Homework 7 – Due Wednesday, October 19 in class and on Canvas (problem 1 only); Thursday, October 20, 2016 on Canvas (problems 2 and 3)

Please refer to HW guidelines from HW1, course syllabus, and collaboration policy.

Problems to be handed in (for each: 10 points, at most 2 pages) (Don't forget to prove correctness and analyze time/space requirements of your algorithm.)

1. (**Bring to class on Wednesday 10/19**) For this part of the homework, you will be assigned to a "group" (via Canvas), and each group will receive a different problem.

   Please submit your typed solution online and **bring a printed version to class** on Wednesday, October 19.

   You may discuss the problem with students in your own group (as usual, you must write your solution on your own). *However, please don't discuss your problem with members of other groups.* Doing so will just make the activity in class less valuable.

2. (**Number of shortest paths**) Chapter 6, problem 22.

3. (**Polynomials**)

   (a) Consider two sets A and B, each containing n integers in the range from 0 to $10n$. The **Cartesian sum** of A and B is defined by

   $$C = \{x + y : x \in A \text{ and } y \in B\}.$$

   Note that the integers in C are in the range from 0 to $20n$. We want to find the elements of C and the number of times each element of C is realized as a sum of elements in A and B. Show that the problem can be solved in $O(n \log n)$ time. (*Hint:* Represent A and B as polynomials of degree at most $10n$.)

   (b) Given a list of values $z_0, z_1, ...z_{n-1}$ (possibly with repetitions), show how to find the coefficients of a polynomial $P(x)$ of degree n that evaluates to zero only at $z_0, z_1, ..., z_{n-1}$. Your algorithm should run in time $O(n \log^2 n)$. (*Hints:* The polynomial $P(x)$ evaluates to zero at z_j if and only if $P(x)$ is a multiple of $(x - z_j)$. Use divide and conquer.)