Problem 1. Euler’s/ Fermat’s Little Theorem.

(a) Find $3^{31} \mod 7$, $2^{35} \mod 7$.

(b) Solve the congruence $x^{103} \equiv 4 \pmod{11}$. 
(c) Suppose that $p$ and $q$ are distinct primes, $a^p \equiv a \pmod{p}$, and $a^q \equiv a \pmod{p}$. Show that $a^{pq} \equiv a \pmod{pq}$.
Problem 2. Prove that an odd integer $n$ is prime if and only if $(n - 2)! \equiv 1 \pmod{n}$. 
Problem 3. Apply the Extended Euclidean’s Algorithm to find the gcd(240, 46) and two integers s and t such that 240s + 46t = gcd(240, 46).

Recall: Let a, b be integers, with a ≥ b ≥ 0. Using the division with remainder property, define the integers \( r_0, r_1, \ldots, r_{\lambda+1} \) and \( q_1, \ldots, q_\lambda \) where \( \lambda \geq 0 \) and integers \( s_0, s_1, \ldots, s_{\lambda+1} \) and \( t_0, t_1, \ldots, t_{\lambda+1} \) as follow:

\[
\begin{align*}
    r_0 &:= a, \quad s_0 := 1, \quad t_0 := 0, \\
    r_1 &:= b, \quad s_1 := 0, \quad t_1 := 1, \\
    r_{i+1} &:= r_{i-1} - q_i r_i, \\
    s_{i+1} &:= s_{i-1} - q_i s_i, \\
    t_{i+1} &:= t_{i-1} - q_i t_i \\
\end{align*}
\]

(\( i = 1, 2, \ldots, \lambda \))

then, for \( i = 0, \ldots, \lambda + 1 \), we have \( a s_i + b t_i = r_i \); in particular, \( a s_\lambda + b t_\lambda = \gcd(a, b) \). See Theorem 4.3 (page 78) for the rest of the properties.