CS591 **May 22, 2014**

Homework set 1

due: Friday, May 30 in class. Solve any five of the six given problems.

1. Prove that if you start with a maximal matching M of a graph then the set of all of the vertices of the edges of M is a vertex cover of the graph. Give an example of a graph where you need to add both vertices of the edges in the matching.

2. Write the pseudocode of your algorithm for finding an (almost) minimum vertex cover of a graph with n nodes and m vertices. Compute the computational complexity of your algorithm in terms of n and m. Implement your algorithm and report its running time for the graphs provided in the class website. (The datasets are given in an edge list format.)

3. We say that a directed graph G is acyclic, if it does not contain any directed cycle. Give an approximation algorithm to find a maximal acyclic subgraph of G. Show that it is a 2-approximation. **Hint:** Number the vertices of the graph in arbitrary order. Then look at the set of forward (an edge is forward if it is directed from a smaller to a larger id vertex) and backward edges.

4. A minimal maximal matching in a graph is a maximal matching with the fewest number of edges. Finding a minimal maximal matching is hard. Find a 2-approximation to solve this problem and prove that your algorithm gives an answer within 2 of the optimal. **Hint:** Use the fact that any maximal matching is at least half the maximum matching.

5. Show that for a graph of n vertices the maximum number of min-cuts is $\frac{n(n-1)}{2}$. Show that this bound is achieved by giving an example of an n vertex graph with $\frac{n(n-1)}{2}$ min cuts.

6. The max-cut problem is that of finding a cut of maximum size in a graph G. Show that the random algorithm `Rand-MaxCut` is a 2-approximation for the max-cut problem.

Algorithm 1 `Rand-MaxCut` algorithm

Input: graph $G = (V, E)$
$V_1 \leftarrow \emptyset$
$V_2 \leftarrow \emptyset$
for $v \in V$ **do**
  Pick a value b in $\{0, 1\}$ randomly
  if $b = 0$ **then**
    $V_1 = V_1 \cup \{v\}$
  end if
  if $b = 1$ **then**
    $V_2 = V_2 \cup \{v\}$
  end if
end for
return V_1, V_2
