Problem 1. Cosets and Quotient Groups:

(a) Find all cosets of subgroup $\langle 4 \rangle$ of $\mathbb{Z}_{12}$.

(b) Find the index of the quotient group $\mathbb{Z}_{24}/\langle 3 \rangle$. What are its elements?
Problem 2. Group Isomorphism: explain why the following pairs of groups are (or are not) isomorphic.

(a) \( \mathbb{Z}_4 \) and \( \mathbb{Z} \)

(b) \( \mathbb{Z}_2 \times \mathbb{Z}_2 \) and \( \mathbb{Z}_4 \)

(c) \( \mathbb{Z}_8^{\ast} \) and \( \mathbb{Z}_{12}^{\ast} \)
Problem 3. Exponent of a Group: find the exponent of the following groups.

(a) $\mathbb{Z}_{35}$ and $\mathbb{Z}_{15} \times \mathbb{Z}_{35}$

(b) $\mathbb{Z}_{15}^*$ and $\mathbb{Z}_{600}^*$. 