

Discussion 5

Date: Tuesday, March 23, 2021.

Problem 1. Abelian Groups

- (a) Write down the Abelian operation tables for \mathbb{Z}_5 .
 (b) Find the order of the following finite groups: \mathbb{Z}_{600} , \mathbb{Z}_{600}^* , and $\mathbb{Z}_{600}\{10\}$.

(a)		+					÷						
		0	1	2	3	4			0	1	2	3	4
0	0	0	1	2	3	4	0	0					
	1	1	2	3	4	0		1					
2	2	2	3	4	0	1	2	2					
	3	3	4	0	1	2		3					
4	4	4	0	1	2	3	4	4					

(b) \mathbb{Z}_n has order n

$\Rightarrow 2_{600}$ has order 600

$Z_{t_n}^*$ has order $4(n)$

χ^* has order 4 (600)

$$600 = 2^3 \cdot 3 \cdot 5^2 \Rightarrow \varphi(2^3 \cdot 3 \cdot 5^2)$$

$$Z_{600} \{10\} = \{0, 60, 120, 180, 240, 300, 360, 420, 480, 540\} \rightarrow \text{order } 10$$

Problem 2. Let H_1 be a subgroup of an abelian group G_1 and H_2 be a subgroup of an abelian G_2 . Show that $H_1 \times H_2$ is a subgroup of $G_1 \times G_2$.

$$\text{Let } a_1, a_2 \in H_1 \Rightarrow (a_1, b_1) \in H_1 \times H_2 \\ b_1, b_2 \in H_2 \Rightarrow (a_2, b_2) \in H_1 \times H_2$$

$$a_1 + a_2 \in H_1 \Rightarrow (a_1 + a_2, b_1 + b_2) \in H_1 \times H_2 \\ b_1 + b_2 \in H_2 \Rightarrow (a_1, b_1) + (a_2, b_2) \in H_1 \times H_2 \\ H_1 \times H_2 \text{ is closed under "}" + - \text{ op}$$

$$\text{Inverse: Let } (a, b) \in H_1 \times H_2 \\ (\text{to show: } -(a, b) = (-a, -b) \in H_1 \times H_2)$$

$(0, 0)$ is the identity of $H_1 \times H_2$

$$a + (-a) = 0, \quad b + (-b) = 0$$

$$\Rightarrow (a, b) + (-a, -b) = (0, 0)$$

$$\Rightarrow -(a, b) = (-a, -b) \text{ is an inverse} \\ \text{of } (a, b) \in H_1 \times H_2$$

Problem 3. Let $S = \mathbb{Z} \setminus \{-1\}$ and define a binary operation on S by $a * b = a + b + ab$. Show that S is closed under the $*$ -operation and S forms an abelian group under $*$.

Assume S is not closed under $*$ - or
 $\exists a, b \in S \Rightarrow a * b = -1$

$$\Rightarrow a+b+ab = -1$$

$$\Rightarrow a+b+ab+1 = 0$$

$$\Rightarrow (a+1)(b+1) = 0$$

$$a+1 = 0 \quad \text{or} \quad b+1 = 0$$

$$a = -1 \quad \text{or} \quad b = -1 \quad (\because)$$

$a * b = a \cdot 1 \cdot b + abc = b * a =$
 $(a * b) * c = (a+b+ab) * c$
 $= a+b+ab+c - (a * b + ab) c$
 $a * (b * c) = a * (b * c - bc)$
 $= a * b * c + bc + (b * c - bc) a$
 \exists identity : 0. $a * 0 = a + 0 + a \cdot 0$
 $= a$

inverse: $a + a' = 0$

$$a' = \frac{-a}{a+1} \hookrightarrow a + a' + aa' = 0$$

$$\Rightarrow a' + a = a + a' = 0$$