Homework set 4

Due: Tuesday, June 24 by 1pm. Solve any five of the six given problems.

1. Consider the following `MultCut` algorithm for the multiway cut problem:

   1. For each $i = 1, 2, \ldots, k$, compute a minimum weight isolating cut for s_i, say C_i.
   2. Discard the heaviest of the cuts and output the union of the rest.

   Show that `MultCut` can be used as a subroutine for finding a k-cut within a factor of $2 - 2/k$ of the minimum k-cut. How many subroutine class are needed?

2. A natural greedy algorithm for computing the multiway cut is the following. Starting with G, compute minimum s_i–s_j cuts for all pairs s_i, s_j that are still connected and remove the lightest of these cuts; repeat this until all pairs s_i, s_j are disconnected. Prove that this algorithm achieves a guarantee of $2 - 2/k$.

3. Consider yelp reviews about a restaurant and assume that the comments of each review R are stored in a multidimensional vector r, where $r(i)$ is a value in $\{-1, -\frac{1}{2}, 0, \frac{1}{2}, 1\}$ that represents whether the user that wrote review R has a positive or a negative experience with respect to attribute i of the restaurant.

   Now assume that the restaurant owner has a budget B (in dollars) and wants to make changes in his restaurant. The changes of attribute i cost him $c(i)$ dollars, and entails in gain $g(i) = 1 - r(i)$. Design an algorithm that will allow the owner to maximize his gain, while staying within his budget. Explain clearly how your algorithm works, you may write pseudo code or explain in your own words.

4. Choose either the FFD or BFD bin packing algorithm and prove that it gives an approximation algorithm with a factor of 3/2. That is, for any instance I of bin packing the algorithm gives an answer that is $\leq 3/2\mathrm{OPT}(I) + 1$.

5. Let Next Fit (NF) be the bin packing algorithm which works like FF but which tries to pack the next object only in the most recently started bin. If the object doesn't fit into the most recently started bin then open a new bin and pack it there. Give an example where the NF algorithm produces a result which is exactly 2 times OPT.

   The monotonic property on Bin Packing algorithms says that the number of bins used for packing a subset of the objects is no more than the number of bins used in packing all n objects. Show that NF has the monotonic property. Show that for some bin packing instance FF is not monotonic.

6. (i) Phrase the bin packing problem as an integer programing (IP) problem. State what each of the variables and the inequalities mean in terms of bin packing.

   (ii) Give a linear problem which is a relaxation of the IP in (i).

   (iii) Finally, write the linear program (DUAL) which is dual to the LP in (ii).