Problem 1. Modular Inverses

(a) Find the modular inverses of 4, 5, and 7 in \( \mathbb{Z}_{11} \) and \( \mathbb{Z}_{17} \).

(b) Determine whether the following congruence has solution(s) or not (and how many). If the congruence has a unique solution, try to solve it using modular inverses.

(i) \( 66x \equiv 100 \pmod{121} \)
(ii) \( 21x \equiv 14 \pmod{91} \)
(iii) \( 3x \equiv 5 \pmod{17} \)
(iv) \( 10x \equiv 3 \pmod{11} \)
Problem 2. More congruence drilling...

(a) Prove that the equation $x^2 - 7y^3 = 3$ has no solution for any $x, y \in \mathbb{Z}$. (Hint: consider mod 7 arithmetic)

(b) Prove the Cancellation Law, namely, if $ac \equiv bc \pmod{n}$ and $\gcd(c, n) = 1$, then $a \equiv b \pmod{n}$. 
Problem 3. Let $p$ be an odd prime. Show that $\Sigma_{\alpha \in \mathbb{Z}_p^*} \alpha^{-1} = \Sigma_{\alpha \in \mathbb{Z}_p^*} \alpha = 0$. 