CS 591: Formal Methods in Security and Privacy

Non-interference

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From the previous classes

Some Examples of Security Properties

- Access Control
- Encryption
- Malicious Behavior Detection
- Information Filtering
- Information Flow Control

Private vs Public

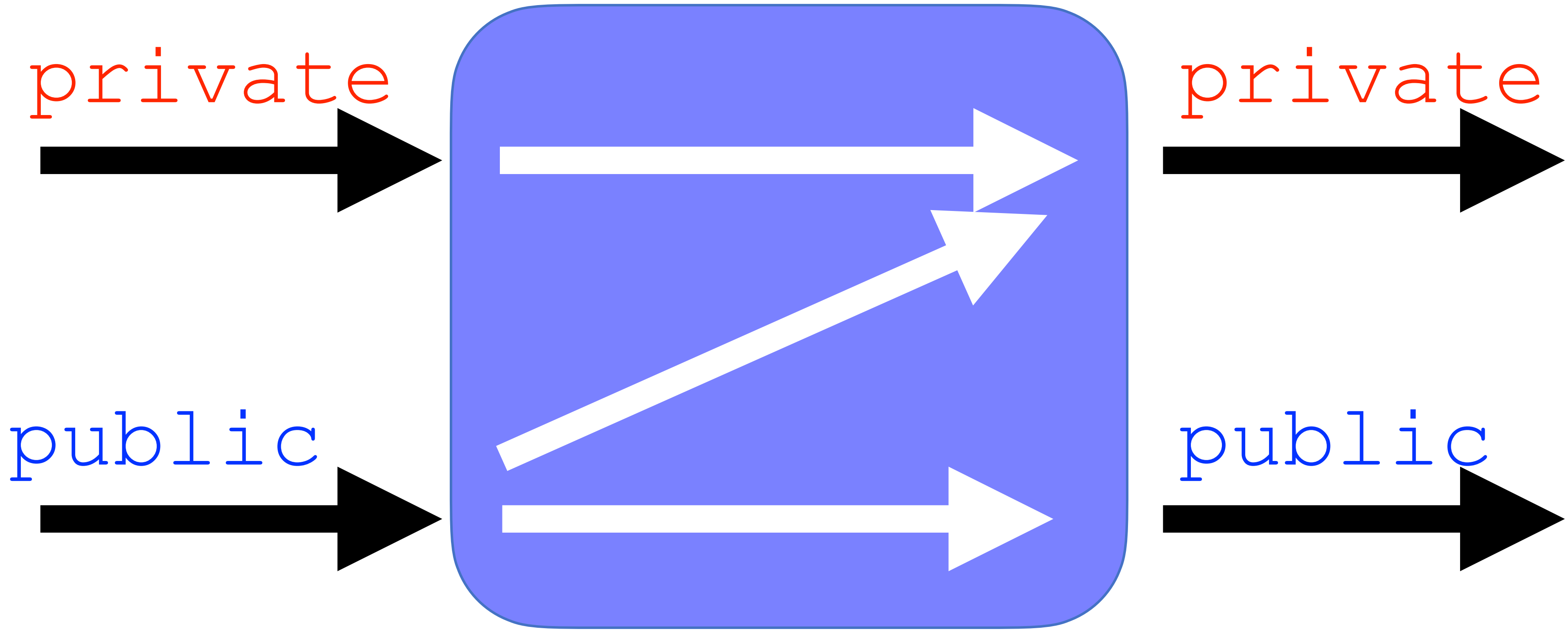
We want to distinguish confidential information that need to be kept secret from nonconfidential information that can be accessed by everyone.

We assume that every variable is tagged with one either public or private.

`x:public` `x:private`

Information Flow Control

We want to guarantee that confidential information do not flow in what is considered nonconfidential.



Is this program secure?

```
x:private
y:public

if y mod 3 = 0 then
  x:=1
else
  x:=0
```

Is this program secure?

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x:private
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Secure

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```

Insecure

Today: Noninterference - Relational Hoare Logic

How can we formulate a policy that forbids flows from private to public?

Low equivalence

Two memories m_1 and m_2 are low equivalent if and only if they coincide in the value that they assign to public variables.

In symbols: $m_1 \sim_{low} m_2$

Noninterference

A program `prog` is noninterferent if and only if, whenever we run it on two low equivalent memories m_1 and m_2 we have that:

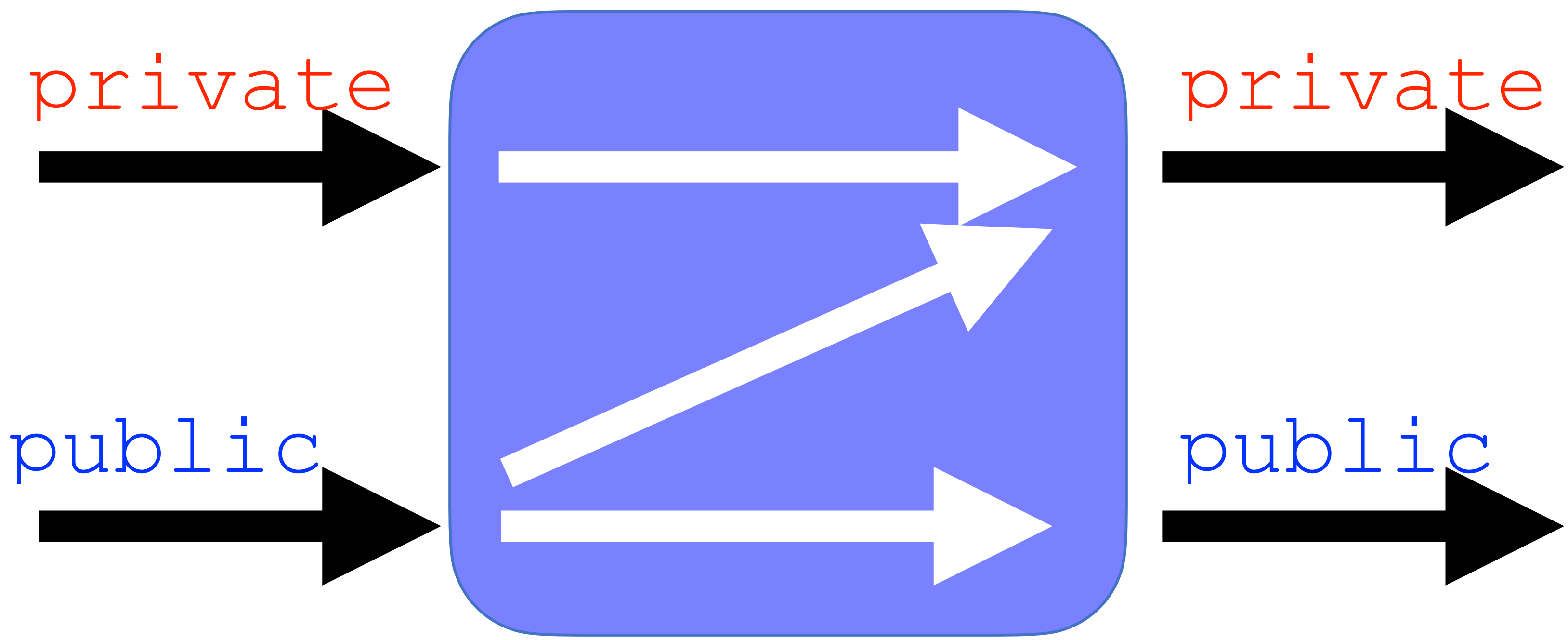
1) Either both terminate or both non-terminate
2) If they both terminate we obtain two low equivalent memories m_1' and m_2'.

Noninterference

In symbols, c is noninterferent if and only if for every $m_1 \sim_{low} m_2$:

1) $\{c\}_{m1}=\perp$ iff $\{c\}_{m2}=\perp$

2) $\{c\}_{m1}=m_1'$ and $\{c\}_{m2}=m_2'$ implies $m_1' \sim_{low} m_2'$



How can we prove our programs noninterferent?

Does this program satisfy noninterference?

```
x:private
y:public

x:=y
```

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Yes

Does this program satisfy noninterference?

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x:private
y:public

x:=y
```

Yes

$m^{in}{}_1=[x=n_1,y=k]$

Does this program satisfy noninterference?

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x:private
y:public

x:=y
```

Yes

$m^{in}_1=[x=n_1,y=k]$

$m^{in}_2=[x=n_2,y=k]$

Does this program satisfy noninterference?

```
x:private
y:public

x:=y
```

Yes

$m^{in}_1=[x=n_1,y=k]$

$m^{out}_1=[x=k,y=k]$

$m^{in}_2=[x=n_2,y=k]$

$m^{out}_2=[x=k,y=k]$

Does this program satisfy noninterference?

```
x:private
y:public

y:=x
```

Does this program satisfy noninterference?

```
x:private
y:public

y:=x
```

No

Does this program satisfy noninterference?

```
x:private
y:public

y:=x
```

No

$m^{in}_1=[x=n_1,y=k]$

Does this program satisfy noninterference?

```
x:private
y:public

y:=x
```

No

$m^{in}_1=[x=n_1,y=k]$

$m^{in}_2=[x=n_2,y=k]$

Does this program satisfy noninterference?

```
x:private
y:public

y:=x
```

$\boxed{\text{No}}$

$m^{in}_1=[x=n_1,y=k]$

$m^{out}_1=[x=n_1,y=n_1]$

$m^{in}_2=[x=n_2,y=k]$

$m^{out}_2=[x=n_2,y=n_2]$

Does this program satisfy noninterference?

```
x:private
y:public

y:=x
y:=5
```

Does this program satisfy noninterference?

```
x:private
y:public

y:=x
y:=5
```

Yes

Does this program satisfy noninterference?

```
x:private
y:public

y:=x
y:=5
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Yes

$m^{in}_1=[x=n_1,y=k]$

Does this program satisfy noninterference?

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```

Yes

$m^{in}_1=[x=n_1,y=k]$

$m^{in}_2=[x=n_2,y=k]$

Does this program satisfy noninterference?

```
x:private
y:public

y:=x
y:=5
```

Yes

$m^{in}_1=[x=n_1,y=k]$

$m^{out}_1=[x=n_1,y=5]$

$m^{in}_2=[x=n_2,y=k]$

$m^{out}_2=[x=n_2,y=5]$

Does this program satisfy noninterference?

```
x:private
y:public
if y mod 3 = 0 then
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Does this program satisfy noninterference?

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if y mod 3 = 0 then
  x:=1
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```

Yes

Does this program satisfy noninterference?

```
x:private
y:public
if y mod 3 = 0 then
  x:=1
else
  x:=0
```

Yes

m^{in}_1=[x=n_1,y=6]

Does this program satisfy noninterference?

```
x:private
y:public
if y mod 3 = 0 then
  x:=1
else
  x:=0
```

Yes

m^{in}_1=[x=n_1,y=6] m^{in}_2=[x=n_2,y=6]

Does this program satisfy noninterference?

```
x:private
y:public
if y mod 3 = 0 then
  x:=1
else
  x:=0
```

Yes

$m^{in}_1=[x=n_1,y=6]$ $m^{in}_2=[x=n_2,y=6]$

$m^{out}_1=[x=1,y=6]$ $m^{out}_2=[x=1,y=6]$

Does this program satisfy noninterference?

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x:private
y:public
if x mod 3 = 0 then
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Does this program satisfy noninterference?

```
x:private
y:public
if x mod 3 = 0 then
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else
  y:=0
```

No

Does this program satisfy noninterference?

```
x:private
y:public
if x mod 3 = 0 then
  y:=1
else
  y:=0
```

No

$m^{in}_1=[x=6,y=k]$

Does this program satisfy noninterference?

```
x:private
y:public
if x mod 3 = 0 then
  y:=1
else
  y:=0
```

No

m^{in}_1=[x=6,y=k]

m^{in}_2=[x=5,y=k]

Does this program satisfy noninterference?

```
x:private
y:public
if x mod 3 = 0 then
  y:=1
else
  y:=0
```

No

m^{in}_1=[x=6,y=k]

m^{in}_2=[x=5,y=k]

m^{out}_1=[x=6,y=1]

m^{out}_2=[x=5,y=0]

Does this program satisfy noninterference?

```
s1:public
s2:private
r:private
i:public

proc Compare (s1:list[n] bool,s2:list[n] bool)
i:=0;
r:=0;
while i<n /\ r=0 do
 if not(s1[i]=s2[i]) then
     r:=1
 i:=i+1
```

Does this program satisfy noninterference?

```
s1:public
s2:private
r:private
i:public

proc Compare (s1:list[n] bool,s2:list[n] bool)
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No

How can we prove our programs noninterferent?

Noninterference

In symbols, c is noninterferent if and only if for every $m_1 \sim_{low} m_2$:

1) $\{c\}_{m1} = \perp$ iff $\{c\}_{m2} = \perp$

2) $\{c\}_{m1} = m_1'$ and $\{c\}_{m2} = m_2'$ implies $m_1' \sim_{low} m_2'$

Is this condition easy to check?

Can we use the tool we studied so far?

Precondition
(a logical formula)

`Precondition`
`Program`
`Postcondition`

$$c : P \Rrightarrow Q$$

Program

Postcondition
(a logical formula)

Validity of Hoare triple

We say that the triple $c : P \Rightarrow Q$ is valid

if and only if

for every memory m such that $P(m)$

and memory m' such that $\{c\}_m = m'$

we have $Q(m')$.

Validity of Hoare triple

We say that the triple $\texttt{c:P}\Rightarrow\texttt{Q}$ is valid
if and only if
for every memory $\texttt{m}$ such that $\texttt{P(m)}$
and memory m' such that $\texttt{\{c\}}_{\texttt{m}}\texttt{=m'}$
we have $\texttt{Q(m')}$.

Validity talks only about one memory. How can we manage two memories?

Relational Property

In symbols, c is noninterferent if and only if for every $m_1 \sim_{low} m_2$:

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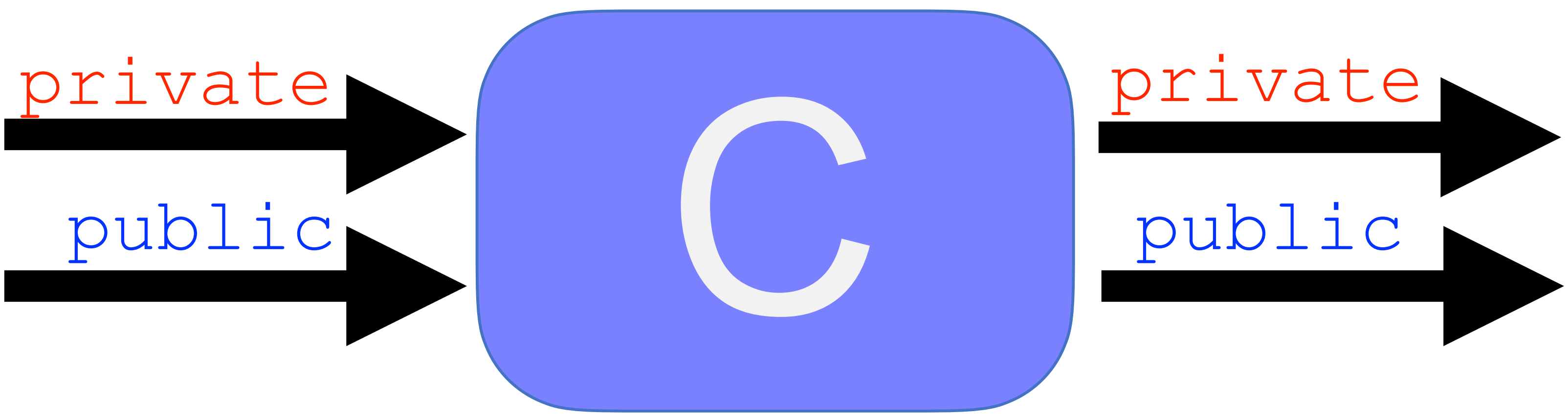

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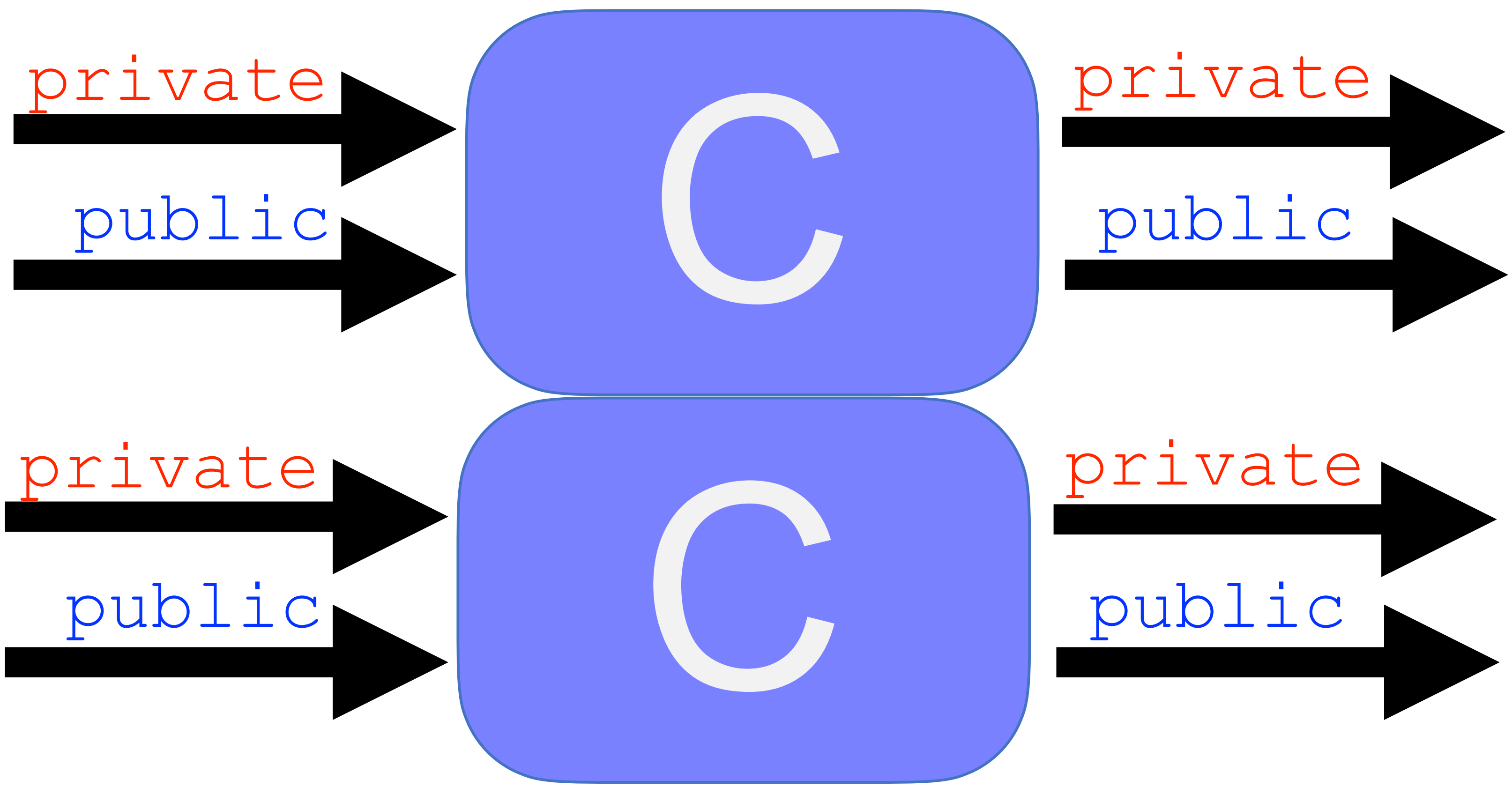

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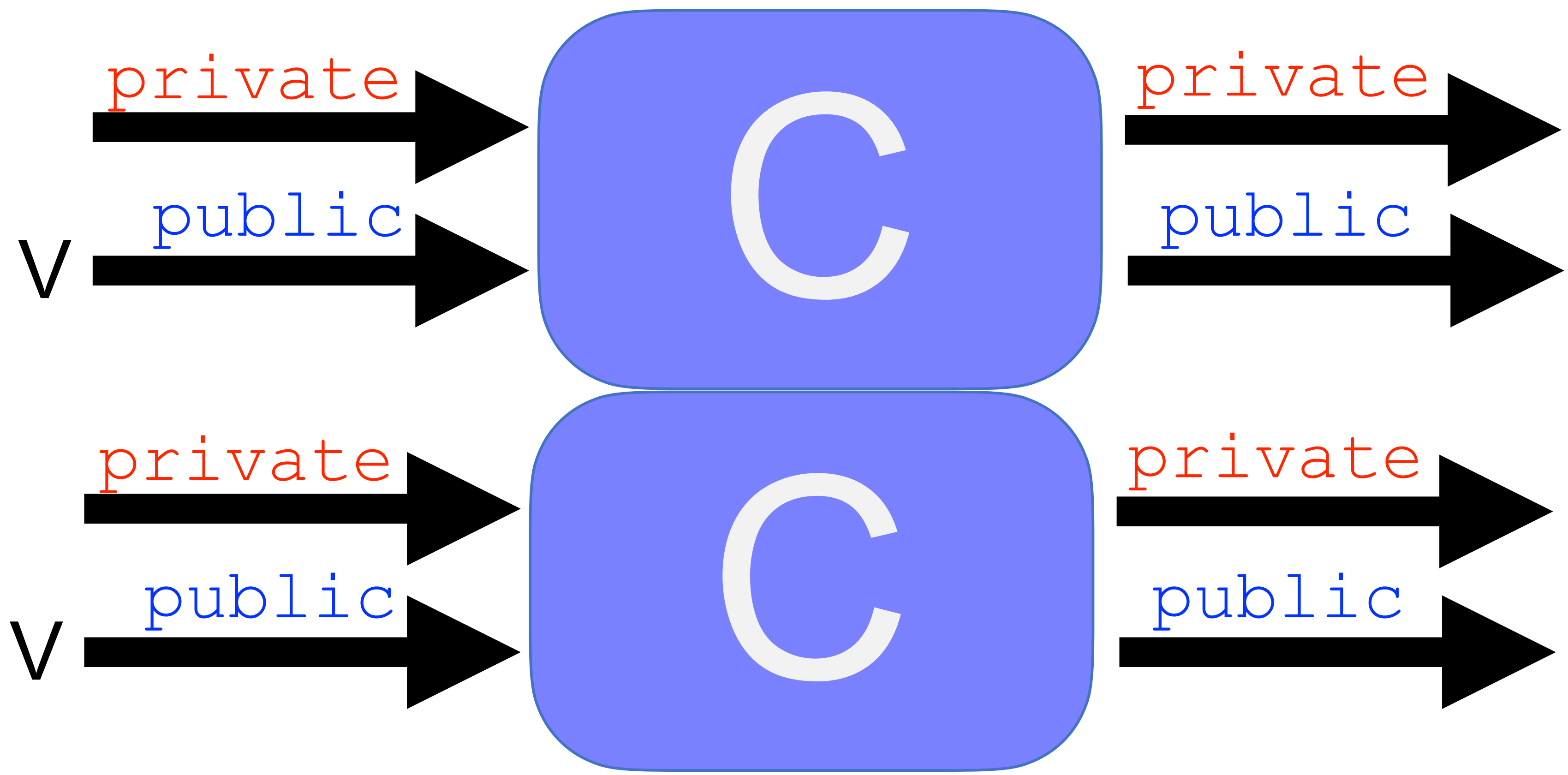

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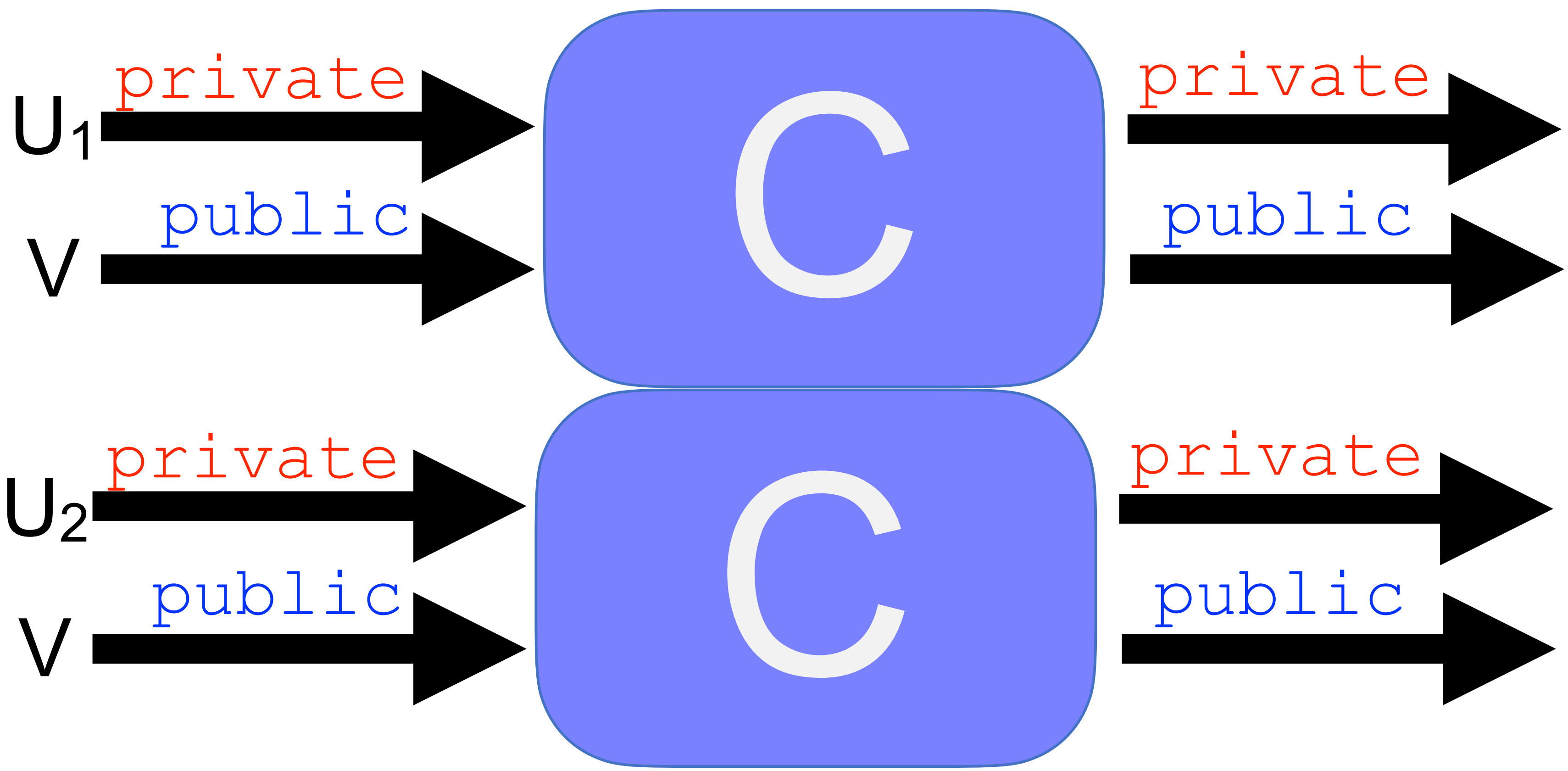

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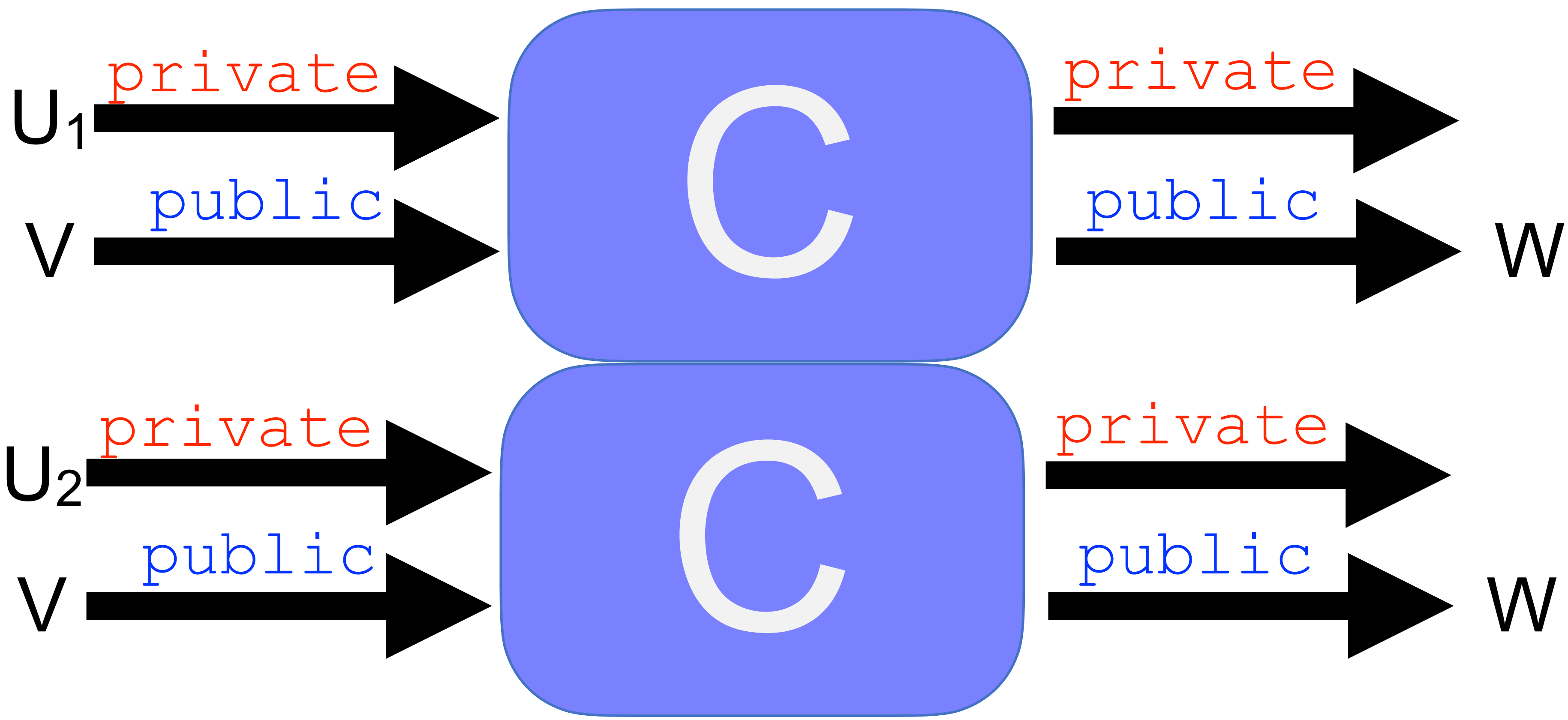

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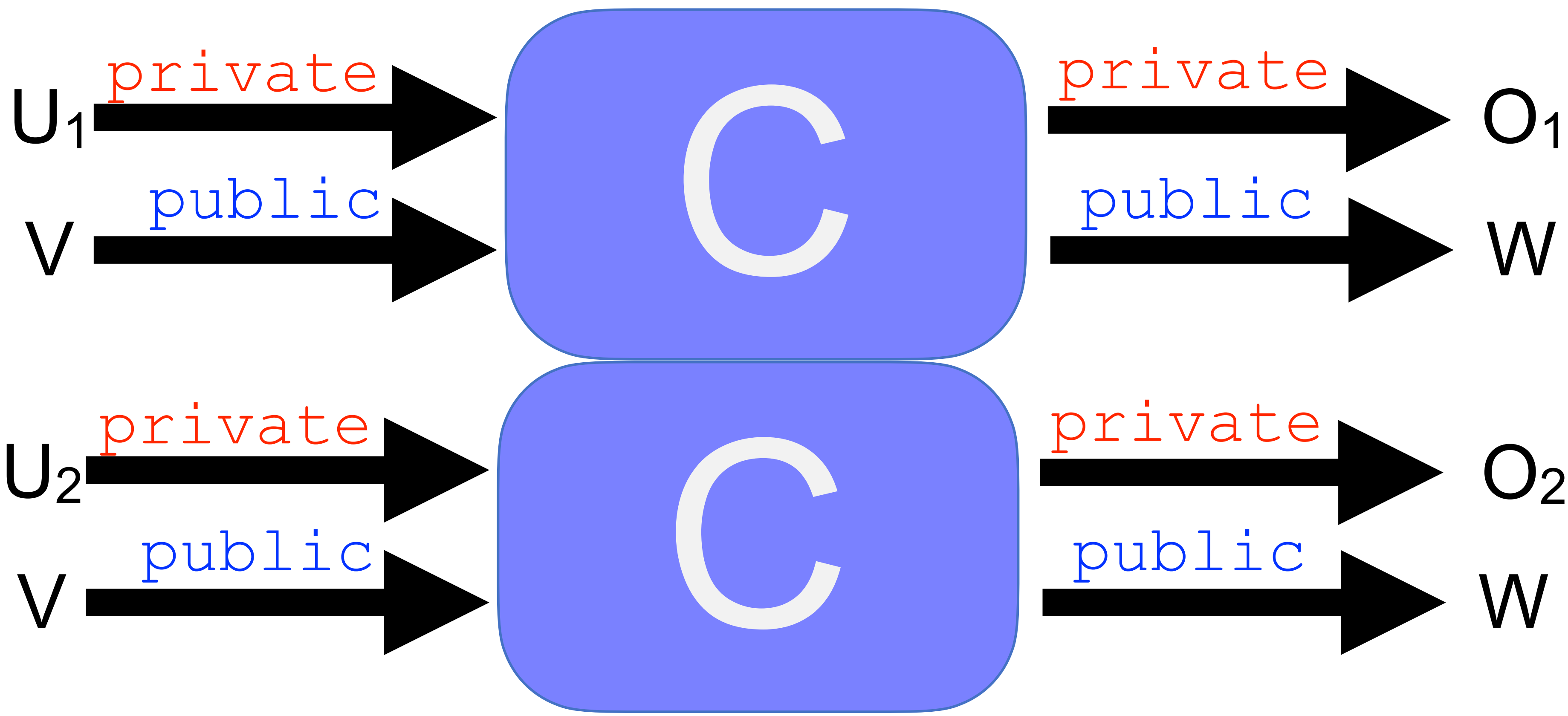

Relational Hoare Logic - RHL

Precondition
(a logical formula)

Precondition

Program$_1$ ~ Program$_2$

Postcondition

$$c_1 \sim c_2 : P \Rightarrow Q$$

Program

Program

Postcondition
(a logical formula)

Relational Assertions

$$c_1 \sim c_2 : P \Rightarrow Q$$

Need to talk about variables
of the two memories

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$$c_1 \sim c_2 : P \Rightarrow Q$$

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$$c_1 \sim c_2 : x\langle 1 \rangle \leq x\langle 2 \rangle \Rightarrow x\langle 1 \rangle \geq x\langle 2 \rangle$$

Relational Assertions

$$c_1 \sim c_2 : P \Rightarrow Q$$

Need to talk about variables
of the two memories

$$c_1 \sim c_2 : x\langle 1\rangle \leq x\langle 2\rangle \Rightarrow x\langle 1\rangle \geq x\langle 2\rangle$$

Tags describing which
memory we are referring to.

Validity of Hoare quadruple

We say that the quadruple $c_1 \sim c_2 : P \Rightarrow Q$ is valid if and only if for every pair of memories m_1, m_2 such that $P(m_1, m_2)$ we have:

1) $\{c_1\}_{m1} = \perp$ iff $\{c_2\}_{m2} = \perp$
2) $\{c_1\}_{m1} = m_1'$ and $\{c_2\}_{m2} = m_2'$ implies $Q(m_1', m_2')$.

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Is this easy to check?

Rules of Relational Hoare Logic
Skip

$$\frac{}{\vdash \texttt{skip} \sim \texttt{skip} : P \Rightarrow P}$$

Correctness of an axiom

We say that an axiom is correct if we can prove the validity of each instance of the conclusion.

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Is this still good for RHL?

Correctness of Skip Rule

$$\frac{}{\vdash \texttt{skip} \sim \texttt{skip} : P \Rightarrow P}$$

To show this rule correct we need to show the validity of the quadruple `skip~skip: P⇒P`.

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For every m_1, m_2 such that $P(m_1, m_2)$ and m_1', m_2' such that $\{skip\}_{m1} = m_1'$ and $\{skip\}_{m2} = m_2'$ we need $P(m_1', m_2')$.

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Follow easily by our semantics:

$\{\texttt{skip}\}_m=m$

Rules of Relational Hoare Logic

Abort

$$\overline{\vdash \texttt{abort} \sim \texttt{abort} : \texttt{true} \Rightarrow \texttt{false}}$$

Rules of Relational Hoare Logic
Abort

$\vdash \texttt{abort} \sim \texttt{abort} : \texttt{true} \Rightarrow \texttt{false}$

To show this rule correct we need to show the validity of the quadruple $\texttt{abort} \sim \texttt{abort} : \texttt{T} \Rightarrow \texttt{F}$.

Rules of Relational Hoare Logic
Abort

$\vdash \texttt{abort} \sim \texttt{abort} : \texttt{true} \Rightarrow \texttt{false}$

To show this rule correct we need to show the validity of the quadruple $\texttt{abort} \sim \texttt{abort} : \texttt{T} \Rightarrow \texttt{F}$.

For every $\texttt{m}_1, \texttt{m}_2$ such that $\texttt{P}(\texttt{m}_1, \texttt{m}_2)$ we can show $\{\texttt{abort}\}_{\texttt{m1}} = \bot$ iff $\{\texttt{abort}\}_{\texttt{m2}} = \bot$.

Rules of Relational Hoare Logic
Abort

$\vdash \texttt{abort} \sim \texttt{abort} : \texttt{true} \Rightarrow \texttt{false}$

To show this rule correct we need to show the validity of the quadruple $\texttt{abort} \sim \texttt{abort} : \texttt{T} \Rightarrow \texttt{F}$.

For every m_1, m_2 such that $P(m_1, m_2)$ we can show $\{\texttt{abort}\}_{m1} = \bot$ iff $\{\texttt{abort}\}_{m2} = \bot$.

Follow easily by our semantics:

$\{\texttt{abort}\}_{m} = \bot$

Rules of Relational Hoare Logic

Assignment

$$\overline{\vdash \texttt{x}_1 \texttt{:=} \texttt{e}_1 \sim \texttt{x}_2 \texttt{:=} \texttt{e}_2 \texttt{:} \quad \texttt{P}[\texttt{e}_1\texttt{<1>}/\texttt{x}_1\texttt{<1>}, \texttt{e}_2\texttt{<2>}/\texttt{x}_2\texttt{<2>}] \Rightarrow \texttt{P}}$$

Rules of Relational Hoare Logic
Composition

$$\dfrac{\vdash c_1 \sim c_2 : P \Rightarrow R \qquad\qquad \vdash c_1' \sim c_2' : R \Rightarrow S}{\vdash c_1 ; c_1' \sim c_2 ; c_2' : P \Rightarrow S}$$

Rules of Relational Hoare Logic Consequence

$$\frac{P \Rightarrow S \qquad \vdash \mathtt{c}_1 \sim \mathtt{c}_2 : S \Rightarrow R \qquad R \Rightarrow Q}{\vdash \mathtt{c}_1 \sim \mathtt{c}_2 : P \Rightarrow Q}$$

We can weaken P, i.e. replace it by something that is implied by P. In this case S.

We can strengthen Q, i.e. replace it by something that implies Q. In this case R.

Rules of Hoare Logic
If then else

$$\vdash c_1 \sim c_2 : e_1\langle 1\rangle \wedge e_2\langle 2\rangle \wedge P \Rightarrow Q$$

$$\vdash c_1' \sim c_2' : \neg e_1\langle 1\rangle \wedge \neg e_2\langle 2\rangle \wedge P \Rightarrow Q$$

$$\frac{\vdash c_1 \sim c_2 : e_1\langle 1\rangle \wedge e_2\langle 2\rangle \wedge P \Rightarrow Q \qquad \vdash c_1' \sim c_2' : \neg e_1\langle 1\rangle \wedge \neg e_2\langle 2\rangle \wedge P \Rightarrow Q}{\vdash \begin{array}{c}\texttt{if } e_1 \texttt{ then } c_1 \texttt{ else } c_1' \\ \sim \\ \texttt{if } e_2 \texttt{ then } c_2 \texttt{ else } c_2'\end{array} : P \Rightarrow Q}$$

Rules of Hoare Logic
If then else

$$\vdash c_1 \sim c_2 : e_1\langle 1\rangle \wedge e_2\langle 2\rangle \wedge P \Rightarrow Q$$

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$$\frac{}{\vdash \begin{array}{c} \texttt{if } e_1 \texttt{ then } c_1 \texttt{ else } c_1' \\ \sim \\ \texttt{if } e_2 \texttt{ then } c_2 \texttt{ else } c_2' \end{array} : P \Rightarrow Q}$$

Is this correct?

An example

$$\vdash \begin{array}{c}\texttt{if true then skip else x:=x+1} \\ \sim \\ \texttt{if false then x:=x+1 else skip}\end{array} : \{x=n\} \Rightarrow \{x=n+1\}$$

Is this a valid quadruple?

An example

$$\vdash \begin{array}{c}\texttt{if true then skip else x:=x+1} \\ \sim \\ \texttt{if false then x:=x+1 else skip}\end{array} : \texttt{\{x=n\}} \Rightarrow \texttt{\{x=n+1\}}$$

Is this a valid quadruple? ✗

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Is this a valid quadruple? ✗

Can we prove it with the rule above?

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Is this a valid quadruple? ✗

Can we prove it with the rule above? ✓

Rules of Relational Hoare Logic
If then else

$$P \Rightarrow e_1\langle 1\rangle = e_2\langle 2\rangle$$

$$\vdash c_1 \sim c_2 : e_1\langle 1\rangle \wedge P \Rightarrow Q$$

$$\vdash c_1' \sim c_2' : \neg e_1\langle 1\rangle \wedge P \Rightarrow Q$$

$$\vdash \begin{array}{c} \texttt{if } e_1 \texttt{ then } c_1 \texttt{ else } c_1' \\ \sim \\ \texttt{if } e_2 \texttt{ then } c_2 \texttt{ else } c_2' \end{array} : P \Rightarrow Q$$

Rules of Hoare Logic
While

$$P \Rightarrow e_1\langle 1\rangle = e2\langle 2\rangle$$

$$\frac{\vdash c_1 \sim c_2 \;:\; e_1\langle 1\rangle \wedge P \Rightarrow P}{\vdash \begin{array}{c}\texttt{while } e_1 \texttt{ do } c_1 \\ \sim \\ \texttt{while } e_2 \texttt{ do } c_2\end{array} : P \Rightarrow P \wedge \neg e_1\langle 1\rangle}$$

Invariant

How can we prove this?

```
x:private
y:public

x:=y
```

$: \quad =_{\mathtt{low}} \Rightarrow \; =_{\mathtt{low}}$

How can we prove this?

```
x:private
y:public

y:=x
```

$: \; =_{low} \Rightarrow \neg(=_{low})$

How can we prove this?

```
x:private
y:public

y:=x
```

$: \; =_{\mathtt{low}} \Rightarrow \neg(=_{\mathtt{low}})$

Can we prove it?

How can we prove this?

```
x:private
y:public

y:=x
y:=5
```

$: \quad =_{\mathtt{low}} \Rightarrow \quad =_{\mathtt{low}}$

How can we prove this?

```
x:private
y:public

if y mod 3 = 0 then
 x:=1
else
 x:=0


:  =low ⇒ =low
```

How can we prove this?

```
x:private
y:public

if x mod 3 = 0 then
  y:=1
else
  y:=1
```

$: \; =_{low} \Rightarrow \; =_{low}$

How can we prove this?

```
x:private
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if x mod 3 = 0 then
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Can we prove it?

$: \quad =_{low} \Rightarrow \; =_{low}$

Rules of Relational Hoare Logic
If then else

$$\frac{\begin{array}{c} P \Rightarrow e_1\langle 1\rangle = e_2\langle 2\rangle \\ \vdash c_1 \sim c_2 : e_1\langle 1\rangle \wedge P \Rightarrow Q \\ \vdash c_1' \sim c_2' : \neg e_1\langle 1\rangle \wedge P \Rightarrow Q \end{array}}{\vdash \begin{array}{c} \texttt{if } e_1 \texttt{ then } c_1 \texttt{ else } c_1' \\ \sim \\ \texttt{if } e_2 \texttt{ then } c_2 \texttt{ else } c_2' \end{array} : P \Rightarrow Q}$$

Rules of Relational Hoare Logic
If then else - left

$$\vdash c_1 \sim c_2 : e\langle 1\rangle \wedge P \Rightarrow Q$$

$$\vdash c_1' \sim c_2 : \neg e\langle 1\rangle \wedge P \Rightarrow Q$$

$$\vdash \begin{array}{c} \texttt{if } e \texttt{ then } c_1 \texttt{ else } c_1' \\ \sim \\ c_2 \end{array} : P \Rightarrow Q$$

Rules of Relational Hoare Logic
If then else - left

$$\vdash c_1 \sim c_2 : e\langle 2\rangle \wedge P \Rightarrow Q$$

$$\vdash c_1 \sim c_2' : \neg e\langle 2\rangle \wedge P \Rightarrow Q$$

$$\vdash \begin{array}{c} c_1 \\ \sim \\ \texttt{if } e \texttt{ then } c_2 \texttt{ else } c_2' \end{array} : P \Rightarrow Q$$

How can we prove this?

```
x:private
y:public

if x mod 3 = 0 then
  y:=1
else
  y:=1
```

$: \; =_{low} \Rightarrow \; =_{low}$

How can we prove this?

```
x:public
z:public
y:private

y:=0
z:=0
if x=0 then z:=1;
if z=0 then y:=1
```

: $=_{low} \Rightarrow \; =_{low}$

How can we prove this?

```
x:private
z:public
y:private

y:=0
z:=0
if x=0 then z:=1;
if z=0 then y:=1
```

$: \; =_{low} \Rightarrow \neg(=_{low})$

How can we prove this?

```
s1:public
s2:private
r:private
i:public

proc Compare (s1:list[n] bool,s2:list[n] bool)
i:=0;
r:=0;
while i<n /\ r=0 do
 if not(s1[i]=s2[i]) then
     r:=1
 i:=i+1

: n>0 /\ =low ⇒ ¬(=low)
```

How can we prove this?

```
s1:public
s2:private
r:private
i:public

proc Compare (s1:list[n] bool,s2:list[n] bool)
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```