- Towards identity-anonymization on graphs K. Liu \& E. Terzi, SIGMOD 2008
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## Growing Privacy Concerns

## - Person specific information is being routinely collected.

"Detailed information on an individual's credit, health, and financial status, on characteristic purchasing patterns, and on other personal preferences is routinely recorded and analyzed by a variety of governmental and commercial organizations."

- M. J. Cronin, "e-Privacy?" Hoover Digest, 2000.



## Proliferation of Graph Data



## Linked in.

## facebook

http://www.touchgraph.com/

## Privacy breaches on graph data

- Identity disclosure
- Identity of individuals associated with nodes is disclosed
- Link disclosure
- Relationships between individuals are disclosed
- Content disclosure
- Attribute data associated with a node is disclosed


## Identity anonymization on graphs

- Question
- How to share a network in a manner that permits useful analysis without disclosing the identity of the individuals involved?
- Observations
- Simply removing the identifying information of the nodes before publishing the actual graph does not guarantee identity anonymization.
L. Backstrom, C. Dwork, and J. Kleinberg, "Wherefore art thou R3579X?: Anonymized social netwoks, hidden patterns, and structural steganography," In WWW 2007.
J. Kleinberg, "Challenges in Social Network Data: Processes, Privacy and Paradoxes, " KDD 2007 Keynote Talk.
- Can we borrow ideas from k-anonymity?


## What if you want to prevent the following from happening

- Assume that adversary A knows that B has 327 connections in a social network!
- If the graph is released by removing the identity of the nodes
- A can find all nodes that have degree 327
- If there is only one node with degree 327, A can identify this node as being $B$.


## Privacy model

[ $k$-degree anonymity] A graph $G(V, E)$ is $k$-degree anonymous if every node in $V$ has the same degree as $k$ - 1 other nodes in $V$.

[Properties] It prevents the re-identification of individuals by adversaries with a priori knowledge of the degree of certain nodes.

## Problem Definition

Given a graph $G(V, E)$ and an integer $k$, modify $G$ via a minimal set of edge addition or deletion operations to construct a new graph $G^{\prime}\left(V^{\prime}, E^{\prime}\right)$ such that

1) $G^{\prime}$ is $k$-degree anonymous;
2) $V^{\prime}=V$;
3) The symmetric difference of $G$ and $G^{\prime}$ is as small as possible

- Symmetric difference between graphs $G(V, E)$ and $G^{\prime}\left(V, E^{\prime}\right)$ :
$\operatorname{SymDiff}\left(G^{\prime}, G\right)=\epsilon^{\prime} \backslash E \searrow \mathbb{C} \backslash E^{\prime}$,


## GraphAnonymization algorithm

Input: Graph $G$ with degree sequence $d$, integer $\boldsymbol{k}$
Output: $\boldsymbol{k}$-degree anonymous graph $\mathbf{G}^{\prime}$
[Degree Sequence Anonymization]:

- Contruct an anonymized degree sequence $d^{\prime}$ from the original degree sequence d
[Graph Construction]:
[Construct]: Given degree sequence $d^{\prime}$ ', construct a new graph $G^{0}\left(V, E^{0}\right)$ such that the degree sequence of $G^{0}$ is $d^{6}$
[Transform]: Transform $G^{0}\left(V, E^{0}\right)$ to $G^{\prime}\left(V, E^{\prime}\right)$ so that SymDiff $\left(G^{\prime}, G\right)$ is minimized.


## Degree-sequence anonymization

[k-anonymous sequence] A sequence of integers $d$ is $k$-anonymous if every distinct element value in $d$ appears at least $k$ times.
[100,100, 100, 98, 98,15,15,15]

> [degree-sequence anonymization] Given degree sequence $d$, and integer $k$, construct $k$-anonymous sequence $d^{\prime}$ such that $/ / d^{\prime}-d / /$ is minimized

Increase/decrease of degrees correspond to additions/deletions of edges

## Algorithm for degree-sequence anonymization



## DP for degree-sequence anonymization

- $\boldsymbol{d}(1) \geq \boldsymbol{d}(2) \geq \ldots \geq \boldsymbol{d}(i) \geq \ldots \geq \boldsymbol{d}(n)$ : original degree sequence.
- $d^{\prime}(1) \geq d^{\prime}(2) \geq \ldots \geq d^{\prime}(i) \geq \ldots \geq d^{\prime}(n): k$-anonymized degree sequence.
- $C(i, j)$ : anonymization cost when all nodes $i, i+1, \ldots, j$ are put in the same anonymized group, i.e.,

$$
C(i, j)=\sum_{\ell=i}^{j}(i)-d^{*}
$$

- DA(1, n) : the optimal degree-sequence anonymization cost
- Dynamic Programming with $\mathrm{O}\left(\mathrm{n}^{2}\right)$

$$
D A \backslash i=\min _{k \leq \leq i-k} \nexists A \backslash, t_{+}+C\left(+1, i^{-}\right.
$$

- Dynamic Programming with $\mathbf{O}(\mathbf{n k})$
- Dynamic Programming can be done in $\mathrm{O}(\mathrm{n})$ with some additional bookkeeping


## GraphAnonymization algorithm

Input: Graph $G$ with degree sequence $d$, integer $\boldsymbol{k}$ Output: $k$-degree anonymous graph $G^{\prime}$

Degree Sequence Anonymization]:

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## Are all degree sequences realizable?

- A degree sequence $d$ is realizable if there exists a simple undirected graph with nodes having degree sequence $d$.
- Not all vectors of integers are realizable degree sequences
$-d=\{4,2,2,2,1\}$ ?
- How can we decide?


## Realizability of degree sequences

[Erdös and Gallai] A degree sequence $\boldsymbol{d}$ with $\boldsymbol{d}(1) \geq \boldsymbol{d}(2) \geq \ldots \geq \boldsymbol{d}(i) \geq \ldots \geq \boldsymbol{d}(n)$ and $\Sigma \boldsymbol{d}(i)$ even, is realizable if and only if

$$
\sum_{i=1}^{l} \boldsymbol{d}(i) \leq l(l-1)+\sum_{i=l+1}^{n} \min \{l, \boldsymbol{d}(i)\}, \text { for every } 1 \leq l \leq n-1
$$

Input: Degree sequence $d^{\prime}$
Output: Graph $G^{0}\left(V, E^{0}\right)$ with degree sequence $d^{\prime}$ or $N O!$
$\rightarrow$ If the degree sequence $d^{\prime}$ is NOT realizable?
-Convert it into a realizable and $k$-anonymous degree sequence

## GraphAnonymization algorithm

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## Graph-transformation algorithm

- GreedySwap transforms $G^{0}=\left(V, E^{0}\right)$ into $G^{\prime}\left(V, E^{\prime}\right)$ with the same degree sequence $d^{\prime}$, and min symmetric difference SymDiff( $\left.G^{\prime}, G\right)$.
- GreedySwap is a greedy heuristic with several iterations.
- At each step, GreedySwap swaps a pair of edges to make the graph more similar to the original graph $\mathbb{G}$, while leaving the nodes' degrees intact.


## Valid swappable pairs of edges



A swap is valid if the resulting graph is simple

## GreedySwap algorithm

Input: A pliable graph $G^{0}\left(V, E^{0}\right)$, fixed graph $G(V, E)$
Output: Graph $G^{\prime}\left(V, E^{\prime}\right)$ with the same degree sequence as $G^{0}\left(V, E^{0}\right)$
i=0

## Repeat

find the valid swap in $\boldsymbol{G}^{i}$ that most reduces its symmetric difference with $\boldsymbol{G}$, and form graph $\boldsymbol{G}^{i+1}$
i++

## Experiments

- Datasets: Co-authors, Enron emails, powergrid, Erdos-Renyi, small-world and power-law graphs
- Goal: degree-anonymization does not destroy the structure of the graph
- Average path length
- Clustering coefficient
- Exponent of power-law distribution


## Experiments: Clustering coefficient and Avg Path Length

- Co-author dataset
- APL and CC do not change dramatically even for large values of $\boldsymbol{k}$




## Experiments: Edge intersections

Edge intersection achieved by the GreedySwap algorithm for different datasets.

Parenthesis value indicates the original value of edge intersection

| Synthetic datasets |  |
| :--- | :--- |
| Small world graphs* | $0.99(0.01)$ |
| Random graphs | $0.99(0.01)$ |
| Power law graphs** $^{\text {R }}$ | $0.93(0.04)$ |
| Real datasets | $0.95(0.16)$ |
| Enron | $0.97(0.01)$ |
| Powergrid | $0.91(0.01)$ |
| Co-authors |  |

${ }^{(*)}$ L. Barabasi and R. Albert: Emergence of scaling in random networks. Science 1999.
${ }^{(* *)}$ Watts, D. J. Networks, dynamics, and the small-world phenomenon. American Journal of Sociology 1999

## Experiments: Exponent of power law distributions

| Original | 2.07 |
| :--- | :--- |
| $\mathrm{k}=10$ | 2.45 |
| $\mathrm{k}=15$ | 2.33 |
| $\mathrm{k}=20$ | 2.28 |
| $\mathrm{k}=25$ | 2.25 |
| $\mathrm{k}=50$ | 2.05 |
| $\mathrm{k}=100$ | 1.92 |

Co-author dataset

Exponent of the powerlaw distribution as a function of $k$

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## What is privacy risk score and why is it useful?

- What?
- It is a credit-score-like indicator to measure the potential privacy risks of online social-networking users.
- Why?
- It aims to boost public awareness of privacy, and to reduce the cognitive burden on end-users in managing their privacy settings.
- privacy risk monitoring \& early alarm
- comparison with the rest of population
- help sociologists to study online behaviors, information propagation


## Privacy Score Overview

Privacy Score measures the potential privacy risks of online social-networking users.


## How is Privacy Score Calculated? - Basic Premises

- Sensitivity: The more sensitive the
mother's maiden name is more sensitive than mobile-phone number
privacy risk.
home address known by everyone poses higher risks than by friends only
- Visibility: The wider the information about a user spreads, the higher his privacy risk.


## Privacy Score Calculation



## Privacy Score Calculation



## The Naïve Approach



## The Naïve Approach



Sensitivity: $\quad \beta_{i}=\frac{N-\left|R_{i}\right|}{N}$

## The Naïve Approach



Sensitivity: $\quad \beta_{i}=\frac{N-\left|R_{i}\right|}{N}$
Visibility: $\quad V(i, j)=\operatorname{Pr}\{R(i, j)=1\}$

$$
P_{i j}=\operatorname{Pr}\{R(i, j)=1\}=\frac{\left|R_{i}\right|}{N} \times \frac{\left|R^{j}\right|}{n}=\left(1-\beta_{i}\right) \times \frac{\left|R^{j}\right|}{n}
$$

## Advantages and Disadvantages of Naïve

- Computational Complexity O (Nn) - best one can hope
- Scores are sample dependent
- Studies show that Facebook users reveal more identifying information than MySpace users
- Sensitivity of the same information estimated from Facebook and from MySpace are different
- What properties do we really want?
- Group Invariance: scores calculated from different social networks and/or user base are comparable.
- Goodness-of-Fit: mathematical models fit the observed user data well.


## Item Response Theory (IRT)

- IRT (Lawley,1943 and Lord,1952) has its origin in psychometrics.
- It is used to analyze data from questionnaires and tests.
- It is the foundation of Computerized Adaptive Test like GRE, GMAT



## Item Characteristic Curve (ICC)

$$
\text { ICC: } P_{i j}=\operatorname{Pr}\{R(i, j)=1\}=\frac{1}{1+e^{-\alpha_{i}\left(\theta_{j}-\beta_{i}\right)}}
$$

- $\theta_{j}$ (ability): is an unobserved hypothetical variable such as intelligence, scholastic ability, cognitive capabilities, physical skills, etc.
- $\beta_{i}$ (difficulty): is the location parameter, indicates the point on the ability scale at which the probability of correct response is . 50
- $\alpha_{i}$ (discrimination): is the scale parameter that indexes the discriminating power of an item




## Mapping from PRS to IRT



## Computing PRS using IRT



## Calculating Privacy Score using IRT



## Advantages of the IRT Model

- The mathematical model fits the observed data well
- The quantities IRT computes (i.e., sensitivity, attitude and visibility) have intuitive interpretations
- Computation is parallelizable using e.g. MapReduce


## Estimating the Parameters of a Profile Item

Attitude leve

| $\theta_{-} 1$ |
| :--- |
| $\theta_{-} 2$ |
| $\theta_{-} 3$ |
| $\ldots \ldots$ |
| $\theta_{-} g$ |
| $\ldots \ldots$ |
| $\ldots \ldots$ |
| $\theta_{-} K$ |

Share

| $r_{-} i 1$ | $f_{-} 1-r_{-} i 1$ |
| :--- | :--- |
| $r_{-} i 2$ | $f_{-} 2-r_{-} i 2$ |
| $r_{-} i 3$ | $f_{-} 3-r_{-} i 3$ |
| $\ldots \ldots$ | $\ldots \ldots$ |
| $r_{-} i g$ | $f_{-} g-r_{-} i g$ |
| $\ldots \ldots$ | $\ldots \ldots$ |
| $\ldots \ldots$ | $\ldots \ldots$ |
| $r_{-} i K$ | $f_{-} g-r_{-} i K$ |

\# of users at this attitude level

| $f_{-} 1$ |
| :--- |
| $f_{-} 2$ |
| $f_{-} 3$ |
| $\ldots \ldots$ |
| $f_{-} g$ |
| $\ldots \ldots$ |
| $\ldots \cdots$ |
| $f_{-} K$ |

$$
\sum_{g=1}^{K} f_{g}=N
$$

Log likelihood: $L=\log \left[\prod_{g=1}^{K}\binom{f_{g}}{r_{i g}} P_{i g}{ }^{r_{i g}} 1-P_{i g}{ }^{f_{g}-r_{i g}}\right], \quad$ where $P_{i g}=\frac{1}{1+e^{-\sigma_{i g}\left(\theta_{g}-\beta_{i}\right)}}$
Newton-Raphson: $\left[\begin{array}{c}\alpha_{i} \\ \beta_{i}\end{array}\right]_{t+1}=\left[\begin{array}{c}\alpha_{i} \\ \beta_{i}\end{array}\right]_{t}-\left[\begin{array}{cc}L_{11} & L_{12} \\ L_{21} & L_{22}\end{array}\right]_{t}^{-1} \times\left[\begin{array}{c}L_{1} \\ L_{2}\end{array}\right]_{t} \quad$ where $L_{1}=\frac{\delta L}{\delta \alpha_{i}}, L_{2}=\frac{\delta L}{\delta \beta_{i}}, \quad L_{11}=\frac{\partial^{2} L}{\partial \alpha_{i}^{2}}, L_{22}=\frac{\partial^{2} L}{\partial \beta_{i}^{2}}, L_{12}=L_{21}=\frac{\partial^{2} L}{\partial \alpha_{i} \beta_{i}}$.

## Experiments - Interesting Results

Favorite Music<br>college/University Job Descriptio<br>Favorite Books Networks You Belong to<br>Favorite movies Zip Code Your Photo Albumspolitical Views<br>Time Period When You Work ThereReligious ViewsPersonal Website/Blog<br>Emails Job Position/Title Whom You Are a Fan/Supporter of Groups You Belong to<br>Events You're Invited to or Associated With Your Marketplace Listings<br>City/Town where You Work Birthday (Month/Day only)<br>Birthday (Month/Day Only) Degree ${ }^{\text {college class Year }}$ Work Phone Mother's Maiden Name<br>Residedence Address (Street) Residence Address (City/Town)<br>High School Class YearLooking for <Friendship, Dating, A Relationship, Networking><br>Concentration (Major) Relationship Status Birthday (Year/Month/Day) IM Screen Name<br>Interested in <Men, Women> Full List of Your Friends<br>EmployerHome Phone Spouse's Name<br>${ }^{\text {Favorite }}$ Quotations Mobile Phone<br>Activities




```
facebook Home Profile Friends Inbox
Invite your friends | My Profile < (please update all fields) > | My Privacy | Contact us | FAQs Help

B Privacy-aware Market Place
\begin{tabular}{|l|l|l|l|l|}
\hline Home & Add a new post & My posts & Admin & About \\
\hline
\end{tabular}Choose Privacy Settings


User Profile


Items for posting

\section*{EPrivacy Score}

The Recommended Privacy Score is provided for you. Note that if your current privacy score is lower than your recommended privacy then that implies your current settings are more private.

\section*{Current}

\section*{Recommended}
```

