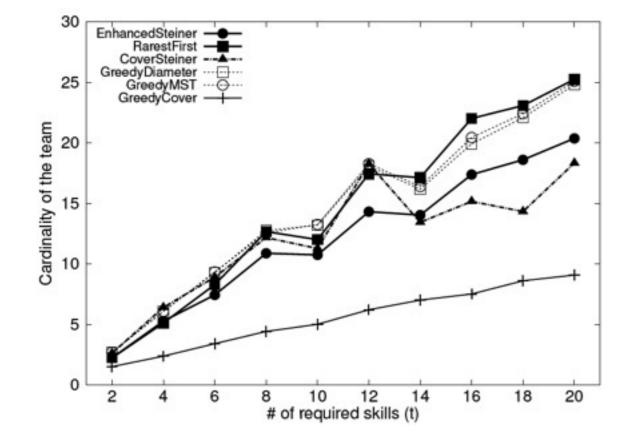


 $T = \{algorithms, java, graphics, python\}$





Experiments - Cardinality of teams



Dataset

DBLP graph (DB, Theory, ML, DM)

~6000 authors

~2000 features

Features: keywords appearing in papers

Tasks: Subsets of keywords with different cardinality k



Example teams (I)

- S. Brin, L. Page: The anatomy of a large-scale hypertextual Web search engine
 - Paolo Ferragina, Patrick Valduriez, H. V. Jagadish, Alon Y. Levy, Daniela Florescu Divesh Srivastava, S. Muthukrishnan
 - P. Ferragina ,J. Han, H. V.Jagadish, Kevin Chen-Chuan Chang, A. Gulli, S. Muthukrishnan, Laks V. S. Lakshmanan



Example teams (II)

- J. Han, J. Pei, Y. Yin: Mining frequent patterns without candidate generation
 - F. Bronchi
 - A. Gionis, H. Mannila, R. Motwani



Extensions

Skill attribution

	Team	Skill Attribution
Experts' skills	Known	Unknown
Participation of experts in	Unknown	Known
Network structure	Known	Irrelevant

• Team chemistry as a factor of success



Example teams (II)

- J. Han, J. Pei, Y. Yin: Mining frequent patterns without candidate generation
 - F. Bronchi
 - A. Gionis, H. Mannila, R. Motwani

