# Scalable Algorithms for Correlation Clustering on Large Graphs

#### Nathan Cordner

Boston University

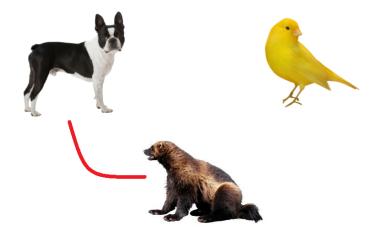
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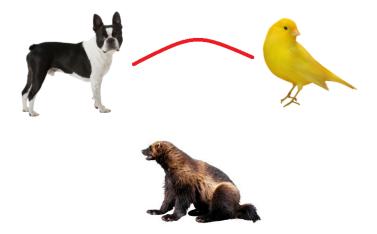
## Correlation Clustering Introduction

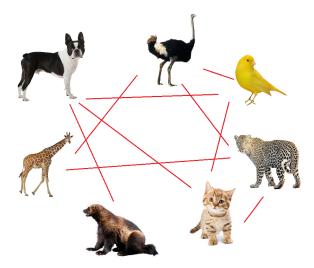
## Scalable Algorithms for

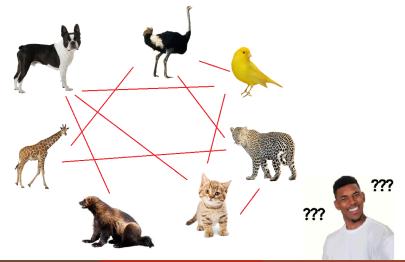
- Cluster Improvement
- Constrained Cluster Sizes
- Constrained Number of Clusters
- Consensus Clustering











# Correlation Clustering [BBC04]

## Given a complete graph G = (V, E) $E = E^+ \cup E^-$

Want to cluster + edges and separate - edges

- Maximize Agreements
- Minimize Disagreements
- Some Applications
  - Classification
  - Entity Resolution
  - Communities in Social Networks

# The Pivot Algorithm [ACN08, AL09]

Neighborhood Oracle  $N(u) = \{v \in V \mid \{u, v\} \in E^+\}$ 

 $\mathsf{Pivot}(V, E = E^+ \cup E^-):$ 

 $\blacksquare$  Pick random pivot node  $u \in V$ 

• Set 
$$C = \{u\}$$

For all 
$$v \in N(u)$$
:

If 
$$v \in V$$
: Add  $v$  to  $C$ 

• Repeat on  $V = V \setminus C$  until empty

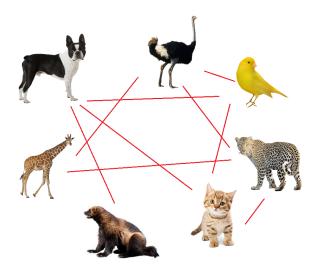
Return completed clustering

Runs in  $O(|V| + |E|^+)$  time

Randomized expected 3-approximation

# The Pivot Algorithm [ACN08, AL09]

Example:



# Other Linear Time Methods

## The Vote Algorithm [ES09]

- Pick unclustered nodes one at a time
  - First node creates its own cluster
  - All others: add to existing cluster, or create own
  - Greedily minimize increase in clustering cost
- $\blacksquare$  Runs in  $\Theta(|V|+|E|^+)$  time

## LocalSearch: given a clustering,

- Each node decides whether to stay or move clusters
- Iterate until improvements converge
- Each *iteration* is  $\Theta(|V| + |E|^+)$

# LP Methods

Linear Program for Correlation Clustering:

$$\min \sum_{(u,v)\in E^+} x_{uv} + \sum_{(u,v)\notin E^+} (1-x_{uv})$$
$$x_{uv} + x_{vw} \geq x_{uw} \quad \text{for all } u, v, w \in V$$
$$x_{uv} \in [0,1] \quad \text{for all } u, v \in V$$

•  $O(|V|^2)$  variables,  $O(|V|^3)$  constraints!

Rounding Methods:

- 2.5-approx [ACN08]
- 2.06-approx [CMSY15]

• (1.994+
$$\epsilon$$
)-approx [CLN22]

# One Algorithm to Rule Them All?

Pivot has been successfully used to cluster

- Social network graphs [KPT11]
- Protein-protein interaction graphs [KPT11; HWH15]

Event graphs generated from news stories [CMB17] Pivot has been adapted for

- Probabilistic graphs [KPT11; MTG20]
- Chromatic correlation clustering [KSZC21]
- Fair correlation clustering [AEKM20]
- Data streaming and online settings [ACGM15; LMVW21]
- Query constraints [GKBT20]
- Cluster size constraints [PM15]

Deterministic and parallel versions [ZW09; CDK14; PORJ15]

- How do Pivot and the other linear time algorithms perform when tested against slower algorithms with better approximation guarantees?
- Can we boost Pivot's clustering quality in various settings without diminishing its run time advantages?
- What practical improvements can we make for CC algorithms applied to consensus clustering?

How do Pivot and the other linear time algorithms perform when tested against slower algorithms with better approximation guarantees?

- We show experimentally that Pivot, Pivot with LocalSearch, and Vote perform well when compared against state-of-the-art algorithms
  - Clustering costs are close to, or even lower than, state-of-the-art algorithms
  - Running times are much, much quicker!
- We also show that adaptations of these algorithms perform well in other CC settings

Can we boost Pivot's clustering quality in various settings without diminishing its run time advantages?

- We develop a lightweight LocalSearch method (InnerLocalSearch) that show experimentally converges much faster than LocalSearch while still providing a significant reduction in clustering cost
- We also demonstrate InnerLocalSearch's applicability to constrained cluster sizes and the related consensus clustering problem

What practical improvements can we make for CC algorithms applied to consensus clustering?

- We develop a memory-efficient implementation of Pivot and other CC algorithms to use on larger graphs
- We also show a clustering-sampling method that improves running time while only incurring small increases of clustering cost

# Outline

## Correlation Clustering Introduction

- Scalable Algorithms for
  - Cluster Improvement
  - Constrained Cluster Sizes
  - Constrained Number of Clusters
  - Consensus Clustering

Goals:

- Compare Pivot, LS, and Vote with state-of-the-art methods
- Develop the InnerLocalSearch method and show its competitiveness with LS on larger data sets

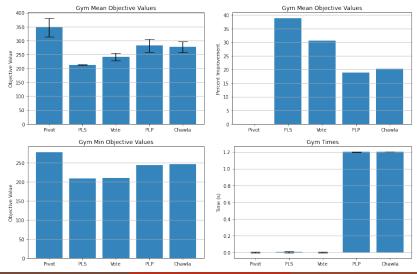
How does Pivot compare to the state-of-the-art approximation algorithms?

Methods tested:

- Pivot and Vote
- **PLS**: Pivot with LocalSearch
- **PLP**: 2.5-approx LP rounding [ACN08]
- **Chawla**: 2.06-approx LP rounding [CMSY15]

# **Pivot Comparison**

**Gym**: |V| = 94,  $|E|^+ = 465$ 

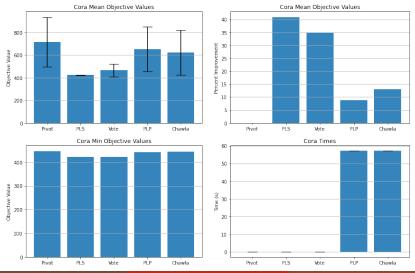


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# **Pivot Comparison**

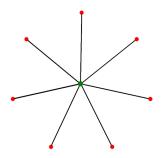
#### **Cora200**: |V| = 190, $|E|^+ = 1,588$



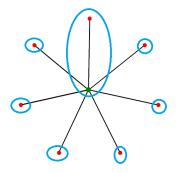
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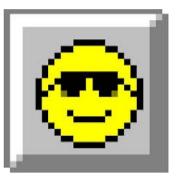
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Pivot can form sparse, star-shaped clusters



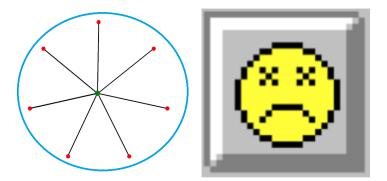
Pivot can form sparse, star-shaped clusters





## Drawbacks of Pivot

Pivot can form sparse, star-shaped clusters



#### Can we do better and still maintain scalability?

LocalSearch: given a clustering,

- Each node decides whether to stay or move clusters
- Iterate until improvements converge
- Slow: each iteration is  $\Theta(|V| + |E|^+)$
- Somewhat popular though [MTG20; AEKM20]

New Idea: run LS inside clusters only

 Will not generate same level of improvement as a full LS, but will converge much faster

# **InnerLocalSearch**

InnerLocalSearch: on graph G

- Obtain clusters  $C_1, \ldots, C_k$  from  $\mathsf{Pivot}(G)$
- Let  $G_i$  be the graph induced by  $C_i$
- Return LocalSearch $(G_1), \ldots, LocalSearch(G_k)$

Properties

- Iteration time:  $O(\min\{|V|d, |V| + |E|^+\})$ 
  - *d* is size of largest Pivot cluster
  - Tends to converge much faster than LS
- Easily run in parallel
- Improves cluster costs from Pivot (nearly like LS)
- Immediately applies to CC generalizations

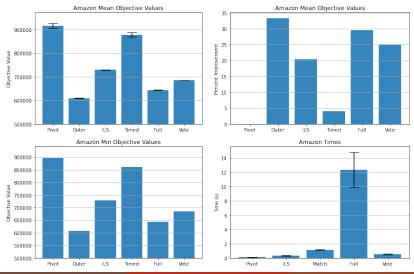
How does ILS compare to LS and other methods?

Methods tested:

- Pivot, ILS, and Vote
- Outer: inter-cluster cost from Pivot result (benchmark for ILS improvement)
- Timed: Pivot with LocalSearch, using time limit set by ILS convergence
- Match: time required for Pivot with LocalSearch to reach same level of improvement as ILS
- **Full**: Pivot with LocalSearch run to convergence

## LocalSearch Comparison

#### **Amazon**: |V| = 334, 863, $|E|^+ = 925, 872$ , d = 107.1

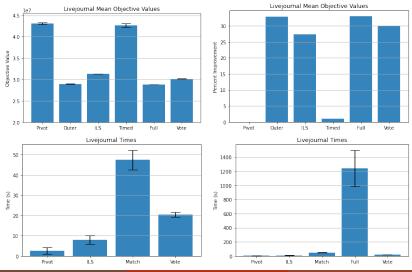


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# LocalSearch Comparison

**Livejournal**: |V| = 3,997,962,  $|E|^+ = 34,681,189$ , d = 717.9

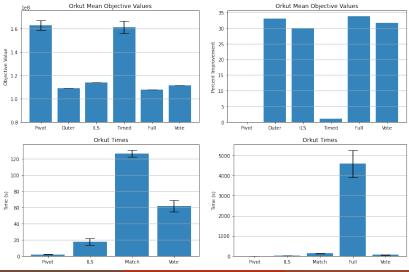


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## LocalSearch Comparison

**Orkut**: |V| = 3,072,441,  $|E|^+ = 117,185,083$ , d = 1679.8



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## Outline

## Correlation Clustering Introduction

- Scalable Algorithms for
  - Cluster Improvement
  - Constrained Cluster Sizes
  - Constrained Number of Clusters
  - Consensus Clustering

Goals:

 Compare Pivot, LS, and Vote with state-of-the-art methods

 Compare performance of Pivot, ILS, LS, and Vote on larger data sets Two kinds of constraints:

- Uniform [PM15]: given K ≥ 1, all clusters must have size ≤ K
- Non-Uniform [JXLW20]: every node v can only be in a cluster of size at most K<sub>v</sub>

Soft constraints: some violations of clusters sizes allowed **Hard constraints**: all size constraints must be observed

# Uniform Constrained Cluster Sizes

Linear Program for Uniform Size-Constrained CC:

$$\min \sum_{(u,v)\in E^+} x_{uv} + \sum_{(u,v)\notin E^+} (1-x_{uv})$$
$$x_{uv} + x_{vw} \geq x_{uw} \quad \text{for all } u, v, w \in V$$
$$\sum_{v\in V} (1-x_{uv}) \leq K \quad \text{for all } u \in V,$$
$$x_{uv} \in [0,1] \quad \text{for all } u, v \in V$$

LP rounding algorithms 6-approx [PM15] 5.37-approx [JCTZ21]

## Pivot adaptations [PM15]

- 7-approx by removing a smallest set of + edges
- 11-approx for random removal

## Our Approach:

- **Pivot**: pivot node chooses *K* − 1 neighbors at random if full Pivot cluster is too large
- Vote and LocalSearch: only add nodes to clusters that are not yet at capacity
- InnerLocalSearch: no changes!

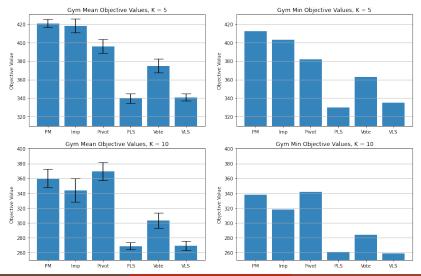
How does Pivot compare to the state-of-the-art approximation algorithms?

Methods tested:

- Pivot and Vote
- **PLS**: Pivot with LocalSearch
- **VLS**: Vote with LocalSearch
- **PM**: 6-approx LP rounding [PM15]
- Imp: 5.37-approx LP rounding [JCTZ21]

# **Pivot Comparison**

**Gym**: |V| = 94,  $|E|^+ = 465$ 

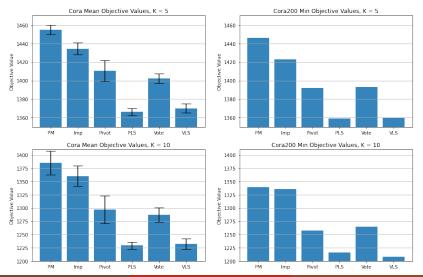


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### **Pivot Comparison**

#### **Cora200**: |V| = 190, $|E|^+ = 1,588$



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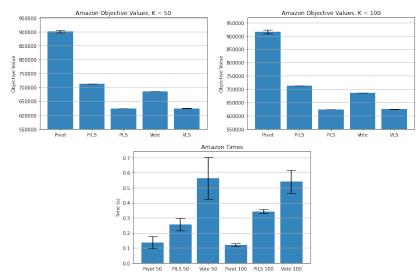
Scalable CC Algs

How does ILS compare to LS and other methods?

Methods tested:

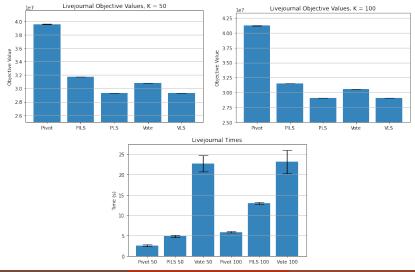
- Pivot, ILS, and Vote
- **PLS**: Pivot with LocalSearch (5-minute time limit)
- **VLS**: Vote with LocalSearch (5-minute time limit)

#### **Amazon**: $|V| = 334, 863, |E|^+ = 925, 872$

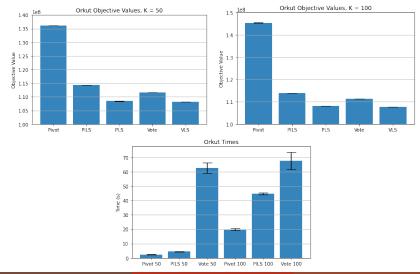


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#### Livejournal: |V| = 3,997,962, $|E|^+ = 34,681,189$



#### **Orkut**: $|V| = 3,072,441, |E|^+ = 117,185,083$



# Non-Uniform Constrained Cluster Sizes

Linear Program for Non-Uniform Size-Constrained CC:

$$\min \sum_{(u,v)\in E^+} x_{uv} + \sum_{(u,v)\notin E^+} (1-x_{uv})$$
$$x_{uv} + x_{vw} \geq x_{uw} \quad \text{for all } u, v, w \in V$$
$$\sum_{v\in V} (1-x_{uv}) \leq \frac{K_u}{K_u} \quad \text{for all } u \in V,$$
$$x_{uv} \in [0,1] \quad \text{for all } u, v \in V$$

LP rounding algorithms [JXLW20]

Soft:  $(2/\alpha)$ -approximation where every cluster satisfies  $|C| \leq 1/(1-\alpha) \min_{u \in C} \{K_u\}$ ,  $\alpha \in (0, 1/2]$ 

• Hard: 2K-approximation where  $K = \max_{u \in V} \{K_u\}$ 

# Non-Uniform Constrained Cluster Sizes

### Our Approach:

- Pivot: set initial size bound of new cluster equal to pivot node; include neighbors at random, adjusting size bound after each one and rejecting neighbors once size limit is reached
- Vote and LocalSearch: only add nodes to clusters that are not yet at capacity, adjusting capacity as needed
  - Ordered Vote first sorts nodes by increasing size of  $K_u$
  - LocalSearch also maintains (min) priority queue of size constraints for each cluster in order to update size constraints quickly when nodes leave
- InnerLocalSearch: no changes!

How does Pivot compare to the state-of-the-art approximation algorithms?

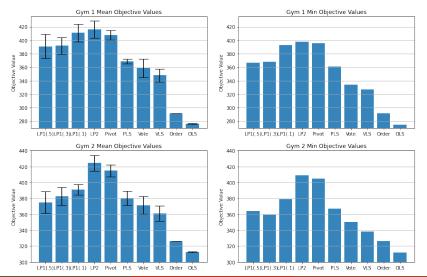
Methods tested:

- **Pivot**, **Vote**, and Ordered Vote (**Order**)
- PLS, VLS, OLS: Pivot, Vote, and Ordered Vote with LocalSearch
- LP1( $\alpha$ ): Soft constraint  $(2/\alpha)$ -approx LP rounding
- LP2: Hard constraint 2K-approx LP rounding

Size bounds: randomly chosen from 1 to  $\left|N(v)\right|+1$  for each node v

# **Pivot Comparison**

**Gym**: |V| = 94,  $|E|^+ = 465$ 



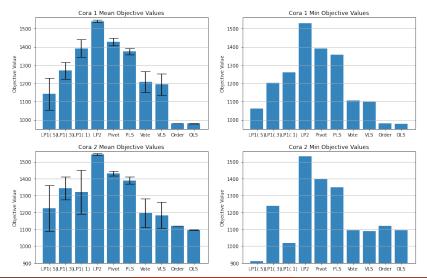
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Scalable CC Algs

### **Pivot Comparison**

#### **Cora200**: |V| = 190, $|E|^+ = 1,588$



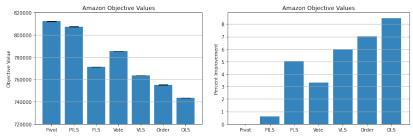
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Scalable CC Algs

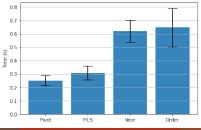
How does ILS compare to LS and other methods? Methods tested:

- **Pivot**, **Vote**, and Ordered Vote (**Order**)
- PLS, VLS, OLS: Pivot, Vote, and Ordered Vote with LocalSearch (5-minute time limit)
- **PILS**: Pivot with InnerLocalSearch

#### **Amazon**: $|V| = 334, 863, |E|^+ = 925, 872$

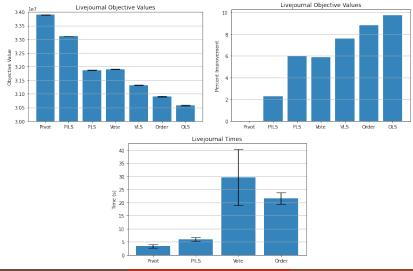


Amazon Times

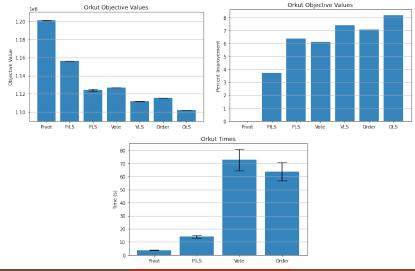


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#### Livejournal: |V| = 3,997,962, $|E|^+ = 34,681,189$



#### **Orkut**: |V| = 3,072,441, $|E|^+ = 117,185,083$



# Outline

# Correlation Clustering IntroductionScalable Algorithms for

- Cluster Improvement
- Constrained Cluster Sizes
- Constrained Number of Clusters
- Consensus Clustering

### Goals:

- Adapt Pivot and Vote methods to work in new constrained setting
- Compare Pivot, LS, and Vote with state-of-the-art methods

### Compare performance of Pivot, LS, and Vote on larger data sets

# Constrained Number of Clusters

Minimize cost with clustering of size  $\leq K$ 

- K = 2:
  - Pivot-like 3-approximation [BBC04]
  - LocalSearch 2-approximation [CSW08]
- General case:
  - PTAS [GG06]
  - Improved PTAS [KS09]
  - *K*-approximation [IN16]
  - *K*-LocalSearch heuristic [C17, TCD19]
- Proposed Algorithms:
  - K-Pivot, K-Vote
  - Blend (start with Pivot, finish with Vote)

### PTAS approaches:

- $\ \ \, \bullet \ \ \, (1+\epsilon) \text{-approximation factor}$
- **Giotis**, Guruswami:  $|V|^{O(9^K/\epsilon^2)} \log |V|$  running time
- Karpinski, Schudy:  $|V|^2 2^{O((K^6 \log K)/\epsilon^2)}$  running time
- Both methods rely on brute-force searches to identify best possible clusterings on large (O(log |V|), O(K<sup>4</sup> log K)) subsets of nodes

K-LocalSearch

- $\blacksquare$  Randomly assign nodes to one of K clusters
- Run LocalSearch until convergence

### **Previous Methods**

Bansal et al. 3-approximation (K = 2):

- For every node, generate one Pivot cluster and assign remaining nodes to a second cluster
- Return 2-clustering with minimum cost
- II'ev and Navrotskaya K-approximation:
  - For every node, generate K 1 Pivot clusters and assign remaining nodes to cluster K
  - Run LocalSearch to convergence on each clustering and return the one with minimum cost
  - Running time: O(|V|<sup>3</sup>I) where I is the largest number of iterations required by LS

# **Proposed Methods**

K-Pivot:

- Form *K* Pivot clusters
- Merge additional Pivot clusters to the *current* smallest cluster

K-Vote:

- **\blacksquare** Run Vote until K clusters are formed
- Continue assigning nodes to existing clusters that minimize increase of clustering cost

Blend:

- Form *K* Pivot clusters
- Continue assigning nodes to existing clusters that minimize increase of clustering cost

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### Priority Queues

- Once K clusters are formed, we initialize a PQ to track cluster sizes
- K-Pivot merges new clusters to current smallest
- K-Vote and K-LS methods can either merge a node u to a cluster that contains  $v \in N(u)$  or to the current smallest cluster
- $O(|V|\log K + |E|^+)$  running time

How does Pivot compare to the state-of-the-art approximation algorithms?

Methods tested:

Rand: random K-clustering, used as baseline
PTAS: the Giotis and Guruswami PTAS

Sample size limited to 13 (14) nodes

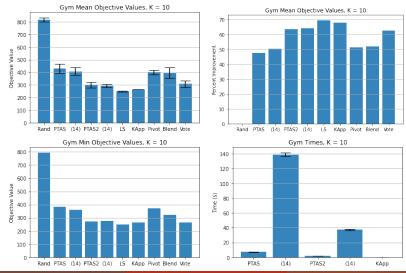
PTAS2: the Karpinski and Schudy PTAS

Sample size limited to 13 (14) nodes

KApp: the II'ev and Navrotskaya K-approximation
Pivot. Vote, Blend, and LS

### **Pivot Comparison**

**Gym**: |V| = 94,  $|E|^+ = 465$ 

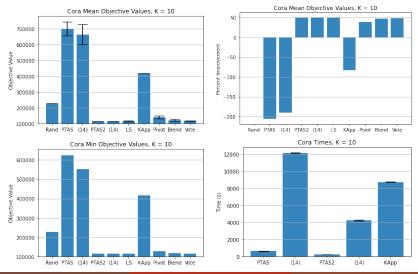


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Scalable CC Algs

### **Pivot Comparison**

**Cora**: |V| = 1,879,  $|E|^+ = 64,955$ 



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Scalable CC Algs

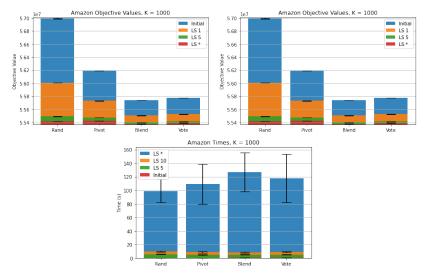
How do the baseline methods compare against each other before and after LS improvements?

Methods tested:

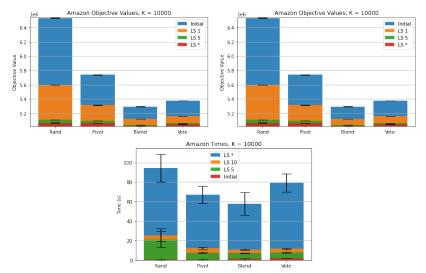
- **Pivot**, **Vote**, **Blend**, and **Rand**
- LS is applied to each baseline result with various time constraints (LS \* runs to convergence)

Note that ILS does not work here!

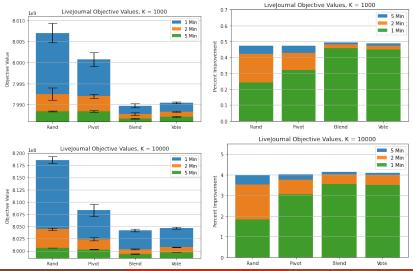
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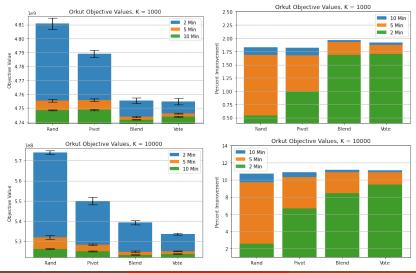


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Scalable CC Algs

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Scalable CC Algs

### Outline

#### Correlation Clustering Introduction

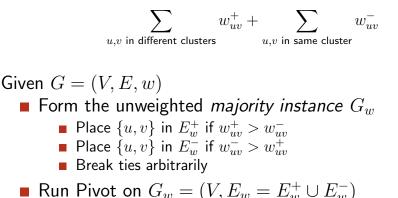
- Scalable Algorithms for
  - Cluster Improvement
  - Constrained Cluster Sizes
  - Constrained Number of Clusters
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### Goals:

- Develop a memory-efficient implementation of Pivot and other CC algorithms for larger graphs
- Demonstrate a clustering-sampling method to improve running time without too much impact on clustering cost

# Weighted CC [BBC04, ACN08]

Every pair of nodes u, v has weights  $w_{uv}^+, w_{uv}^- \ge 0$ Clustering Cost:



# Weighted CC [ACN08, KPT11]

**Probability Constraints**:  $w_{uv}^+ + w_{uv}^- = 1$ Notation:  $p(u, v) = w_{uv}^+$ ,  $1 - p(u, v) = w_{uv}^-$ 

Relation to original CC problem

$$\{u, v\} \in E^+ \Leftrightarrow p(u, v) = 1$$
$$\{u, v\} \in E^- \Leftrightarrow p(u, v) = 0$$

Pivot Approximation Results

- 5-approx with probability constraints
- 2-approx with PC and the triangle inequality

$$w_{uv}^- \le w_{ux}^- + w_{xv}^-$$

Given clusterings  $\mathcal{C}_1 \dots, \mathcal{C}_k$  of node set V

Find clustering C minimizing  $\sum_{i=1}^{k} \text{Disagree}(C, C_i)$ 

■ Disagree(C, C<sub>i</sub>) = number of node pairs (u, v) clustered together in only one input clustering

### Relation to Correlation Clustering

- p(u,v) = number of input clusterings where u, v are clustered together divided by k
- $\blacksquare$  Edge weights (1-p) satisfy the triangle inequality

Previous Pivot Problems [ACN08; GF08]

- Time inefficiency:  $O(k|V|^2)$  to compute all edges
- Space inefficiency:  $O(|V|^2)$  to store all edges

Improvement # 1: only compute edges as needed■ Precompute cluster labels for each node

Improvement # 2: reduce number of input clusterings

- Picking one input clustering at random: 2-approx
- Pivot on full set of inputs: 1.57-approx [ACN08]
- What about in between?

### Improvement 1 Examples

Mushrooms

- 22 input clusterings, 8124 nodes
- Edges: 57.59 s; average Pivot run: 0.0082 s
- Labels: 0.029 s; average Pivot run: 0.0129 s

Facebook Government

- 100 input clusterings, 7057 nodes
- Edges: 272.52 s; average Pivot run: 0.054 s
- Labels: 0.121 s; average Pivot run: 144.85 s

### Sampling input clusterings

- Assume k large and sample R < k input clusterings
- p = true probability u, v are clustered together (assume p < 1/2)
- Let *X* = number of sampled clusters where *u*, *v* are clustered together
- $\blacksquare$  Model X as a Binomial rv with R trials and success probability p
- Pivot algorithm "makes a mistake" when X > R/2

# Consensus Clustering

What is  $\mathbb{P}(X > R/2)$ ? Normalize:  $Z = (X - pR)/\sqrt{Rp(1-p)}$ Estimate  $\mathbb{P}(X > R/2)$  using

$$\mathbb{P}\left(Z > \frac{R/2 - pR}{\sqrt{Rp(1-p)}}\right) = \mathbb{P}\left(Z > \frac{\sqrt{R}(1/2 - p)}{\sqrt{p(1-p)}}\right)$$

Let  $f(R,p) = \sqrt{R}(1/2 - p)/\sqrt{p(1-p)}$ Find  $\mathbb{P}(Z > f(R,p))$  by evaluating

$$\operatorname{Err}(R,p) := 1 - \Phi(f(R,p)),$$

where  $\Phi$  is the standard normal CDF

**Lemma**: the expected cost multiple of edge (u, v) due to error in a Pivot clustering is

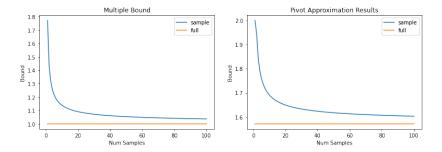
$$p \cdot (1 - \mathsf{Err}(R, p)) + (1 - p) \cdot \mathsf{Err}(R, p)$$

Cost multiple upper bound:

$$g(R) = \max_{p \in [0, 1/2]} \{ [p \cdot (1 - \mathsf{Err}(R, p)) + (1 - p) \cdot \mathsf{Err}(R, p)] / p \}$$

# Consensus Clustering

### **Theorem**: Pivot is a ((6g(R) + 5)/7)-approx algorithm

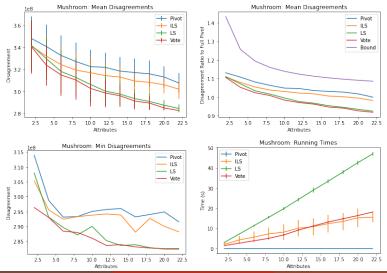


Methods tested:

- **Pivot**, Pivot with LocalSearch (LS), Pivot with InnerLocalSearch (ILS), and Vote
- Different attribute levels are tested for each algorithm
- **Bound** shows the theoretical cost increase limit
- Edge weights are computed on-the-fly
- LS and ILS restricted to one iteration only

### Experiments

#### **Mushrooms**: k = 22, |V| = 8,214



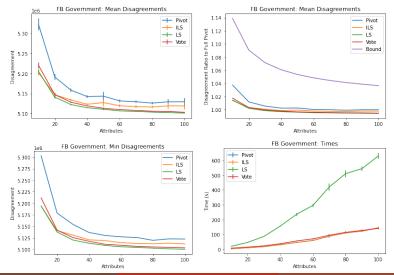
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Scalable CC Algs

### Experiments

#### Facebook Government: k = 100, |V| = 7,057



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### Accomplishments

- We demonstrated that Pivot, LS, and Vote were competitive with current state-of-the-art algorithms in general and constrained CC settings
- We developed and demonstrated the usefulness of ILS for Pivot on larger graphs
- We developed two practical improvements for applying CC algorithms to consensus clustering

### Future Work

- Can we demonstrate improved approximation bounds for Pivot and related methods for constrained CC settings?
- How well do these methods perform in other CC settings?

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