Comp115: Databases

Relational Query Optimization

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Query Optimization

Overview

Readings: Chapter 12.4

Query optimization

Cost estimation

Plan enumeration and costing

System R strategy
Review of Query Processing

Implementation of single Relational Operations

Choices depend on indexes, memory, stats,…

Joins

- Blocked nested loops:
  - simple, exploits extra memory

- Indexed nested loops:
  - best if one relation small and one indexed

- Sort/Merge Join
  - good with small amount of memory, bad with duplicates

- Hash Join
  - fast (enough memory), bad with skewed data
Query Optimization

Typically many methods of executing a given query, all giving the same answer

Cost of alternative methods often varies enormously

Desirable to find a low-cost execution strategy

We will cover:

– Relational algebra equivalences
– Cost estimation
  • Result size estimation and reduction factors
  • Statistics and Catalogs
– Enumerating alternative plans

Will focus on “System R”-style optimizers
Refresh: Query execution

Query

Select *
From Blah B
Where B.blah = "foo"

Usually there is a heuristics-based rewriting step before the cost-based steps.

Query Parser

Query Optimizer

Plan Generator

Plan Cost Estimator

Catalog Manager

Schema

Statistics

Query Plan Evaluator
Query Plans

A tree, with relational algebra operators as nodes
Each operator labeled with choice of algorithm

Plan:

\[ \pi_{\text{sname}} \]

\[ \sigma_{\text{bid}=100 \land \text{rating} > 5} \]

\[ \bowtie \]

(sid=sid)

(sailors)

(reserves)

(On-the-fly)

(Page-Oriented
Nested loops)

By convention, outer is on left.
A note on implementation:

Relational operators at nodes support uniform *iterator* interface:

\[ \text{open()}, \text{get}\_\text{next}(), \text{close}() \]

**Unary Operators** – On open() call open() on child

**Binary Operators** – call open() on left child then on right
Query Optimization Overview

A Query:  

```sql
SELECT S.sname
FROM Reserves R, Sailors S
WHERE R.sid=S.sid AND
    R.bid=100 AND S.rating>5
```

To optimize:

1. Query first broken into “blocks”
2. Each block converted to relational algebra
3. Then, for each block, several alternative query plans are considered
4. Plan with lowest estimated cost is selected
A Familiar Schema for Examples

Sailors \((\text{sid}: \text{integer}, \text{sname}: \text{string}, \text{rating}: \text{integer}, \text{age}: \text{real})\)
Reserves \((\text{sid}: \text{integer}, \text{bid}: \text{integer}, \text{day}: \text{dates}, \text{rname}: \text{string})\)
Boats \((\text{bid}: \text{integer}, \text{bname}: \text{string}, \text{color}: \text{string})\)
Query Optimization

Overview

Query optimization

Readings: Chapters 15.1 and 15.3

Cost estimation

Plan enumeration and costing

System R strategy
Step 1: Break query into Query Blocks

Query block = unit of optimization

Nested blocks are usually treated as calls to a subroutine, made once per outer tuple

(This is an over-simplification, but serves for now)

Outer block

```
SELECT S.sname
FROM Sailors S
WHERE S.age IN
    (SELECT MAX (S2.age)
     FROM Sailors S2
     GROUP BY S2.rating)
```
Step 2: Converting query block into relational algebra expression

```
SELECT S.sid
FROM Sailors S, Reserves R, Boats B
WHERE S.sid = R.sid AND R.bid = B.bid AND B.color = "red"
```

\[ \pi_{S.sid}(\sigma_{B.color = "red"}(\text{Sailors} \bowtie \text{Reserves} \bowtie \text{Boats})) \]
SELECT S.sid, MIN(R.day)
FROM Sailors S, Reserves R, Boats B
WHERE S.sid = R.sid AND R.bid = B.bid AND B.color = "red"
AND S.rating = (SELECT MAX(S2.rating) FROM Sailors S2)
GROUP BY S.sid
HAVING COUNT(*) >= 2

For each sailor with the highest rating (over all sailors), and at least two reservations for red boats, find the sailor id and the earliest date on which the sailor has a reservation for a red boat.
Example translated to relational algebra

\[
\pi_{S.sid, \text{MIN}(R.day)} (H \text{AVING COUNT} (> 2) (G \text{ROUP BY S.Sid} (S \text{.color} = "red" \land S \text{.rating} = \text{val} (S \text{ailors} \bowtie R \text{eserves} \bowtie B \text{oats}))))
\]

SELECT S.sid, MIN (R.day) FROM Sailors S, Reserves R, Boats B WHERE S.sid = R.sid AND R.bid = B.bid AND B.color = "red" AND S.rating = (SELECT MAX (S2.rating) FROM Sailors S2) GROUP BY S.sid HAVING COUNT (*) >= 2
Select-Project-Join Optimization

Core of every query is a select-project-join (SPJ) expression

Other aspects, if any, carried out on result of SPJ core:
  - Group By (either sort or hash)
  - Having (apply filter on-the-fly)
  - Aggregation (easy once grouping done)
  - Order By (sorting is the name of the game)

Not much room to exploit equivalences on non-SPJ parts

Focus on optimizing SPJ core
Relational Algebra Equivalences

- **Selections:** \( \sigma_{c_1 \land \ldots \land c_n}(R) \equiv \sigma_{c_1} \left( \ldots \left( \sigma_{c_n}(R) \right) \right) \) (Cascade)
  \[ \sigma_{c_1} \left( \sigma_{c_2}(R) \right) \equiv \sigma_{c_2} \left( \sigma_{c_1}(R) \right) \] (Commute)

- **Projections:** \( \pi_{a_1}(R) \equiv \pi_{a_1} \left( \ldots \left( \pi_{a_n}(R) \right) \right) \) (Cascade)
  \[ a_i \text{ is a set of attributes of } R \text{ and } a_i \subseteq a_{i+1} \text{ for } i = 1 \ldots n-1 \]

- These equivalences allow us to ‘push’ selections and projections ahead of joins.
Examples ...

\[ \sigma_{\text{age}<18 \land \text{rating}>5} \text{ (Sailors)} \]

\[ \iff \sigma_{\text{age}<18} (\sigma_{\text{rating}>5} \text{ (Sailors)}) \]

\[ \iff \sigma_{\text{rating}>5} (\sigma_{\text{age}<18} \text{ (Sailors)}) \]

\[ \pi_{\text{age, rating}} \text{ (Sailors)} \iff \pi_{\text{age}} (\pi_{\text{rating}} \text{ (Sailors)}) \]

\[ \pi_{\text{age, rating}} \text{ (Sailors)} \iff \pi_{\text{age, rating}} (\pi_{\text{age, rating, sid}} \text{ (Sailors)}) \]
Another Equivalence

A projection commutes with a selection that only uses attributes retained by the projection

\[ \pi_{\text{age}, \text{rating}, \text{sid}} \left( \sigma_{\text{age} < 18} \land \text{rating} > 5 \right) \text{ (Sailors)} \leftrightarrow \sigma_{\text{age} < 18} \land \text{rating} > 5 \left( \pi_{\text{age}, \text{rating}, \text{sid}} \text{ (Sailors)} \right) \]
Equivalences Involving Joins

\[ R \bowtie (S \bowtie T) \equiv (R \bowtie S) \bowtie T \] (Associative)

\[ (R \bowtie S) \equiv (S \bowtie R) \] (Commutative)

These equivalences allow us to choose different join orders
Mixing Joins with Selections & Projections

Converting selection + cross-product to join

$$\sigma_{S.\text{sid} = R.\text{sid}} \ (\text{Sailors} \times \text{Reserves})$$

$$\leftrightarrow \ \text{Sailors} \ \Join_{S.\text{sid} = R.\text{sid}} \ \text{Reserves}$$

Selection on just attributes of S commutes with R\Join S

$$\sigma_{S.\text{age}<18} \ (\text{Sailors} \ \Join_{S.\text{sid} = R.\text{sid}} \ \text{Reserves})$$

$$\leftrightarrow \ (\sigma_{S.\text{age}<18} \ (\text{Sailors})) \ \Join_{S.\text{sid} = R.\text{sid}} \ \text{Reserves}$$

We can also “push down” projection (**but be careful**...)

$$\pi_{S.\text{name}} \ (\text{Sailors} \ \Join_{S.\text{sid} = R.\text{sid}} \ \text{Reserves})$$

$$\leftrightarrow \ \pi_{S.\text{name}} \ (\pi_{\text{name, sid}} \ (\text{Sailors}) \ \Join_{S.\text{sid} = R.\text{sid}} \ \pi_{\text{sid}} \ (\text{Reserves}))$$
What do you think? True or False?

1. $R \times S = S \times R$
2. $(R \times S) \times T = R \times (S \times T)$
3. $\sigma_p(R \cup S) = \sigma_p(R) \cup S$
4. $R \cup S = S \cup R$
5. $\sigma_p(R - S) = R - \sigma_p(S)$
6. $R \cup (S \cup T) = (R \cup S) \cup T$
7. $\sigma_{R.p \lor S.q} (R \bowtie S) = (\sigma_p R) \bowtie S \cup R \bowtie (\sigma_q S)$
Query Rewriting

Modern DBMS’s may **rewrite** queries before the optimizer sees them.

Main purpose: **de-correlate** and/or **flatten** nested subqueries.

**De-correlation:**
- Convert correlated subquery into uncorrelated subquery.

**Flattening:**
- Convert query with nesting into query w/o nesting.
Example: Decorrelating a Query

```
SELECT  S.sid
FROM    Sailors S
WHERE   EXISTS
        (SELECT  *
         FROM    Reserves R
         WHERE   R.bid=103
         AND     R.sid=S.sid)
```

Equivalent uncorrelated query:
```
SELECT  S.sid
FROM    Sailors S
WHERE   S.sid IN
        (SELECT  R.sid
         FROM    Reserves R
         WHERE   R.bid=103)
```

**Advantage:** nested block only needs to be executed *once* (rather than once per S tuple)
Example: “Flattening” a Query

```
SELECT S.sid
FROM Sailors S
WHERE S.sid IN
  (SELECT R.sid
   FROM Reserves R
   WHERE R.bid=103)
```

Equivalent non-nested query:
```
SELECT S.sid
FROM Sailors S, Reserves R
WHERE S.sid=R.sid
AND R.bid=103
```

**Advantage**: can use a join algorithm + optimizer can select among join algorithms & reorder freely
Query transformations: Summary

Before optimizations, queries are flattened and de-correlated

Queries are first broken into blocks

Blocks are converted to relational algebra expressions

Equivalence transformations are used to push down selections and projections
Query Optimization

Overview

Query optimization

Cost estimation
  Readings: Chapter 15.2

Plan enumeration and costing

System R strategy
Recall: Query Optimization Overview

1. Query first broken into “blocks”
2. Each block converted to relational algebra
3. Then, for each block, several alternative query plans are considered
4. Plan with lowest estimated cost is selected

\[
\begin{align*}
\text{SELECT } & \text{S.sname} \\
\text{FROM } & \text{Reserves R, Sailors S} \\
\text{WHERE } & \text{R.sid=S.sid AND} \\
& \text{R.bid=100 AND S.rating>5}
\end{align*}
\]

\[
\pi_{\text{sname}} \sigma_{\text{bid}=100 \land \text{rating}>5} (\text{Reserves} \bowtie \text{Sailors})
\]
Cost-based Query Sub-System

Queries

Select *
From Blah B
Where B.blah = "foo"

Usually there is a heuristics-based rewriting step before the cost-based steps.

Steps 3 & 4

Catalog Manager

Plan Generator

Plan Cost Estimator

Query Plan Evaluator

Query Optimizer

Query Parser
Two Main Issues

1. For a given query, **what plans are considered?**
   Algorithm to search plan space for cheapest (estimated) plan.

2. How is the **cost of a plan estimated?**

*Ideally:* Want to find best plan.

*Reality:* Avoid worst plans!
Highlights of System R Optimizer

Impact:
- Most widely used currently; works well for < 10 joins

Cost estimation:
- Very inexact, but works okay in practice
- Statistics, maintained in system catalogs, used to estimate cost of operations and result sizes
- Considers combination of CPU and I/O costs
- More sophisticated techniques known now

Plan Space: Too large, must be pruned
- Only the space of left-deep plans is considered
- Cross products are avoided
Schema for Examples

Sailors (\textit{sid}: integer, \textit{sname}: string, \textit{rating}: integer, \textit{age}: real)
Reserves (\textit{sid}: integer, \textit{bid}: integer, \textit{day}: dates, \textit{rname}: string)

Reserves:
- Each tuple is 40 bytes long, 100 tuples per page, 1000 pages, 100 distinct bids

Sailors:
- Each tuple is 50 bytes long, 80 tuples per page, 500 pages, 10 Ratings, 40,000 sids
Cost Estimation

For each plan considered, must estimate cost:

- Must *estimate cost* of each operation in plan tree.
  - Depends on input cardinalities
  - We’ve already discussed how to estimate the cost of operations (sequential scan, index scan, joins, etc.)

- Must *estimate size of result* for each operation in tree!
  - Use information about the input relations
  - For selections and joins, assume independence of predicates

- In System R, cost is boiled down to a single number consisting of 
  \#I/O + \textit{factor} \times \#CPU instructions
Statistics and Catalogs

Need information about the relations and indexes involved. **Catalogs** typically contain at least:

- # tuples (**NTuples**) and # pages (**NPages**) per relation
- # distinct key values (**NKeys**) for each index
- low/high key values (**Low/High**) for each index
- Index height (**IHeight**) for each tree index
- # index pages (**INPages**) for each index

Statistics in catalogs are updated periodically

- Updating whenever data changes is too expensive; lots of approximation anyway, so slight inconsistency is OK

More detailed information (e.g., histograms of the values in some field) are sometimes stored
Size Estimation and Reduction Factors

Consider a query block:

```
SELECT attribute list
FROM relation list
WHERE term1 AND ... AND termk
```

Maximum # tuples in result is the product of the cardinalities of relations in the FROM clause

*Reduction factor (RF)* associated with each *term* reflects the impact of the *term* in reducing result size

RF is usually called “selectivity”
Result Size Estimation for Selections

Result cardinality =
Max # tuples * product of all RF’s
(Implicit assumption that values are uniformly distributed and terms are independent!)

Term col=value (given index I on col)
RF = 1/NKeys(I)

Term col>value
RF = (High(I)-value)/(High(I)-Low(I))

Note: if missing indexes, assume RF = 1/10
Result Size Estimation for Joins

Q: Given a join of R and S, what is the range of possible result sizes (in #of tuples)?

- Hint: what if \( R_{cols} \cap S_{cols} = \emptyset \)?

- \( R_{cols} \cap S_{cols} \) is a key for R (and a Foreign Key in S)?
Result Size Estimation for Joins

General case: $R\_cols \cap S\_cols = \{A\}$ (and A is key for neither)

- If $N\text{Keys}(A,S) > N\text{Keys}(A,R)$
  
  - Assume S values are a superset of R values, so each R value finds a matching value in S
  
  - Estimate each tuple r of R generates $NTuples(S)/N\text{Keys}(A,S)$ result tuples, so...

  $$\text{est\_size} = NTuples(R) \times NTuples(S)/N\text{Keys}(A,S)$$

- Else, if $N\text{Keys}(A,R) > N\text{Keys}(A,S)$ ... symmetric argument, yielding:

  $$\text{est\_size} = NTuples(R) \times NTuples(S)/N\text{Keys}(A,R)$$

- Overall:

  $$\text{est\_size} = NTuples(R)\times NTuples(S)/\text{MAX}\{N\text{Keys}(A,S), N\text{Keys}(A,R)\}$$
On the Uniform Distribution Assumption

Assuming uniform distribution is rather crude
Histograms

For better estimation, use a histogram

**Equiwidth histogram**

```
Bucket 1: Count=8
Bucket 2: Count=4
Bucket 3: Count=15
Bucket 4: Count=3
Bucket 5: Count=15
```

**Equidepth histogram**

```
Bucket 1: Count=9
Bucket 2: Count=10
Bucket 3: Count=10
Bucket 4: Count=7
Bucket 5: Count=9
```
Cost estimation: Summary

The costs of possible strategies vary widely

Estimate result sizes using statistics

Estimate costs of each operator

Focus on optimizing select-project-join (SPJ) blocks
Query Optimization

Overview

Query optimization

Cost estimation

Plan enumeration and costing

Readings: Chapter 15.4

System R strategy
Enumeration of Alternative Plans

There are two main cases:

- Single-relation plans
- Multiple-relation plans

For queries over a single relation:

- Each available access path (file scan / index) is considered, and the one with the least estimated cost is chosen
- The different operations are essentially carried out together (e.g., if an index is used for a selection, projection is done for each retrieved tuple)
Cost Estimates for Single-Relation Plans

Index I on primary key matches selection:
- Cost is $Height(I) + 1$ for a B+ tree, about 2.2 for hash index.

Clustered index I matching one or more selects:
- $(NPages(I) + NPages(R)) \times \text{product of RF’s of matching selects.}$

Non-clustered index I matching one or more selects:
- $(NPages(I) + NTuples(R)) \times \text{product of RF’s of matching selects}$

Sequential scan of file:
- $NPages(R)$

**Note:** Must also charge for duplicate elimination if required.
Reminder: Sailors has 500 pages, 40000 tuples

If we have an index on rating:

- Cardinality: \( \frac{1}{N_{Keys(I)}} \times NTuples(S) = \frac{1}{10} \times 40000 \) tuples retrieved
- Clustered index: cost = \( \frac{1}{N_{Keys(I)}} \times (N_{Pages(I)}+N_{Pages(S)}) = \frac{1}{10} \times (50+500) = 55 \) pages retrieved.
- Unclustered index: cost = \( \frac{1}{N_{Keys(I)}} \times (N_{Pages(I)}+NTuples(S)) = \frac{1}{10} \times (50+40000) = 4005 \) pages retrieved

If we have an index on sid:

- Would have to retrieve all tuples/pages. With a clustered index, the cost is 50+500, with unclustered index, 50+40000

Doing a file scan:

- We retrieve all file pages (500)

\[
\text{Example}
\]

```
SELECT S.sid
FROM Sailors S
WHERE S.rating=8
```
Queries Over Multiple Relations

As number of joins increases, number of alternative plans grows rapidly → *need to restrict search space*

Fundamental decision in System R: *only left-deep join trees* are considered

- Left-deep trees allow us to generate all *fully pipelined* plans
  - Intermediate results are not written to temporary files
  - Not all left-deep trees are fully pipelined (e.g., SM join)
Plan Enumeration – The Hard Way

1. Select order of relations (the only degree of freedom for left-deep plans)
   - maximum possible orderings = N! (but no X-products)
2. For each join, select join algorithm
3. For each input relation, select access method

Q: How many plans for a query over N relations?

Back-of-envelope calculation:

- With 3 join algorithms, I indexes per relation:
  \[ \# \text{ plans} \approx [N!] \times [3^{(N-1)}] \times [(I + 1)^N] \]
- Suppose N = 3, I = 2: \# plans \approx 3! \times 3^2 \times 3^3 = 1458 plans

- For each candidate plan, must estimate cost

Query optimization is NP-complete
Plan Enumeration Example

```
SELECT  S.sname, B.bname, R.day
FROM    Sailors S, Reserves R, Boats B
```

Let’s assume:

- Two join algorithms to choose from:
  - Hash-Join / NL-Join (page-oriented or Index-NL-Join)
- Unneeded columns removed at each stage
- Non-clustered B+Tree index on R.sid; no other indexes
- R.sid index has 50 pages
- S has 500 pages, 80 tuples/page
- R has 1000 pages, 100 tuples/page
- B has 10 pages
- 100 R S tuples fit on a page
1. Enumerate relation orderings:

Candidate Plans

```
SELECT S.sname, B.bname, R.day
FROM Sailors S, Reserves R, Boats B
```

Prune plans with cross-products immediately!
2. Enumerate join algorithm choices:

Candidate Plans

```
SELECT S.sname, B.bname, R.day
FROM Sailors S, Reserves R, Boats B
```
3. Enumerate **access method choices:**

```sql
SELECT S.sname, B.bname, R.day
FROM Sailors S, Reserves R, Boats B
```

Candidate Plans

- **NLJ**
- **B**
- **NLJ**
- **S**
- **R**

+ do same for other plans

(heapscan)

(INDEX scan on R.sid)

(heap scan)
Now estimate the cost of each plan

Example:

Cost to join S with R
\[ |S| + (\left( |S| \times p_s \right) \times \text{cost of finding matching } R \text{ tuples}) \]
\[ 500 + 500 \times 80 \times (1/40000)(50[\text{idx}]+100,000) = 100,050 \]

Size of \( S \bowtie R \) = \( \text{NTuples}(S) \times \text{NTuples}(R) / \text{distinct keys}(\text{sid}) \) = 100,000 tuples;
\[ 100,000/100 = 1000 \text{ pages} \]

Cost to NL join with B = 1000 \times 10 = 10000 (pipelined)

\[ \text{Total estimated cost} = 500 + 100,050 + 10000 = 110,550 \]
Now You Try ...

Estimate the cost of each of these plans:

1) NLJ

2) HJ

3) NLJ

4) HJ

Join algorithms:

**NLJ = page-oriented NL Join**
- Scan left input + scan right input once per page in left input

**HJ = hash-join (assume 2 passes)**
- Scan both inputs + write both inputs in buckets + read all buckets

**Relevant stats:**
- **S** has 500 pages, 80 tuples/page
- **R** has 1000 pages, 100 tuples/page
- **B** has 10 pages
- 100 S R tuples fit on a page

S = Sailors
R = Reserves
B = Boats
Answers ...

Plan 1:

S \bowtie R \text{ size } = 100,000 \text{ tuples; 1000 pages}

Estimated cost = $500 + 500(1000) + 1000(10)\$

\begin{align*}
& \text{scan S} \\
& \text{join w/R} \\
& \text{join w/B}
\end{align*}

Plan 2:

S \bowtie R \text{ size } = 100,000 \text{ tuples; 1000 pages}

Estimated cost = $500 + 500(1000) + 2*1000 + 3*10\$

\begin{align*}
& \text{scan S} \\
& \text{join w/R} \\
& \text{join w/B}
\end{align*}
Answers ...

Plan 3:

$S \bowtie R$ size = 100,000 tuples; 1000 pages

Cost = $500 + 2 \times 500 + 3 \times 1000 + 1000(10)$

- scan $S$
- join with $R$
- join with $B$

Plan 4:

$S \bowtie R$ size = 100,000 tuples; 1000 pages

Cost = $500 + 2 \times 500 + 3 \times 1000 + 2 \times 1000 + 3 \times 10$

- scan $S$
- join with $R$
- join with $B$
Much of the computation is redundant

Idea: when we estimate costs & result sizes of sub-plans, remember them.
Enumerated Plans (just the S-R-B ones)

Observe that many plans share common sub-plans (i.e., only upper part differs)
Query Optimization

Overview

Query optimization

Cost estimation

Plan enumeration and costing

System R strategy

Readings: Chapter 15.6
Improved Strategy (used in System R)

Shared sub-plan observation suggests a better strategy:

Enumerate plans using N passes (N = # relations joined):

- **Pass 1:** Find best 1-relation plans for each relation
- **Pass 2:** Find best ways to join result of each 1-relation plan as outer to another relation *(All 2-relation plans.)*
- **Pass N:** Find best ways to join result of a (N-1)-relation plan as outer to the Nth relation *(All N-relation plans.)*

For each subset of relations, retain only:

- Cheapest subplan overall (possibly unordered), plus
- Cheapest subplan for each *interesting order* of the tuples

For each subplan retained, remember cost and result size estimates
A Note on ”Interesting Orders”

An intermediate result has an “interesting order” if it is sorted by any of:

- ORDER BY attributes
- GROUP BY attributes
- Join attributes of other joins
System R Plan Enumeration

A N-1 way plan is not combined with an additional relation unless there is a join condition between them (unless all predicates in WHERE have been used up)

– i.e., avoid Cartesian products if possible

Always push all selections & projections as far down in the plans as possible

– Usually a good strategy, as long as these operations are cheap
System R Plan Enumeration Example

```
SELECT S.sname, B.bname, R.day
FROM Sailors S, Reserves R, Boats B
```

This time let’s assume:

- Two join algorithms to choose from:
  - Sort-Merge-Join / NL-Join (page-oriented or Index-NL-Join)
  - Clustered B+Tree on S.sid (height=3; 500 leaf pages)
- S has 10,000 pages, 5 tuples/page
- R has 10 pages, 10 tuples/page
- B has 10 pages, 20 tuples/page
- $10 R \bowtie S$ tuples fit on a page
- $10 R \bowtie B$ tuples fit on a page
Pass 1 (single-relation subplans)

S: (a) heap scan or (b) scan index on S.sid
   a) heap scan cost = 10,000
   b) index scan cost = 500 + 10,000 = 10,500
   Retain both, since (b) has “interesting order” by sid

R: heap scan only option
   Cost = 10

B: heap scan only option
   Cost = 10
Pass 2 (2-relation subplans)

Starting with S as outer

Heap scan-S as outer:

a) NL-Join with R, cost = 10,000 + 10,000(10) = 110,000
b) SM-Join with R, cost = 10,000 + 2*10,000 + 3*10 = 30,030

Index scan-S as outer:

c) NL-Join with R, cost = 10,500 + 10,000(10) = 110,500
d) SM-Join with R, cost = 10,500 + 3*10 = 10,530

Retain (d) only

Note: best S⋈ R plan exploits “interesting order” of non-optimal subplan!
Pass 2 (continued)

Starting with R as outer

Join with S:

a) NL-Join with S, cost = 10 + 10(10,000) = 100,010
b) Index-NL-Join with Index-S, cost = 10 + 100*4 = 410
c) SM-Join with S, cost = 10 + 2*10 + 3*10,000 = 30,030

Join with B:

a) NL-Join with B, cost = 10 + 10(10) = 110
b) SM-Join with B, cost = 10 + 2*10 + 3*10 = 60
Pass 2 (continued)

Starting with B as outer

Join with R:

a) NL-Join with R, cost = 10 + 10(10) = 110
b) SM-Join with R, cost = 10 + 2*10 + 3*10 = 60
Further pruning of 2-relation subplans

\[
\begin{align*}
S \bowtie R: & \quad \text{cost} = 10,530 \text{, order} = \text{sid} \\
& \quad \text{(INDEX scan)} \quad \text{(heap scan)} \\
& \quad \text{Index-NLJ} \quad \text{cost} = 410, \text{order} = \text{none} \\
& \quad \text{(heap scan)} \quad \text{(INDEX lookup)}
\end{align*}
\]

\[
\begin{align*}
B \bowtie R: & \quad \text{cost} = 60, \text{order} = \text{bid} \\
& \quad \text{(heap scan)} \quad \text{(heap scan)} \\
& \quad \text{SMJ} \quad \text{cost} = 60, \text{order} = \text{bid} \\
& \quad \text{(heap scan)} \quad \text{(heap scan)}
\end{align*}
\]
Pass 3 (3-relation subplans)

S \bowtie R subplan:  
- cost = 410  
- order = none  
- result size = 10 pages

\[
\text{cost} = 410 + 10(10) = 510
\]

\[
\text{Index-NLJ} \quad \text{NLJ} \quad \text{B}
\]

R \quad S

- (heap scan)  
- (INDEX lookup)

\[
\text{cost} = 410 + 2*10 + 3*10 = 460
\]
Pass 3 (continued)

B ▷▷ R subplan:
cost=60, order=bid
result size = 100 tuples (10 pages)

Index-NLJ

SMJ

S

NLJ

SMJ

S

(heap scan)  (heap scan)

(heap scan)  (heap scan)

cost = 60 + 10(10,000) = 100,060

cost = 60 + 100*4 = 460

(heap scan)  (heap scan)

(Index lookup)

Index-

NLJ

SMJ

S

(heap scan)  (heap scan)

(heap scan)  (heap scan)

cost = 60 + 10*2 + 3*10,000 = 30,080  cost = 60 + 10*2 + 10,500 = 10,580
And the Winner is ...

Observations:

- Best plan mixes join algorithms
- Worst plan had cost > 100,000
  (exact cost unknown due to pruning)

Optimization yielded ~ **1000-fold improvement** over worst plan!
Some notes w.r.t. reality...

In spite of pruning plan space, this approach is still exponential in the # of tables

- Rule of thumb: works well for < 10 joins

In real systems, COST considered is:

\[ \#\text{IOs} + \text{factor} \times \#\text{CPU Instructions} \]
System R strategy: Summary

Enumerate plans using N passes (N = # relations joined):

For each subset of relations, retain only:

- Cheapest subplan overall (possibly unordered), plus
- Cheapest subplan for each interesting order of the tuples

For each subplan retained, remember cost and result size estimates