1. We say that a relation $\triangleright$ satisfies the diamond property if whenever $a \triangleright b$ and $a \triangleright c$ then there exists a $d$ such that $b \triangleright d$ and $c \triangleright d$. Show that $\rightarrow$ does not satisfy the diamond property. (Hint: find terms $a$, $b$ and normal form $c$ such that $a \rightarrow b \rightarrow c$ and $a \rightarrow c$ and use the fact that $\rightarrow$ is non-reflexive).

2. Prove the following typing judgements:

(FOb_{1} )

(a) $\emptyset \vdash [x = 0, y = 0, add = \varsigma(s : A)s.x + s.y] : A$ where $A \equiv [x : Nat, y : Nat, add : Nat]$. 
(b) $\emptyset \vdash [x = 0, y = 0, add = \varsigma(s : A)s.x + s.y].add \iff \varsigma(s : A)0 : A$ where $A \equiv [x : Nat, y : Nat, add : Nat]$. 
(c) $\emptyset \vdash [\ell_1 = \varsigma(s_1 : A_1)[\ell = \varsigma(s_2 : A_2)s_1.\ell_2], \ell_2 = []] : A_1$ where $A_1 \equiv [\ell_1 : A_2, \ell_2 : []]$ and $A_2 \equiv [\ell : []]$. 

(FOb_{1<} )

(a) $\emptyset \vdash [x = 0, y = 0, add = \varsigma(s : A)s.x + s.y] : A'$ where $A' \equiv [x : Nat]$ and $A' \equiv [x : Nat]$. 
(b) $\emptyset \vdash [\ell_1 = \varsigma(s_1 : A_1)[\ell = \varsigma(s_2 : A_2)[], \ell_2 = []], \ell_1 \iff \varsigma(s_1 : A_1)[] : A_1$ where $A_1 \equiv [\ell_1 : [], \ell_2 : []]$ and $A_2 \equiv [\ell : []]$. 
(c) $\emptyset \vdash [x = 0, getx = \varsigma(s : A)s.x, setx = \varsigma(s : A)\lambda(n : Nat)s.x := n] : A$ where $A \equiv [x : Nat, getx : Nat, setx : Nat \rightarrow []]$. 

3. Consider the object term $o \triangleq [\ell = \varsigma(s : A)s]$ in Ob_{1<}. 

(a) Is there a type $A$ in Ob_{1<} such that the term $o.\ell$ is typable? 
(b) Is there a type $A$ in Ob_{1<} such that the term $o.\ell \ell$ is typable? 

4. Show that if we remove the type annotations in the system $F_{1<}$, then the minimum-types property is lost. You need to find a type-erased lambda term $a$ such that $\emptyset \vdash a : A$ and $\emptyset \vdash a : A'$ with $A$ and $A'$ unrelated (i.e., with no common super type).