Erasures vs. Errors in Local Decoding and Property Testing
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Overview of Results
- Local list decoding in the presence of erasures
  - Local list decoding: Hadamard Code
  - Erasure vs. $O(\log n)$ separation between erasure-resistant testing and tolerant testing
- Approximate local list decoding in the presence of erasures
  - Constant vs. $n^{1/2}$ separation between erasure-resistant testing and tolerant testing
- Relationship between local decoding in the presence of erasures and in the presence of errors

Locally (Unique or List) Decodable Codes
- Locally decodable codes
  - Each message bit can be decoded with high probability by querying a few bits of the codeword
- Locally list decodable codes
  - Given oracle access to codeword, with high probability, obtain a list of descriptions of local decoders of each codeword message
  - Decodes from a larger fraction of corruption

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Relationships Between Models
- Constructions are explicit
  - [Sudan–Furman–Vladimirov]: standard vs. tolerant
  - [Dvir–Raskhodnikova–Vladimirov]: standard vs. erasure-resistant
- Our Result: erasure-resistant vs. tolerant

Relationships Between Erasure-Resistant and Tolerant Testing

Theorem: There exists a property $R$ of $n$ bit strings such that:
- erasure-resistant testing $R$ has constant query complexity;
- tolerant testing $R$ needs $n^{\Omega(1)}$ queries.

In this paper, we obtain an erasure-resistant testing algorithm with complexity $\tilde{O}(\log n)$.

A Locally List Erasure-Decodable Code
- An error-correcting code $C_2: \mathbb{F}_2^n \rightarrow \mathbb{F}_2^m$, usually $m \geq n$.
- Parameters $y$:
  - $w$: fraction of erased elements
  - $r$: fraction of erasures

Property Testing

Separating Property: Erasure-Resilient Testing
- Idea: If a constant fraction ($\epsilon/4$) of the encoding is erased, we can locally list erasure-decode the encoded part.

3CNF Properties: Hard to Test, Easy to Decide
- Formula $\varphi$: 3CNF on $n$ variables, $O(n)$ clauses
- Property $R_{\varphi}$: $L(\varphi)$ set of satisfying assignments to $\varphi$

Theorem (Boris Bukh, Rani Hod, and Yehuda Meshulam): There exists sufficiently small $\epsilon$, $\epsilon^{\text{-testing}} R_{\varphi}$ requires $O(1)$ queries.

3CNF properties have efficient PCPPs
- Every property-decider of a circuit of size $m$ has PCPP with proof length $O(m)$ and constant query complexity.

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Errors and Erasures in Local Decoding
- Local decoding implies local erasure-decoding
- Locally decodable from at most an $\epsilon/2$ fraction of erasures
- Also holds for local list decoding and approximate local list decoding
- Local erasure-decoding implies local decoding (up to some parameters)
- Locally decodable from at most an $\epsilon$ fraction of erasures using $\tilde{O}(\sqrt{n\log n})$ queries

Open Questions
- Even stronger separation -- constant vs. linear?
- Separation between errors and erasures for a "natural" property?
- Constant-query, constant list size, local list erasure-decodable codes with inverse polynomial rate?