Online-Erasure-Resilient Testers

**Theorem 1.** Linearity and quadraticity can be tested with online erasures with the same query complexity as in standard property testing.

A function \( f: \{0,1\}^d \rightarrow \{0,1\} \) is linear if it is a polynomial of degree at most 1; quadratic if polynomial of degree at most 2.

**Linearity.** BLR tester \([1]\) optimal for no erasures. Repeat \( O(1/\varepsilon) \) times:
- Sample pair \((x,y)\).
- Reject if \( f(x) + f(y) \neq f(x+y) \).

**Issue** with 1-online-erasure oracle: once \( x \) and \( y \) are queried, oracle erases \( x+y \).

Our **tester**
- Sample and query reserve of \( O(t/\varepsilon) \) points.
- Query sums of points sampled from reserve.

**Quadraticity.** Tester of Alon et al. \([2]\) looks for more complicated witnesses.

**Lower Bounds**

**Theorem 2.** Some properties are impossible to test even with a 1-online-erasure oracle: sortedness and Lipschitz property of sequences.

Sorted sequence: \( f(x) \leq f(y) \) for all \( x < y \).

If no erasures, can be tested with \( O(\log n) \) queries \([3]\) or \( O(\sqrt{n}) \) uniform queries \([4]\).

Hard to test instances: