# CS 235: Algebraic Algorithms, Spring 2021 <br> <br> Practice Problems for Final Exam 

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Exam Date: 6:00PM, Tuesday, May $05^{\text {th }}, 2021$.
Problem 1. Merten's Theorem. For each positive integer $k$, let $P_{k}$ denote the product of the first $k$ primes. Show that $\varphi\left(P_{k}\right)=\Theta\left(P_{k} / \log \log P_{k}\right)$.

## Problem 2. Group Theory.

1. List the cosets of $\langle 7\rangle$ in $\mathbb{Z}_{16}^{*}$. Is the quotient group $\mathbb{Z}_{16}^{*} /\langle 7\rangle$ cyclic?
2. Are the groups $\mathbb{Z}_{2} \times \mathbb{Z}_{12} \times \mathbb{Z}_{36}$ and $\mathbb{Z}_{4} \times \mathbb{Z}_{4} \times \mathbb{Z}_{6} \times \mathbb{Z}_{9}$ isomorphic?

## Problem 3. Ring Theory.

1. Let $F$ be a field and let $f(x)$ be a non-zero polynomial in $F[x]$. Show that $f(x)$ is a unit in $F[x]$ if and only if $\operatorname{deg}(f(x))=0$.

2 . Which of the following are subrings of the field $\mathbb{R}$ of real numbers.
a. $A=\{m+n \sqrt{2} \mid m, n \in \mathbb{Z}$, and n is even $\}$
b. $B=\{m+n \sqrt{2} \mid m, n \in \mathbb{Z}$, and n is odd $\}$
3. Prove the following ring isomorphism: $\mathbb{Z}[X] /(n, X) \cong \mathbb{Z}_{n}$, where $(n, X)$ is the principal ideal of $\mathbb{Z}[X]$ generated by $n$ and $X$, for $n \geq 2$.

## Problem 4. Topics at Midterm.

1. Is there a number $x$ which is congruent to $1,2,2,1$ under modulo $2,3,4,5$ respectively?
2. Find an integer $n$ where $n>4 \cdot \varphi(n)$
3. Find integers $x$ and $y$ such that $1064 s+856 t=\operatorname{gcd}(1064,856)$
