Dynamic programming, hash functions

1. Is the sequence (, (, )(), (, )(), ) a valid sequence of parenthesis? Given a sequence of left and right parenthesis, design an algorithm that can decide if this is a valid sequence.

2. Given matrices $A_{p \times q}$ and $B_{r \times s}$, for what values of $p, q, r$ and $s$ can you multiply $A$ and $B$? What is the size of the matrix that you get by multiplying an $n \times m$ matrix with an $m \times k$ matrix? How many computations does this involve?

3. Use dynamic programming to decide the best order for multiplying a chain of matrices with sizes $5 \times 7, 7 \times 10, 10 \times 3, 3 \times 8, 8 \times 4$. What is the best order for multiplying matrices in the chain $10 \times 60, 60 \times 3, 3 \times 100, 100 \times 2, 2 \times 50, 50 \times 6, 6 \times 20$?

4. Let $T$ be an array of length 11. Let $h_a$ be a universal hash function defined by the formula

$$h_a(k) = 4 + \sum_{i=1}^{4} a_i k_i \mod 11$$

where $a_1 = 2, a_2 = 3, a_3 = 7, a_4 = 5$. Let $k_i$ denote the $i$th digit of key $k$. Store the following keys in $T$ using $h_a$ as the hash function:

1234, 5261, 7711, 1276, 2222, 1111, 8961, 3489, 1571, 2240, 3001, 7621

5. Come up with four random numbers $c_1, c_2, c_3, c_4$ in the range $0, \ldots, 10$. Repeat the previous exercise, again with array $T$ being of size 11 and universal hash function $h_c(k) = \sum_{i=1}^{4} c_i k_i \mod 11$. What is the number of collisions in this case? Compare to the number of collisions for $h_a$. What does the theorem about universal hash functions say about the probability of two different keys colliding?