Linear programming

1. Mr. Brown is planning a huge new year’s party and he wants to optimize his consumption of beer and hot dogs during the party night. He has to take the following constraints into consideration. To make the evening as jolly as possible he has to drink at least as many beers as he is eating hot dogs. In order not to get too drunk he can have at most twice as many beers as hot dogs. Nevertheless, he has to drink at least two beers to look cool in the eye of his friends. And finally, he can only consume ten units of food and drink before he gets full. Write a linear program that describes Mr. Brown’s consumption preferences if his goal is to maximize the amount of beer he is drinking. First obtain a graphical solution for this linear program. Then apply the simplex algorithm to solve it.

2. Bring the following linear programs to standard form and then to slack form.

\[
\begin{align*}
\min & \quad -2x_1 + 3x_2 \\
\text{s.t.} & \quad x_1 + x_2 = 7, \\
& \quad x_1 - 2x_2 \leq 4, \\
& \quad x_1 \geq 0.
\end{align*}
\]

\[
\begin{align*}
\max & \quad 3x_1 + x_2 + 2x_3 \\
\text{s.t.} & \quad x_1 + 3x_3 \leq 30 - x_2, \\
& \quad -2x_1 - 2x_2 - 5x_3 \geq -24, \\
& \quad x_2 + 2x_3 \leq -4x_1 + 36, \\
& \quad x_1, x_2, x_3 \geq 0.
\end{align*}
\]

\[
\begin{align*}
\max & \quad x_1 + 3x_2 \\
\text{s.t.} & \quad x_1 - x_2 \leq 8, \\
& \quad -x_1 - x_2 \leq -3, \\
& \quad x_1 + 4x_2 \leq 2, \\
& \quad x_1, x_2 \geq 0.
\end{align*}
\]

3. Find an optimal solution to the above linear programs with help of the simplex algorithm.

4. Find the optimal solution to Mr. Brown’s optimization problem in exercise 1. if instead of drinking as much beer as he can his objective is to a.) maximize the amount of hot dogs he is eating c.) minimize the total amount he is consuming.

5. Draw a graphical representation of the linear program described by

\[
\begin{align*}
2x_1 - x_2 & \geq 0, \\
0.5x_1 + x_2 & \geq 2
\end{align*}
\]

where the goal is to maximize \(x_1 + 7x_2\). How many feasible solutions does this polyhedron have? How can you see this when applying the simplex algorithm?