Implementing regularization implicitly via approximate eigenvector computation

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1. REGULARIZATION AND IMPLICIT REGULARIZATION

Regularization is a fundamental technique in the study of mathematical optimization. It allows us to induce a generalization capability in our models, which is important because most problems that we tackle involve noisy data, which can have many different solutions, and we need to choose the one that performs well in unseen data.

Regularization has applications in many fields and, in its simplest form, can be thought of as a mechanism that penalizes solutions that are too complex or too specific to the data at hand. This helps to avoid overfitting, where a model performs well on the training data but poorly on new, unseen data.

2. MOTIVATION: COMMUNITY STRUCTURE IN NETWORKS

In many cases, a network can be modeled as an undirected weighted graph, and significant information can be found in the relative ordering of vertices' eigenvectors. For example, in spectral clustering, we consider the second smallest (i.e., the Neumann) eigenvalue of the adjacency matrix of the graph, and its corresponding eigenvector is often used to distinguish between different communities in the graph.

3. REGULARIZED SPECTRAL OPTIMIZATION

In this presentation, we assume that the graph G = (V,E) is known. Our argument is easily extensible to general graphs. Our setting, where the eigenvector can be obtained in polynomial time by an exact algorithm, is a stronger assumption than implicit regularization for general graphs.

4. DIFFERENT FUNDAMENTAL RANDOM WALKS

Different random walks can be used to obtain different approximations of the eigenvector, yielding a different optimization program. In this work, we focus on the heat random walk, which promises superior performance in the study of community detection and graph partitioning.

5. THREE REGULARIZERS AND MAIN THEOREM

In the previous sections, we have established a useful set of techniques for approximating eigenvectors. In this section, we show how to construct the regularized spectral optimization program using the three regularizers defined in the previous section.

6. DISCUSSION: APPLICATIONS TO GRAPH PARTITIONING

Recently, the regularization of the eigenvector computation by using random walks has found application in community detection and partitioning problems. We show how to apply the regularized spectral optimization program to these problems.

7. DISCUSSION: REGULARIZATION AND LOCALIZATION

The more effective that better-crafted and well-understood regularization is likely to be. The intuition is that the solutions to the optimization problems do not change much in the presence of regularization, which means that the optimization problem is effectively reduced to a problem of finding a good approximate eigenvector.

8. DISCUSSION: RESEARCH DIRECTIONS

Our work suggests that regularized spectral optimization can be used to find better approximations of the eigenvector in a variety of settings. The key insight is that regularization can help to improve the quality of the eigenvector approximation, which in turn can lead to better performance in applications such as community detection and graph partitioning.