CS 235: Algebraic Algorithms, Spring 2021

Discussion 0

Date: Tuesday, January 26, 2021.

Problem 1. Let $a, b, d \in \mathbb{Z}$ with $d \neq 0$. Show that $a|b$ if and only if $da|db$.

Problem 2. Let I be a non-empty set of integers that is closed under addition (i.e. $a + b \in I \ \forall a, b \in I$). Show that I is an ideal if and only if $-a \in I \ \forall a \in I$. (See page 5 of textbook for the definition of an ideal)

Problem 3. Let p be a prime number. Show that $p!$ is not a perfect square.

Reminder: $n! = n \cdot (n-1) \cdot (n-2) \ldots 2$; Perfect square is a number made by squaring a whole number, ex: $4 = 2^2, 9 = 3^2, \ldots$ are perfect squares.

Hint: how would you write down the prime factorisation of a perfect square number n? Then, can you do the same for the number $p!$?