

Perceptually Based Approach to Color Quantization

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ABSTRACT

The process of selecting a small number of representative colors from an image of higher color resolution is called color image quantization. The ultimate goal of color image quantization is to minimize visible distortion. While its application as a frame buffer technique requires that algorithm efficiency is crucial. In this paper, a significantly faster quantization strategy than previous methods: median cut, variance, or octree-based algorithms, etc., is suggested. The new perceptually algorithm integrated with gamma correction produces result approximately as accurate as previous methods. Overall, the new proposed method is a preferable tradeoff between the quantizer complexity and visible distortion of the quantized image.

Keywords: color image quantization, perception, gamma correction, clustering

1. INTRODUCTION

Traditionally, color image quantization is used to reproduce 24-bit images on graphics hardware with a limited number of simultaneous colors. Even though 24-bits graphics hardware is becoming more affordable, color image quantization will still retain its practical value on relieving valuable frame buffer space for animation, window applications, and other graphics functions.

In 1982, Heckbert[1] proposed the popular median-cut algorithm. The partition point is the median point of the projected color distribution along the coordinate axis with the largest color spread. Later, sweeping a cutting plane perpendicular to the R, G, B axes separately, Wan[2] split the box at the position where the variance of the marginal distribution in the corresponding axis is minimized. Another two relatively fast algorithms are octree algorithm[3] and local K-mean algorithm(abbr. LKM) [4]. The principle of octree algorithm is to sequentially read original color stored in an octree of depth 8 (every leaf at depth 8 represents a distinct color). The LKM algorithm is similar to gradient quantization techniques used in gray scale image coding[5] and much faster than the classical K-means algorithm[6]. Considering the time-consuming transforms twice between RGB color space and perception uniform color space, the implement of above algorithms all work in RGB.

Motivated by the fact that previous algorithms are without regard to the perception of human vision, we proposed a new perceptually scheme integrated with gamma correction, which bypass the predicament that conversion of color spaces may bring and is significantly faster and approximately as accurate as previous methods. This paper is organized as follows. In next section, the formulations of the color quantization problem and a distortion measure to minimize the total quantization error are presented. Then, a quantity -Luma [7], which is roughly perceptually uniform, and gamma correction [7] are introduced. In section 3, the strategy of our algorithm is suggested in detail. Section 4 demonstrates the algorithm complexity, a performance comparison including the quality and RMS[8] error of quantized test image. At last, some related techniques are discussed in section 5.

2. RELEVANT CONCEPTIONS

2.1 Formulation of Color Quantization Problem

Mathematically, color quantization can be formulated as a large-scale clustering problem[9]. A color image of N pixels corresponds to a set S of N points, $c_i = (c_{i,0}, c_{i,1}, c_{i,2})$ $1 \leq i \leq N$, in a three-dimensional color space such as RGB,

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YIQ, $L^*u^*v^*$, HSV, etc. In color image quantization, the point set S is partitioned into K subsets $S_k, 1 \leq k \leq K (S_k \neq \emptyset, \cup_{1 \leq k \leq K} S_k = S)$ where all colors $c \in S_k$ are mapped to or approximated by, a representative color, which represents an entry in the color map. The optimal K -partition of the set S is found to minimize the quantization error E .

$$E(S_1, S_2, \dots, S_k) = \sum_{1 \leq k \leq K} \left\{ \frac{1}{|S_k|} \sum_{c_i, c_j \in S_k, i < j} \|c_i, c_j\| \right\} \quad (1)$$

where $\|c_i, c_j\|$ is the Euclidean L2-norm defined as:

$$\sum_{c_i, c_j \in S_k, i < j} \|c_i, c_j\| = \sum_{c_i, c_j \in S_k, i < j} [(c_{i,0} - c_{j,0})^2 + (c_{i,1} - c_{j,1})^2 + (c_{i,2} - c_{j,2})^2] \quad (2)$$

The representative colors for each region is the average of all the original colors the region encloses.

2.2 Gamma Correction

A meaningful measure $\|c_i, c_j\|$ should be performed in a perceptually uniform color space, in which $\|c_0, c_1\| = \|c_1, c_2\|$ if c_0 and c_1 differ as much as c_1 and c_2 in visual sensation. Thus, a perception uniform $L^*u^*v^*$ space (CIE 1976) is recommended. The lightness L^* is the perceptual response to luminance[7] and defined as following:

$$L^* = \begin{cases} 903.3 \frac{Y}{Y_n} & \frac{Y}{Y_n} \leq 0.008856 \\ 116 \left(\frac{Y}{Y_n} \right)^{\frac{1}{3}} - 16 & 0.008856 < \frac{Y}{Y_n} \end{cases} \quad (3)$$

Here, Y_n is the luminance of the white reference. Y indicates luminance, formed as a weighted sum of R , G , and B components. In fact, in video a nonlinear quantity luma Y' , which is roughly perceptually uniform and formed as a weighted sum of R' , G' , and B' [7], is used instead of CIE luminance Y . Namely, a identical nonlinear transfer function – gamma correction – is applied to each of R , G , and B components then output R' , G' , and B' . Here, we introduce the Rec.709 transfer function[7] based on a power function that mimics the lightness sensitivity of vision. Theoretically, this pure power function suffices for gamma correction. Two non-linear power transfer functions to each of tristimulus are described as following:

$$R' = (R \leq 0.018 ? 4.5R : -0.099 + 1.099R^{0.45}) \quad (4)$$

$$R = (R' \leq 0.081 ? R' / 4.5 : \frac{R' + 0.099}{1.099}^{\frac{1}{0.45}}) \quad (5)$$

Using color systems such as CIE $L^*u^*v^*$ is computationally prohibitive, but in practice it is adequate to use R' G' B' components instead in color quantization.

3. THE PROPOSED ALGORITHM

3.1 The strategy embedded gamma-correction

If gamma correction is adapted to previous methods most of which are estimated in order of second even minute, a fairly considerable time is impractical for implement of real-time system. In addition, uniform algorithm is noticeably quick, easy to implement but does not yield very good results. If this algorithm is applied in a non-uniform manner that the axes are broken on a logarithmic scale instead of linear since the human eye cannot distinguish dark colors as well as bright ones, it will produce only slightly better results for its image independence. Thus, if gamma correction is integrated into calculating the similar division position in image-dependent algorithm, it may quickly generate a fairly nice result.

Motivated by the idea above and using the divide-and-conquer strategy[16] for both spatial and quota divisions, we propose a new pre-clustering approach[6]. To address the implement issue to real-time system, the fast quantization algorithm is still performed in RGB color space. The color box that will be partitioned is "shrunk" to fit tightly around the colors it enclosed by finding the minimum and maximum values of each color coordinate. Introducing gamma correction to calculate the further division position in RGB color space is a practical solution that incurs negligible computational cost but scales the RGB space to partially compensate for its non-uniform nature. And the result is actually satisfied and superior to tricks [17] proposed by G Joy that each color cluster is bounded by its rectangular extent in the RGB color space, with a 2:1:4 proportional cluster size limit for red: green: blue. Moreover, to improve the effect of gradually-shade areas especially in photographic image, each subbox is not binary split one at a time in total isolation from each other. Two new generated subboxes have redundancy on the boundary points.

3.2 The implement of algorithm

The main implement of our quantization algorithm is described as followed:

(1) Always assign the subbox with the largest size in RGB for further partition. It can partly alleviate the problem that the box with a large quantization error may not be chosen for further partition since our partition criterion is without any regard to the contents of the box.

(2) Perpendicular to the RGB axis with the largest R'G'B' color spread by Eq. (4), the partition plane pass through the RGB cut-point corresponding to halved point of that R'G'B' spread. For example, when R axis is considered, the midpoint of the projected color spread along the R' axis is first calculated by (4). Then, the cut-point in R axis is obtained by applying (5) to that midpoint in R' axis.

(3) The subbox is orthogonally divided with redundant boundary points through cut-point obtained above. Then, repeat step (1) to (3) until the number of subbox equal to the number of color map K.

(4) The representative color of each subbox is calculated to fill the color map. Finally, the original image is redrawn by mapping each color pixel to the color map.

After the implementation of the above steps, our quantization algorithm produces approximately as accurate result as previous methods do in a significantly faster way.

4. THE COMPARISON AND ANALYSIS OF TEST RESULT

A significant form of distortion often encountered in a quantization image is usually visible as contouring and loss of image detail, especially in a computer-synthesized image or a high-quality photographic image with gradually shaded areas.

Firstly, the area of the image always takes on a stairstepped appearance as opposed to a smooth ramp. Introducing gamma correction, the algorithm actually trades tone to adapt reproduction to the perception of human vision in gradually-shade area. It is accord with that the human visual system is not able to determine the absolute value of a color and more sensitive to color variations. Furthermore, the redundant boundary color points create a favorable smooth ramp. Here, a ISO test photographic image called "snooker"[see Fig.1] is presented. For K=256, a comparison among the result of our algorithm [see Fig.2] and those of other methods[see Fig. 3-7] including that without gamma correction (i.e. cut-point is the halved point in largest spread in RGB) are given. In Fig.2, the shadow of snooker and the gradually-shade appearance of table are significantly more realistic than Fig.3. Fig.2 is obviously superior to Fig.4 and Fig.6 no matter when the shadow or surface of ball is considered. Moreover, evaluated from the subjective image fidelity, our result is also slightly superior to Fig.5 and approximately as accurate as Fig.7.

Secondly, significant image information is carried by some distinct but "rare" colors. If a quantization algorithm approximates the more popular colors, the average distortion might be small, but the "rare" colors of the original will be lost. Fortunately, the "rare" colors are kept in our algorithm. As following, the quantized image of test image called "dog" are demonstrated by various algorithm for K=16. Our algorithm is visibly superior to others in the region of dog tongue. Beside it, the entire effect of quantized image is also approach to the result of octree algorithm. Finally, there is no delay when it performs in a common PC. In Pentium II, it costs about 0.10 second including pre-quantization from 24 bits to 15 bits for K=256 quantized result of 320*240 ISO test image "snooker". Even it absolutely meets as an on-line algorithm for color quantization, i.e., dynamically update the contents of the color image according to the change of color distribution in time, for instance, during animation. The RMS error and time complexity comparison between our algorithm and others are summarized as follows.

	(K=256) for snooker	(K=16) for dog
our algorithm	11.41	23.48
Median Cut	4.12	16.44
Local K-means	4.33	22.70
Octree	1.74	15.70
Uniform	23.02	117.50

Table 1. RMS error comparison

our algorithm	$O(N)$
Median Cut	$O(m \log K N \log N)$
Local K-means	Can't be quantitatively defined, execution time is longer than that of our algorithm.
Octree	The average case needs to address the complexity of the merging algorithm, best case is $O(N)$.
Uniform	$O(N)$

Table 2. algorithm time complexity comparison

(N is the number of point in source image in a m - dimensional color space, K is the number of entry in color map.)

Here, new algorithm is one of quantization algorithms with the lowest time complexity. And its quantized results are best in all algorithm with time complexity $O(N)$. In the other hand, RMS error of new algorithm is much more

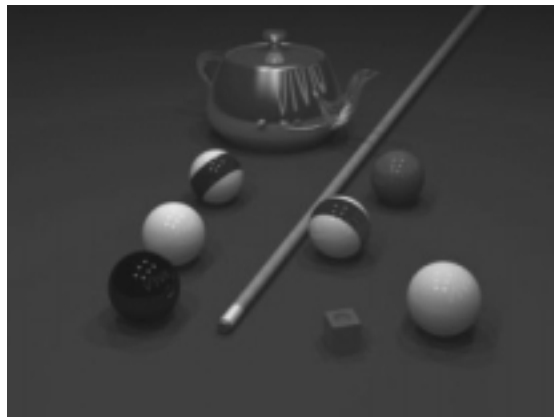


Fig. 1 snooker (24bit/pixel color test image)

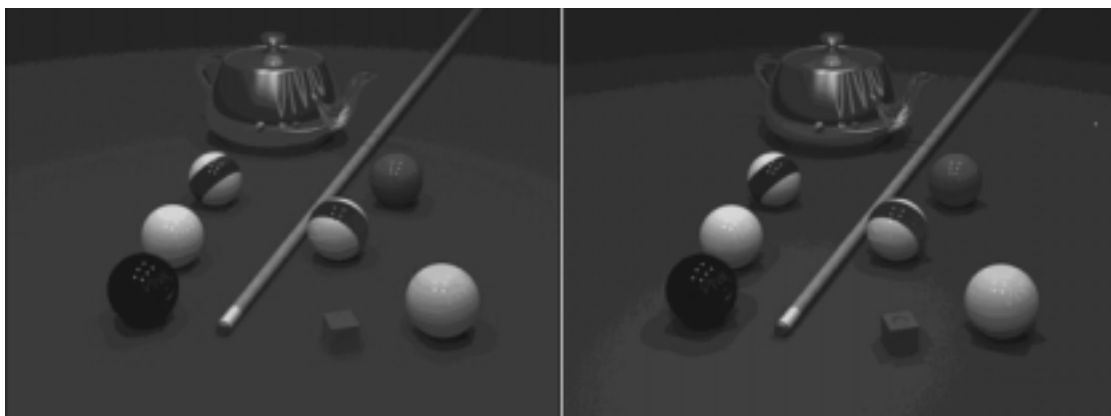


Fig. 2 our algorithm

Fig. 3 without gamma correction

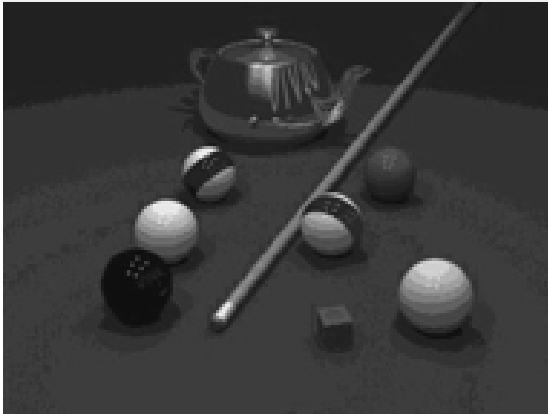


Fig. 4 uniform algorithm



Fig. 5 median-cut algorithm

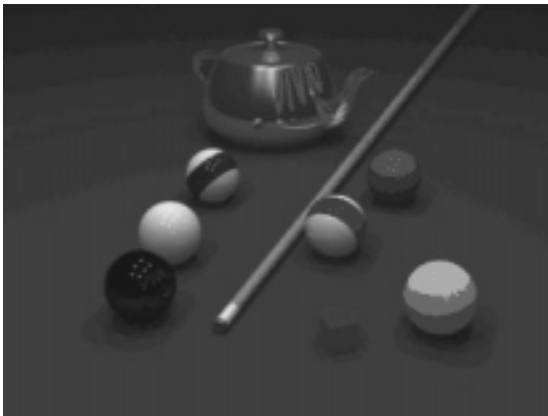


Fig. 6 LKM algorithm

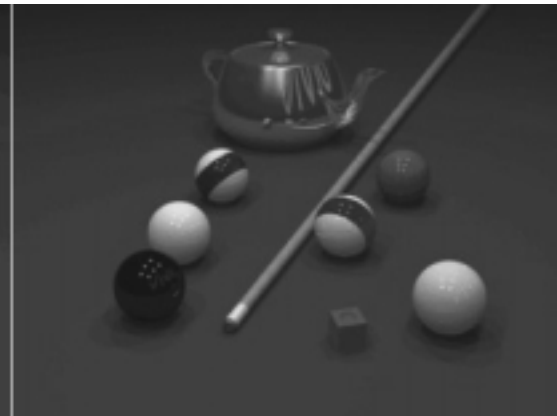


Fig. 7 octree algorithm



Fig. 8 dog(24bit/pixel test color image)



Fig. 9 our algorithm



Fig. 10 median-cut algorithm



Fig. 11 LKM algorithm



Fig. 12 octree algorithm



Fig. 13 uniform algorithm

inferior to the perceived image quality in our algorithm for its trading tone to optimal visual aesthetics. Whereas it is regarded worthy since the ultimate judgement of any quantization algorithm in practice should be subjective image fidelity.

5. DISCUSSION AND CONCLUSION

Currently, our proposed algorithm has been successfully applied to CAPSP[12], an intelligent CAD/CAM Weave System. Some related techniques are discussed below.

(1) The algorithm trade tone to generate a favorable gradually shaded area more meet with visual aesthetics in quantized image. Thus, some compensation according to the calibrated tone of original image can be adjusted to the quantized image.

(2) As the number of available output colors K decreased, perceived image quality is degraded. For $K < 64$, spatial halftoning techniques[13] can be used to increase the number of perceived colors beyond that actually present at the imaging output.

(3) Perceptual color measurement doesn't suffice to quantify perceptual color distance since image context also plays an important role in human vision. Several researchers[14] have been trying to extract and use contextual information to guide the quantization process.

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