CS 235: Algebraic Algorithms, Spring 2021

Practice Problems for Final Exam Exam Date: 6:00PM, Tuesday, May 05th, 2021.

Problem 1. Merten's Theorem. For each positive integer k, let P_k denote the product of the first k primes. Show that $\varphi(P_k) = \Theta(P_k/\log \log P_k)$.

Problem 2. Group Theory.

- 1. List the cosets of $\langle 7 \rangle$ in \mathbb{Z}_{16}^* . Is the quotient group $\mathbb{Z}_{16}^*/\langle 7 \rangle$ cyclic?
- 2. Are the groups $\mathbb{Z}_2 \times \mathbb{Z}_{12} \times \mathbb{Z}_{36}$ and $\mathbb{Z}_4 \times \mathbb{Z}_4 \times \mathbb{Z}_6 \times \mathbb{Z}_9$ isomorphic?

Problem 3. Ring Theory.

- 1. Let F be a field and let f(x) be a non-zero polynomial in F[x]. Show that f(x) is a unit in F[x] if and only if deg(f(x)) = 0.
- 2. Which of the following are subrings of the field \mathbb{R} of real numbers.

a.
$$A = \{m + n\sqrt{2} \mid m, n \in \mathbb{Z}, \text{ and } n \text{ is even}\}\$$

- b. $B = \{m + n\sqrt{2} \mid m, n \in \mathbb{Z} \text{ , and n is odd} \}$
- 3. Prove the following ring isomorphism: $\mathbb{Z}[X]/(n, X) \cong \mathbb{Z}_n$, where (n, X) is the principal ideal of $\mathbb{Z}[X]$ generated by n and X, for $n \ge 2$.

Problem 4. Topics at Midterm.

- 1. Is there a number x which is congruent to 1, 2, 2, 1 under modulo 2, 3, 4, 5 respectively?
- 2. Find an integer n where $n>4\cdot\varphi(n)$
- 3. Find integers x and y such that 1064s + 856t = gcd(1064, 856)