Secure Multi-party Quantum Computing

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Preliminary version presented at NEC workshop on quantum crypto after QIP 2000

Since then:

- Protocols have changed a little.
- Definitions have been found.
- Proofs have changed a lot

Classical Distributed Protocols

- Extensively studied
- Many applications
  - Banking / E-commerce
  - Electronic Voting
  - Auctions / Bidding

Questions for Quantum Protocols

- Do existing protocols remain secure?
  - Not always: factoring, discrete log

Questions for Quantum Protocols

- Do existing protocols remain secure?
- Can we find better / more secure protocols for existing tasks?
  - E.g. Key distribution, coin flipping (?), “quantum voting”

Questions for Quantum Protocols

- Do existing protocols remain secure?
- Can we find better / more secure protocols for existing tasks?
- What new, quantum tasks can we perform?
  - E.g. Quantum Secret-Sharing, Zero-Knowledge, Authentication, Entanglement Purification
  - General trend: do cryptography with quantum data
  - Goal: building blocks for complex protocols

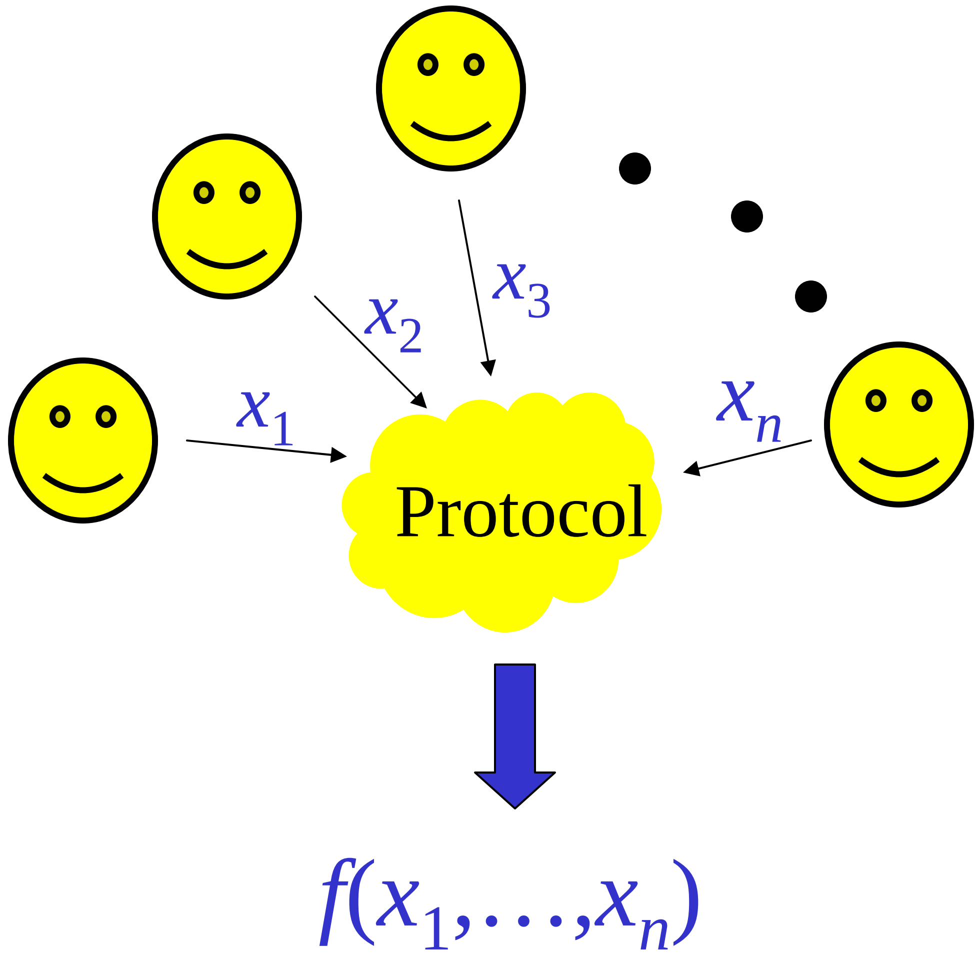
Overview

- What is multi-party (quantum) computing?
- A Sketch of the Protocol
- An Impossibility Result

What is Multi-party Computing?

Classical Multi-party Computing

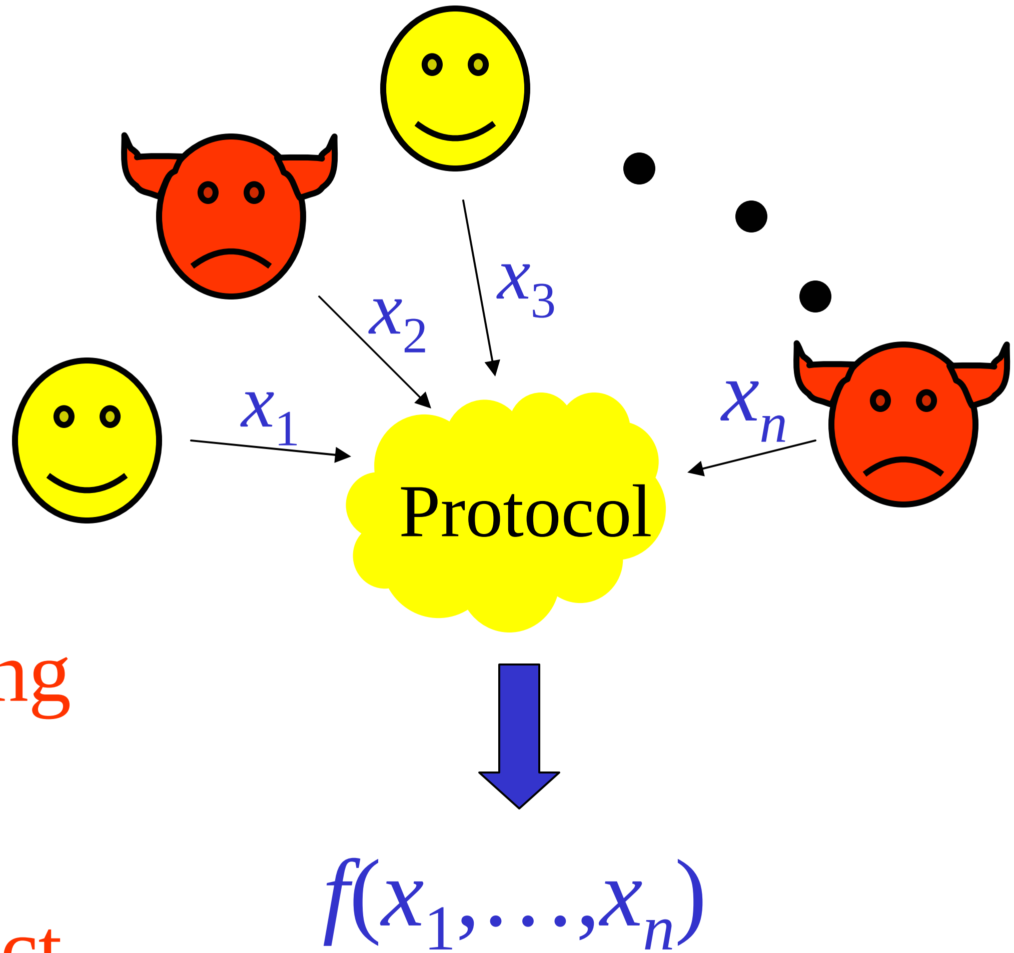
- Network of n players
- Each has input x_i
- Want to compute $f(x_1,\ldots,x_n)$ for some known function f
- *E.g.* electronic voting



Classical Multi-party Computing

Even if t out of n

players try to cheat:



1. Cheaters learn nothing (except output)
2. Cheaters cannot affect output

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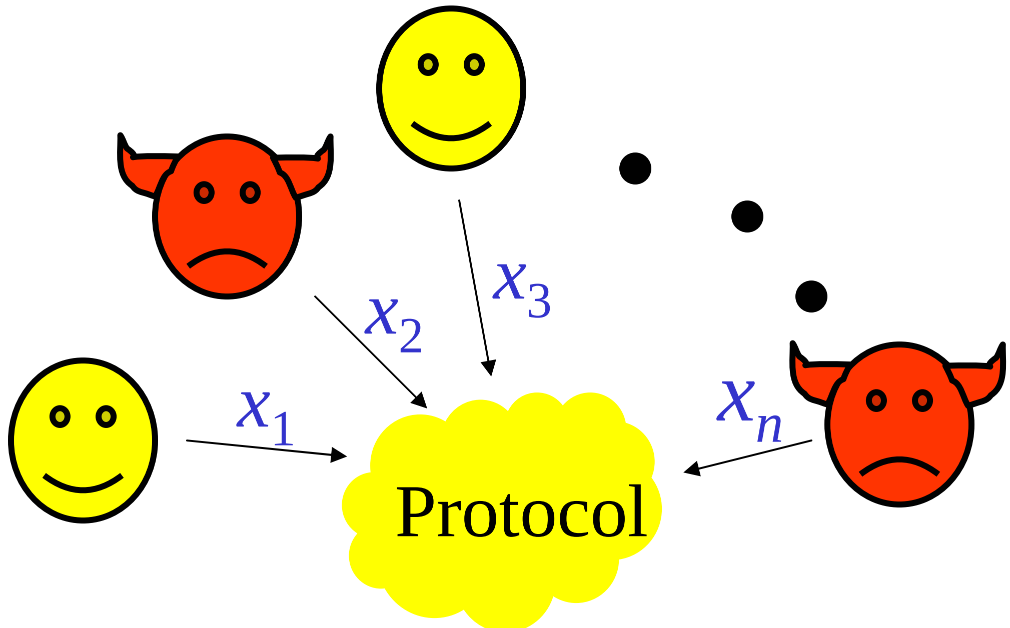
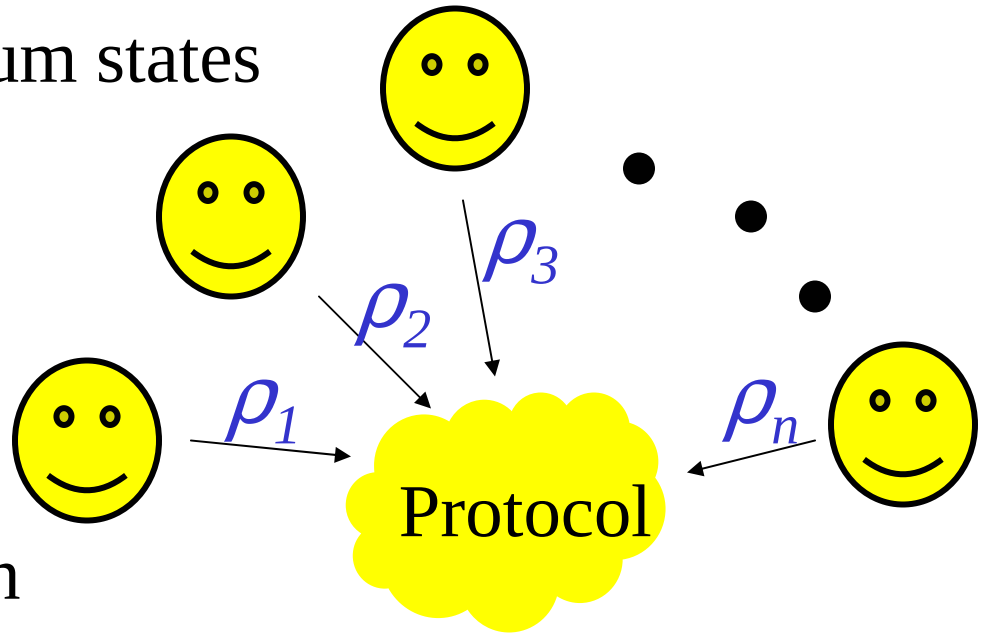


1. Cheaters learn nothing (except output)
2. Cheaters cannot affect output

Even with unbounded computation time

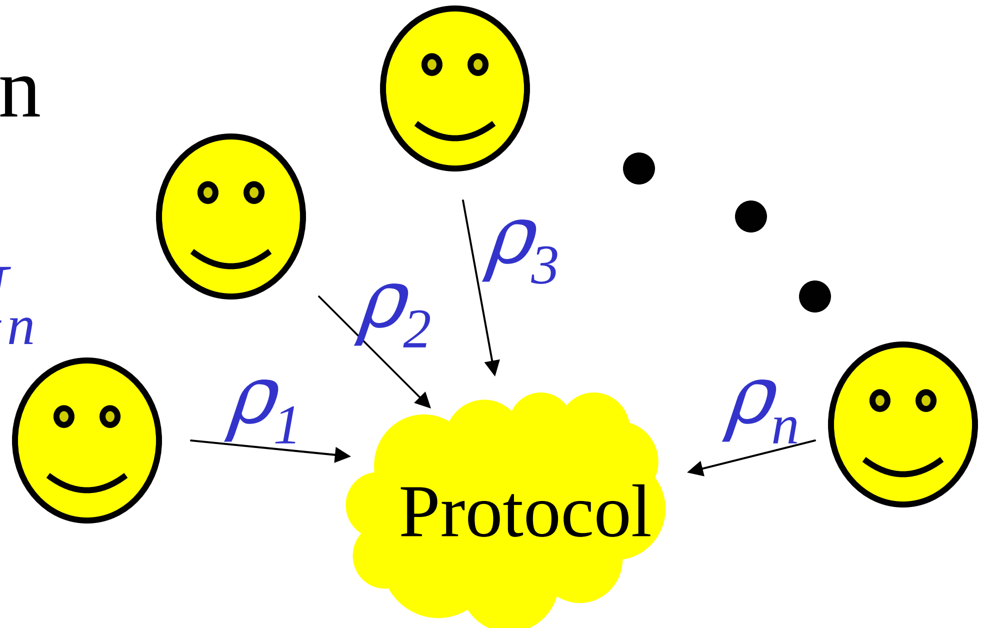
Quantum Multi-party Computing

- Players' inputs are quantum states
  - Possibly entangled
  - No description necessary (protocol is "oblivious")
- Output is quantum
- Want to evaluate a known quantum circuit U
- Player i gets i-th component of output



Quantum Multi-party Computing

- Players' inputs form an arbitrary state
  ρ in $H_1 \otimes H_2 \otimes \ldots \otimes H_n$
- Player *i* holds *i*-th component:
  $\rho_i = \mathrm{tr}_{\{1,\ldots,n\}\setminus i}(\rho)$



Quantum Multi-party Computing

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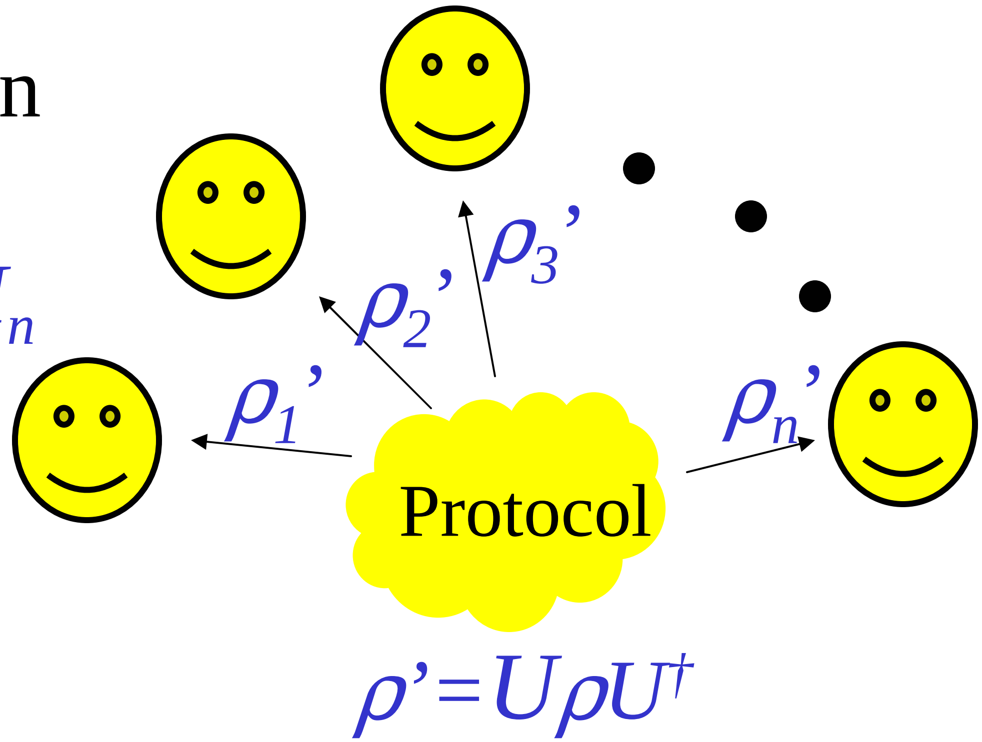
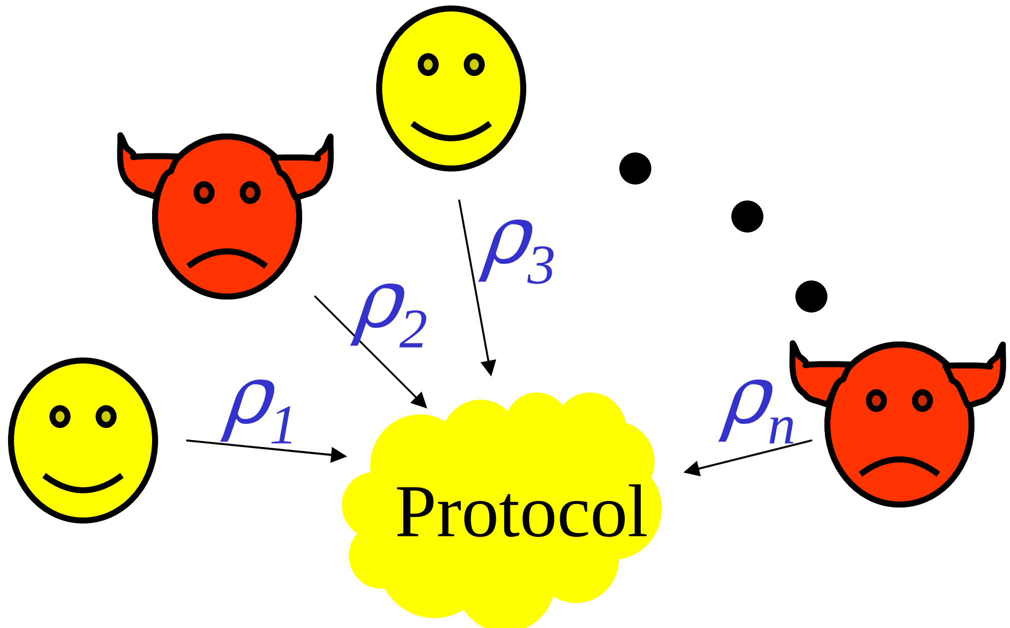
- Player i holds i-th component:
  $\rho_i = \mathrm{tr}_{\{1,\ldots,n\}\setminus i}(\rho)$
- Each player gets one output:
  $\rho_i' = \mathrm{tr}_{\{1,\ldots,n\}\setminus i}(U\rho U^\dagger)$



Quantum Multi-party Computing

Even if t out of n

players try to cheat:

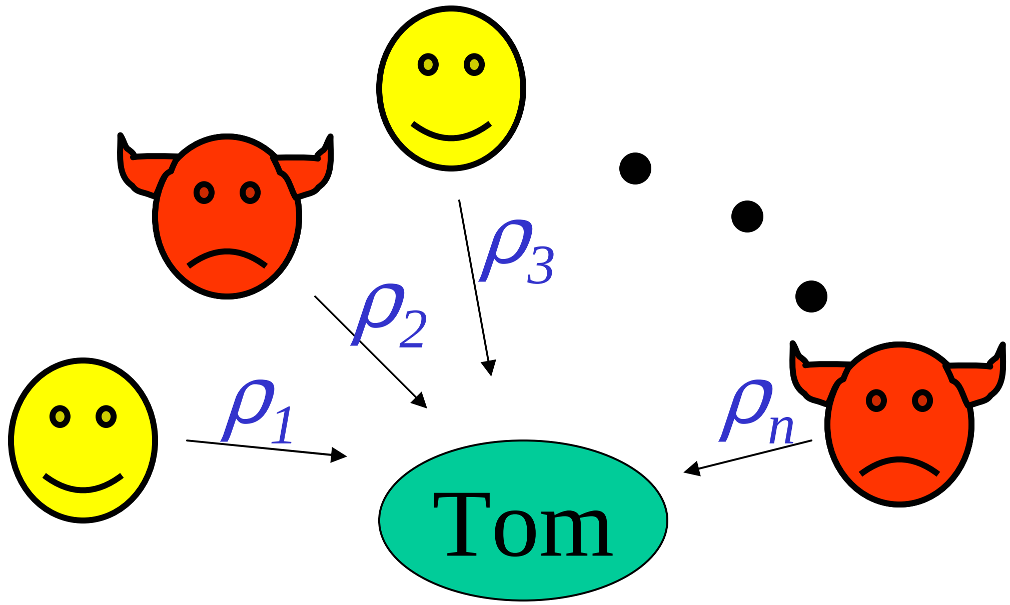


1. Cheaters learn nothing
   (except output)

2. Cheaters cannot affect output
   (except by choice of inputs)

Easy Solution: Trusted Outside Mediator

- If everybody trusts **Tom**
- Send all inputs to **Tom**
- **Tom**:
  - Applies U
  - Distributes outputs



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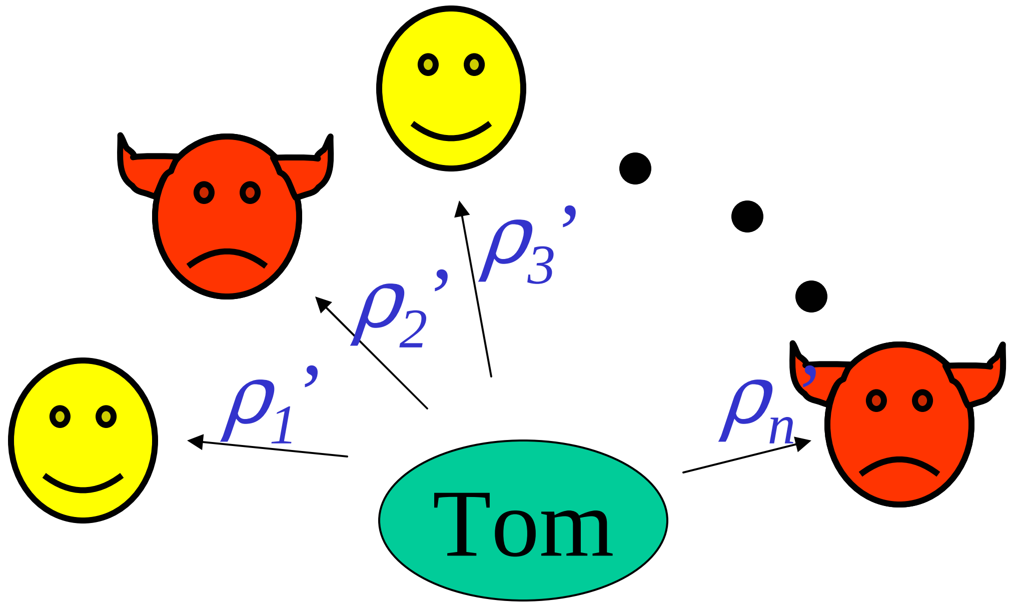


$$\rho' = U \rho U^{\dagger}$$

Challenge: Simulate the presence of Tom

Results

- $t < n/6$:

  Any Multi-party Quantum Computation

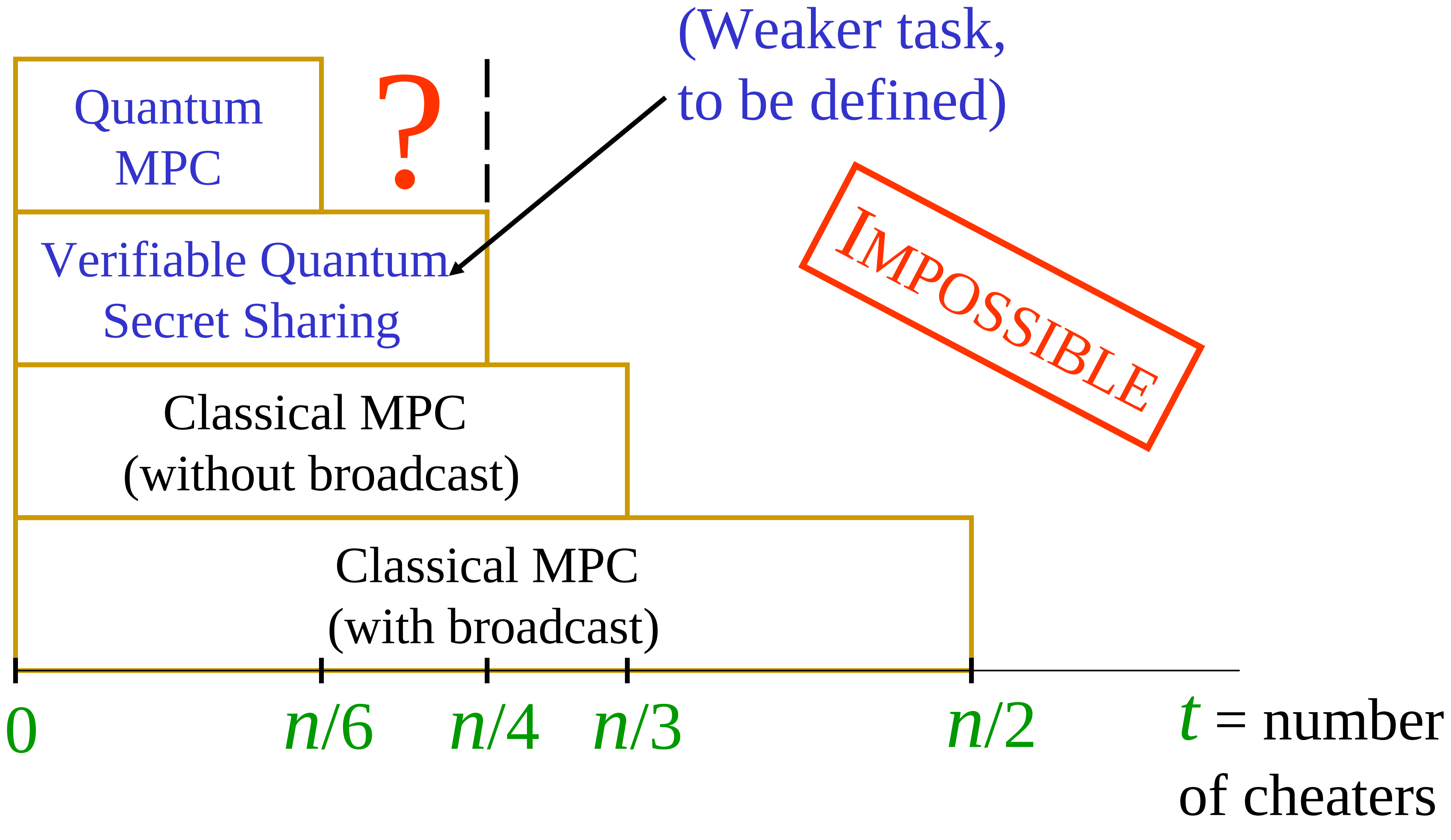
- $t < n/4$:

  Verifiable Secret-Sharing (weaker subtask)

- $t \geq n/4$:

  Even VQSS is impossible

Results


(Weaker task,
to be defined)
Quantum
MPC
?
Verifiable Quantum
Secret Sharing
IMPOSSIBLE
Classical MPC
(without broadcast)
Classical MPC
(with broadcast)
0
n/6
n/4
n/3
n/2
t = number
of cheaters

MPQC and Fault-Tolerant Computing

- MPQC is like FTQC with a different error model...

	FTQC	MPQC
Type of errors	randomly spread, independent	maliciously placed, entangled with data
Error location	Can occur anywhere	At most t positions

  - Similar protocol techniques:

    Classical MPC [BGW,CCD] $\rightarrow$ FTQC [AB99] $\rightarrow$ MPQC [us]
  - Different proof techniques

    (Need different notion of “proximity” to coding subspaces)

A Sketch of the Protocol

Protocol Overview

- Share
  - Each player encodes his input using a QECC
  - Sends *i*-th component to player *i*
  - Proves that sharing was done "correctly"
    i.e. distributed shares form a codeword except on positions held by cheaters
- Compute
  - Use fault-tolerant circuits to apply *U* to encoded inputs
- Distribute
  - Give each player all components of his output

Why is this enough?

- **If**:
  - All players share their input with a “proper” codeword
  - (and) No information is leaked by proof
- **Then** the cheaters:
  - can’t disturb the calculation since QECC and FTQC will tolerate errors in any t locations
  - (Informally:) can’t learn info since they can’t disturb!

An Impossibility Proof

Verifiable Quantum Secret-Sharing

- Idealized "qubit commitment"
- 2-phase protocol
- Sharing: Dealer D shares a secret system ρ such that
  - Cheaters can't learn anything about ρ
  - Dealer can't change ρ
- Recovery: Receiver R specified by context
  - All players send shares to R
  - R reconstructs ρ

Easy Solution: Give ρ to trusted Tom, get it back later.

Verifiable Quantum Secret-Sharing

- Sharing phase of our MPC protocol is a VQSS
- My opinion:

  Most “interesting” MPC protocols will imply VQSS,

  since they should allow simulating Tom’s presence in

  more general tasks

  e.g. qubit commitment
- Theorem: VQSS is impossible for $t \geq n/4$

Theorem: No VQSS tolerates $t \geq n/4$

Lemma:

Any VQSS protocol "is" a QECC correcting t errors

Proof:

- Look at the state $F(|\psi\rangle)$ of protocol at the end of sharing phase when all players are honest, and input is $|\psi\rangle$
- Protocol is oblivious, so $F(|\psi\rangle) = E|\psi\rangle$ for some trace preserving E.
- At this point, arbitrary corruption of t players can't change reconstructed secret $|\psi\rangle$
- Thus E is the encoding operator for a QECC.

Theorem: No VQSS tolerates $t \geq n/4$

Proof:

- No cloning says that no QECC can correct $n/2$ erasures
- Fact: Any QECC which corrects t errors can correct $2t$ erasures
- Thus no QECC tolerates $n/4$ errors
- All these arguments work regardless of dimension of components of QECC
- Thus, no VQSS tolerates $t = n/4$ cheaters.

Conclusions

- Study general cryptographic tasks in distributed setting
- You can do anything you want when $t < n/6$
- You can't do much when $t \geq n/4$
- Along the way:
  - First "zero-knowledge" quantum proofs secure against malicious verifiers
  - Refined notions of "proximity" to QECC's.
  - Wrestled with definitions for malicious quantum adversaries

More Protocol Sketch

How to prove sharing is correct?

- Use Zero-Knowledge Proof techniques due to [Crépeau,Chaum,Damgård1988] (from classical MPC)
- Based on classical Reed-Solomon code:
  - To encode a, pick a random polynomial p of degree $2t$ over Z_q such that $p(0)=a$ and output $(p(1), \ldots ,p(n))$
- We use: "polynomial codes" of [Aharonov,Ben-Or99]

$$E|a\rangle = \sum_{\substack{p:\deg(p)=2t \\ p(0)=a}} |p(1), p(2),\ldots, p(n)\rangle$$

Basic Step

$$A(|x\rangle|y\rangle) = |x\rangle|y+x\rangle$$
$$A^{\otimes n}\left(E|\psi\rangle E\sum|a\rangle\right)$$
$$= E|\psi\rangle E\sum|a\rangle$$

- Prover takes secret $|\psi\rangle$
  - Shares $E|\psi\rangle$ (system #1)
  - Shares $E(\sum|a\rangle)$ (system #2)
- Players together generate random bit b
- If $b=0$ then do nothing

  If $b=1$ then "add in Z_q" System #1 to System #2
- Measure System #2 and broadcast results
- Accept if broadcast vector close to a classical codeword

Properties of Basic Step

- **If** dealer passes test many times in
  - computational basis and
  - Rotated "Fourier basis" (q-ary analogue of $|0\rangle + |1\rangle$, $|0\rangle - |1\rangle$)

  Then shared state is "close" to a quantum codeword
- **If** dealer was honest,

  then no information is leaked and state is not disturbed
- This can be "boosted" to get secure protocol for $t < n/4$

What does "close to a codeword" mean?

- Shared state should differ from a codeword only on positions held by cheaters
- Natural notion of closeness:

  (1) Reduced density matrix of honest players
  = reduced density matrix of some state in coding space Q

- Too strong: Our protocols can't guarantee that.
- Instead:

  (2) Shares held by honest players pass parity checks restricted to those positions

What does "close to a codeword" mean?

- (1) $\neq$ (2)
  - (1) is not even a subspace!
  - Basic problem: errors and data can be entangled
- Analysis of fault-tolerant protocols only requires (1)
- We can only guarantee notion (2)
- Nonetheless, our protocols are secure:
  - Notion (2) strong enough to ensure well-defined decoding: changes made by cheaters to a state in (2) cannot affect output
  - Fault-tolerant procedures work for states in (2)