MATCHING 3D MODELS WITH GLOBAL GEOMETRIC FEATURE MAP

Donghui Wang[1], Chenyang Cui[2] and Zheng Wu[1]

[1]College of Computer Science, Zhejiang University
dhwang@zju.edu.cn, wuzheng@zju.edu.cn
[2]National Key Lab. Of CAD&CG, Zhejiang University
ccy@cad.zju.edu.cn

ABSTRACT

Measuring the similarity between 3D models is a fundamental task in 3D models retrieval. In this paper, we propose a new method based on Global Geometric Feature Map (GGFM) to represent arbitrary polygonal 3D models. Since 3D polygonal model can be expressed a set of facets, the GGFM can fast constitute a spherical histogram about the normal orientation and area and position of every facet on the surface of the model. By computing the spherical correlation between the GGFMs of the matched models, similarity of two models can be obtained. Experimental results show that the proposed method performs well in 3D model similarity matching and is invariant to the translation and rotation and scaling of 3D model. Comparing to the existing methods, this method is fast and needs low computation and storage cost since each facet of the model needs to be computed only once in GGFM.

1. INTRODUCTION AND PREVIOUS WORK

Recent development in 3D modeling tools and 3D scanning device are making acquisition of 3D models easier and less expensive, the World Wide Web is enabling access to 3D models constructed by people all over the world, which leads to a high demand for finding needed 3D models from a wide range of sources. So, 3D model retrieval has become the recent focus of research efforts. Since 3D models are usually defined as the collection of vertex and polygon, a similarity matching between two 3D models can't be done directly upon such the representations. Indeed, the feature extraction of 3D models and similarity measurement are the fundamental problems in 3D model retrieval. There are many techniques for shape matching and feature extraction. Funkhouser et.al.[1,2,3,4] represent the model by a set of 2D views, but the choice of the number and viewpoints of the views which characterize 3D model is a main problem in this approach, and the curvature and moment of the boundary of the views are sensitive to the noise, simultaneously, this method needs too high computation cost. Since 3D models are usually given in arbitrary units of measurement and in unpredictable positions and orientations, Wang et.al.[5,6] firstly transform 3D models to a canonical coordinate frame by using PCA and extract the feature and match 3D models. Sometimes, PCA can not always provide the correct principal axis of 3D model. Hilaga et.al.[7] use reeb graph based on geodesic distance to represent the topology of 3D model, however, the reeb graph doesn't cover full geometric information and the topology of the graph is dependent on the choice of interval size, and the geodesic distance is unsuitable for all 3D models. Osada et.al.[8] propose a continuous probability distribution as a signature for 3D model based on the shape functions such as the angle between 3 random points on the surface , the advantage of this method is that no complex feature extraction is necessary, and robust to small perturbation on the model boundary. However, such distributions do not grasp the intrinsic features of the model since it use the surface feature of the model instead of the volume feature of the model. Novotni M. et.al.[9] propose a specific distance histograms that directly define a measure of geometric similarity of the inspected models, this technique is limited to the specific applications, and inadequate for a general 3D model search.

To avoid above the mentioned shorting of the existing method, this paper propose a new method based on Global Geometric Feature Map to represent 3D object and similarity between 3D models is obtained by using spherical correlation. Generally, an object can be represented as a large number of vertex and facets, GGFM can map a facet of 3D model to a complex point on the unit sphere, the spherical coordinate of the complex point represents the normal orientation of the facet , and the real part and imaginary part of the complex point represent the area and space position of the facet. So, the GGFM constitutes a spherical histogram about the normal orientation and area and position of every facet on the surface of the model, which is described as the shape of 3D model. By computing the correlation between the corresponding GGFMs of two 3D models, similarity of 3D models can be obtained. Experiment results show that our method is invariant to the translation, rotation and scaling of the models, and performs well in 3D model similarity matching. Comparing to the existing methods, our method needs lower computation and storage cost.

2. GLOBAL GEOMETRIC FETAURE MAP

A convex polyhedron is fully specified by the area and normal orientation of its facet on its surface [10]. But 3D polygonal models are usually non-convex polyhedron, and the facets on the surface of 3D model which are in the different position maybe have the same normal orientation and area as shown in Figure1. We have $a^2 = b^2 + c^2$ for (a) and (b), obviously, (a) and (b) are different polyhedron, but (a) and (b) have the same signature about the area and normal orientation of its facet on the surface, so, (a) and (b) can not be correctly matched only according to the normal orientation and area of the facet.

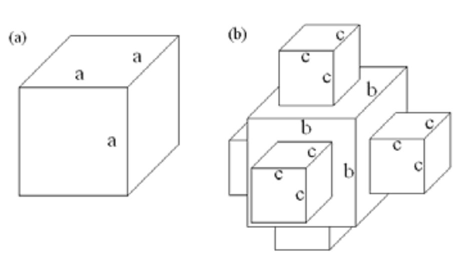

Figure 1. (a) A Cube. (b) A polyhedron with 7 little cubes

In order to improve the accuracy of the extracted feature, Global Geometric Feature Map (GGFM) is presented and the space position of the facet on the surface of 3D model is considered here. A facet of the model can be mapped to a complex point on the unit sphere, the spherical coordinate of this complex point represents the normal orientation of the facet, the real part of the complex point is described as the area of the facet, while the imaginary part of the complex point represents the position of the facet which is defined as the distance from the center of the facet to the origin. Figure 2 and Figure 3 show the GGFMs of two 3D polygonal models respectively.

As shown in Figure 4, ABC represents any facet of 3D mesh model with 3 vertex $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$, the mass center of the facet is $M(x_c, y_c, z_c)$, and the normal orientation of the facet is $\vec{n}_k$, and the space position of the facet is described as the distance D_k which is from the mass center of the facet to the origin, and the area of the facet ABC is defined as A_k ,so, the mapped complex point on the unit sphere can be describe as the following:

$$GGFM_{\vec{n}_k} = A_k + iD_k \qquad (1)$$

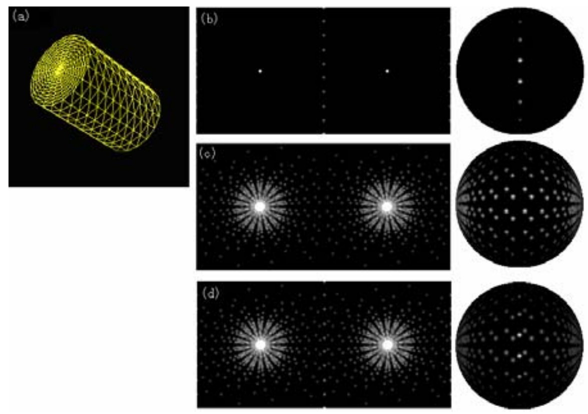

Figure 2. (a) A cylinder model. (b) The real part of GGFM of the cylinder. (c) The imaginary part of GGFM of the cylinder. (d) The GGFM of the cylinder.

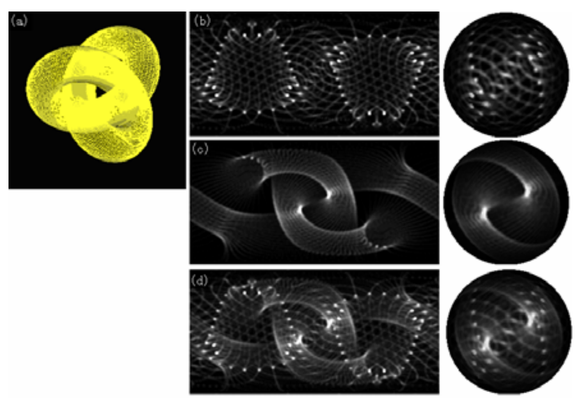

Figure 3. (a) A torus model. (b) The real part of GGFM of the torus. (c) The imaginary part of GGFM of the torus. (d) The GGFM of the torus.

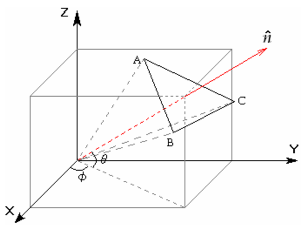

Figure 4. The facet of 3D mesh model.

In Cartesian system, $\vec{n}_k$ is described as (x_k, y_k, z_k) and $x_k^{\ 2} + y_k^{\ 2} + z_k^{\ 2} = 1$, while in spherical system, $\vec{n}_k$ is described as (θ_k, φ_k) :

$$(\theta_k, \varphi_k) = (\sin^{-1}(z_k), \tan(\frac{y_k}{x_k})) \qquad (2)$$

where,

$$x_k = (y_2 - y_1)(z_3 - z_1) - (z_2 - z_1)(y_3 - y_1)$$
$$y_k = (z_2 - z_1)(x_3 - x_1) - (x_2 - x_1)(z_3 - z_1)$$
$$z_k = (x_2 - x_1)(y_3 - y_1) - (y_2 - y_1)(x_3 - x_1)$$

The area of the facet A_k is defined as the following:

$$A_k = \frac{\overline{AB} \cdot \overline{BC} \cdot \sqrt{1 - \cos^2 \alpha}}{2} \qquad (3)$$

where,

$$\overline{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$\overline{AC} = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2}$$
$$\overline{BC} = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2 + (z_3 - z_2)^2}$$
$$\cos \alpha = \frac{\overline{AB}^2 + \overline{BC}^2 - \overline{AC}^2}{2 \cdot \overline{AB} \cdot \overline{BC}}$$

The space position of the facet is defined as the distance D_k as the following:

$$D_k = \sqrt{x_c^{\ 2} + y_c^{\ 2} + z_c^{\ 2}} \qquad (4)$$

here,

(x_c, y_c, z_c) is the mass center of the facet as the following:

$$x_c = \frac{x_1 + x_2 + x_3}{3}, y_c = \frac{y_1 + y_2 + y_3}{3}$$
$$z_c = \frac{z_1 + z_2 + z_3}{3}$$

Since the GGFM is independent of the absolute position of the facet and dependent on the area and normal orientation and the relative distance of the facet, it is invariant to the translation of the object, and it is invariant to the rotation of the object because the normal of the facet rotates with the object and the rotation results in an equal rotation of the GGFM. Simultaneously, the GGFM is also invariant to the scaling of the object. The total mass of the GFM is obviously just equal to the total surface area of the object. If one wishes to deal with objects of the same shape but differing size, one may normalize the GGFM by dividing by the total mass.

In GGFM, the consistence of the position of the facet of the model depends on the choice of the origin of the model. To ensure that signature of 3D model is correctly obtained by GGFM, one fixed geometrical center is considered as the origin of the object. Since each facet of the model needs to be computed only once in GGFM, the computation and storage cost are very low comparing to the existing methods [1-9].

3. SIMILARITY MEASUREMENT USING SPHERICALCORRELATION

The similarity measurement of two matched models can be obtained by computing the spherical correlation of the corresponding GGFMs. The GGFMs of two matched models are defined as the function $f(\omega)$ and $h(\omega)$ on the unit sphere, respectively. We can define the correlation of two functions as the following:

$$K(\alpha, \beta, \gamma) = \int_{\omega \in S^2} f(\omega) \overline{\Lambda_{g(\alpha,\beta,\gamma)} h(\omega)} d(\omega) \qquad (5)$$

Where, $\omega = (\theta, \varphi)$ represents the spherical coordinate of the mapped complex point, $\Lambda_{g(\alpha,\beta,\gamma)}$ is a rotation arithmetic operator on spherical function, $g(\alpha, \beta, \gamma)$ is an element in rotation basis $SO(3)$, and $0 \le \alpha, \gamma \le 2\pi$, $0 \le \beta < \pi$, we have:

$$g(\alpha, \beta, \gamma) = R_z(\alpha) \cdot R_y(\beta) \cdot R_z(\gamma) \qquad (6)$$

where,

$$R_z(A) = \begin{bmatrix} \cos(A) & -\sin(A) & 0 \\ \sin(A) & \cos(A) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_y(A) = \begin{bmatrix} \cos(A) & 0 & \sin(A) \\ 0 & 1 & 0 \\ -\sin(A) & 0 & \cos(A) \end{bmatrix}$$

$R_z(A), R_y(A)$ represent the rotation matrix along the z-axis and y-axis respectively. In order to reduce the complexity of the computation, equation (5) can be transformed to equation (7) according to the shift theorem of Spherical Fourier Transform proposed in [11]:

$$k(\alpha,\beta,\gamma)=\sum_{l}\sum_{mpk}\hat{f}_m^l\,\overline{\hat{h}_p^l}\,p_{pk}^l(0)\,p_{km}^l(0)e^{-i(p\gamma^{'}+k\beta^{'}+m\alpha^{'})} \tag{7}$$

where, $\gamma^{'}=\gamma+\frac{\pi}{2}$, $\beta^{'}=\beta+\pi$, $\alpha^{'}=\alpha+\frac{\pi}{2}$, $p_{pk}^l(0)$ and $p_{km}^l(0)$ are the corresponding Legendre polynomials of spherical harmonic functions, $\hat{f}_m^l$ and $\hat{h}_p^l$ are spherical Fourier Transform of the function $f(\omega)$ and $h(\omega)$ in finite bandwidth B:

$$\hat{f}_m^l=\frac{\sqrt{2\pi}}{2B}\sum_{j=0}^{2B-1}\sum_{k=0}^{2B-1}\alpha_j^{(B)}f(\theta_j,\varphi_k)\overline{Y_m^l(\theta_j,\varphi_k)} \tag{8}$$

In equation (8), we have:

$$Y_m^l(\theta,\varphi)=(-1)^m\sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}}\cdot p_m^l\cdot\cos(\theta)\cdot e^{im\varphi}$$

According to [12], 3D Fourier Transform of the correlation defined in equation (7) is shown as:

$$\hat{k}_{abc}=\sum_{l}\hat{f}_a^l\,\overline{\hat{h}_b^l}\,p_{bc}^l(0)\,p_{ca}^l(0) \tag{9}$$

Here, $\hat{k}_{abc}$ is the matrix with size $(2l+1)^3$,whose elements can be computed by directly from the spherical Fourier coefficients of $f(\omega)$ and $h(\omega)$, and $k(\alpha,\beta,\gamma)$ can be obtained by reverse of 3D Fourier Transform of $\hat{k}_{abc}$, $\hat{f}_m^l$, and $\hat{h}_p^l$ can be fast computed by using the algorithm proposed in [12].

4. EXPERIMENT RESULTS

To Test the performance of the proposed method for 3D model similarity matching,, we used about 300 3DS models which are collected from the internet. One model is selected as the search key from the 300 models, the similarities between the search key and the other remaining models are calculated, and the models are sorted according to the resulting similarity. Figure 4 and Figure 5 show the corresponding GGFMs of two 3D mesh head models, the head model in Figure 5 is the rotated one of the head model in Figure 4, and Figure 6 shows the correlation result in (α,β) and (β,γ) between two head models shown in Figure 4 and Figure 5, the peak value in Figure 6 represents the similarity matching score, the higher the peak is , the more similar the two model is. Here, the matching score is close to "1", which shows that two head models are nearly identical. Figure 7 shows a 3D fish mesh model and the corresponding GGFM, Figure 8 shows the spherical correlation result in (α,β) and (β,γ) between 3D head model in Figure 4 and 3D fish model shown in Figure 7, obviously, no peak appears in Figure 8, the peak value is close to zero, which shows no similarity in head model and fish model. Some matched results are shown in Figure 9. The selected model is shown as the search key and the models returned with highest similarities are shown under the searched models. 300 models are classified into about 41 categories. The experimental results show that the method based on GGFM can accurately identify models, especially not be influenced by the rotation, translation, scaling of the model. The similarity matching agrees with general human intuition.

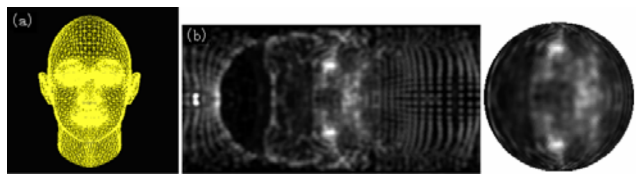

Figure 4. (a) 3D head model (b) The GGFM of head model.

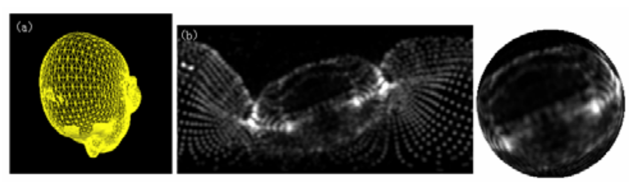

Figure 5. (a) A rotated 3D head model. (b) The GGFM of the rotated head model

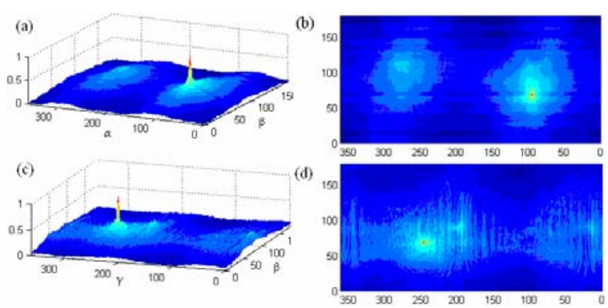

Figure 6. The spherical correlation result between two head models shown in Fig.4 and Fig.5.

Figure 7. (a) 3D fish mesh model. (b) The GGFM of the fish model.

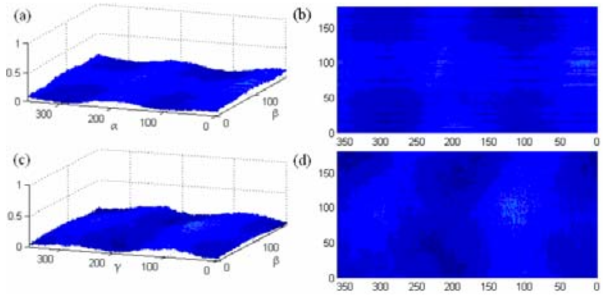

Figure 8. The spherical correlation result between fish model in Figure 7 and head model in Figure 4

5. CONCLUSION

Experimental results show that the proposed method in this paper performs well in 3D model similarity matching and is invariant to the translation and rotation and scaling of the model. Comparing to the existing method, since each facet of the model needs to be computed only once in GGFM, our method is fast to obtain the signature of the model and needs lower computation and storage cost. In the future, we hope to find the best initial pose of 3D model in order to reduce the computation cost in similarity measurement using the spherical correlation.

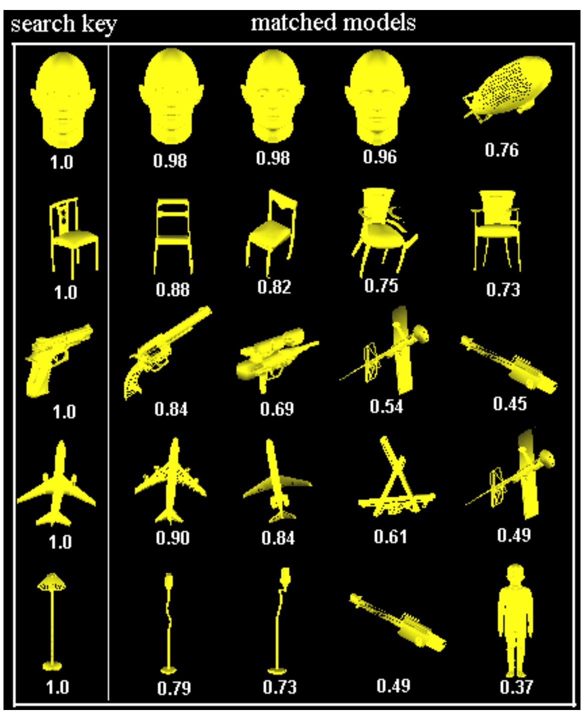

Figure 9. The partial results of experiment

6. REFERENCES

[1] Funkhouser T., Min P. Kazhdan M.,et.al, "A Search Engine for 3D Models", ACM Transactions on Graphics. 22(1):83~105,2003.

[2] Mahmoudi S., Daoudi M., "3D Models Retrieval by Using Characteristic Views", Proc. of 16th International Conference on Pattern Recognition(ICPR2002),02:457 ~460,2002.

[3] Chen D-Y.,Tian X-P.,et.al., "On visual similarity based 3D model retrieval", Proc.of Eurographics 2003, 22(3),2003.

[4] Cyr Ch.M.,Kimia B.B,. " 3D Object Recognition Using Shape Similiarity-Based Aspect Graph", Proc. of ICCV 2001.

[5] Wang D.H., Cui C.Y., "3D model similarity measurement with geometric feature map based on phase-encoded range image", Proc. of Pacific Rim Conference on Multimedia (PCM 2004),103~110,2004.

[6] Vranic D., Saupe D., "Tools for 3D Objec retrieval: Karhunen-Loeve Transform and Spherical Harmonics", Proc. of Multmedia Signal Processing of IEEE(MMSP 2001),293~298,2001.

[7] Hilaga M., Shinagawa U., Kohmura T., and Kunii T.L, "Topology matching for fully automatic similarity estimation of 3D shapes", Proc. of Siggraph 2001,203~212,2003.

[8] Osada R., Funkhouser T., and Chazelle B, "Shape Distributions", PACM Transcations on Graphics, 21(4):807~ 832,2002.

[9] Novotni M., Klein R. ,"Geometric 3D comparison and application",Proceedings of ECDL WS Generalized Documents, 39~44,2001.

[10] Berthold K.P.Horn. "Extended Gaussian Images", Proc. of the IEEE, 72(12):1671~1686, 1984.

[11] Makadia A.,Daniilidis and K.Direct, "3D-Rotation estimation from shperical images via a generalized shift theorem", Proc. of CVPR 2003,217-226,2003.

[12]Makadia A.,Sorgi L.,and Daniilidis K., "Rotation estimation from spherical images", Proc. of ICPR 2004,590~593,2004.