1. For each of the following terms find a way of filling in the placeholders “?” with appropriate types so that term becomes typable, or briefly explain why there exist no types that make the term typable.

\[ M_1 \triangleq (\lambda x:?.x)(\lambda x:?.x) \]
\[ M_2 \triangleq \text{let } y:.? = \lambda x:?.x \text{ in } yy \]
\[ M_3 \triangleq \lambda f:\text{nat} \to ?.\lambda g:\text{bool} \to ?.\lambda x:?.f(x) \]

If \( M_i \) for \( i \in 1..3 \) is typable, provide a complete derivation for the judgement \( \emptyset \vdash M_i : \sigma_i \) where \( \sigma_i \) is the type that follows from the annotations you have chosen.

2. Suppose we decide include \( \text{let } x: \sigma = M \text{ in } N \) as a primitive of \( \text{PCF} \) (instead of as syntactic sugar). What would be a “sound” typing rule for this construct?

3. Exercise 2.2.12 on page 59. Avoid de-sugaring a term into another term that has substitutions, i.e. subterms of the form \( M[x := N] \) for some \( M \) and \( N \).

4. Exercise 2.2.14 on page 64. Show the reduction for \( \text{fib}(2) \) instead of \( \text{fib}(4) \).

5. Exercise 2.3.2 on page 72.

6. (Extra Credit) Suppose we drop the premise \( E \cup \{ x : \sigma \} \vdash \diamond \) from the rule \( \text{(Exp Var)} \) in the type system presented in class. Show that the resulting system does not have a Subject Reduction property (\( \text{Hint: find a simple counterexample for the theorem} \)).