# Midterm practice

cs235

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1. Prove that if $gcd(n,(n-1)!) = 1$ i.f.f $n$ is prime.

   **Solution.** If $n$ is prime, then $(n-1)!$ has no factors in common with $n$ because they are all smaller. Therefore they have no common divisors so $gcd(n,(n-1)!) = 1$.

   If $gcd(n,(n-1)!) = 1$, then $n$ has no common divisors with any number below it. Otherwise $gcd(n,(n-1)!) > 1$. The only way this is possible is if $n$ is prime. Therefore $gcd(n,(n-1)!) = 1$ i.f.f $n$ is prime.

2. Let $p$ be a prime number. Show that $p!$ is not a perfect square.

   **Solution.** The prime factorization for a perfect square is $\prod_p p_i^2$, i.e. all prime factors are at a power of 2. $p! = p(p-1)!$. Thus in order for $p!$ to be a perfect square, $p|(p-1)!$. This is obvously not the case. Therefore $p!$ cannot be a perfect square because $p$ is a prime factor but it is not to the power of 2.

3. Let $n$ be an integer. Show that if $a, b$ are relatively prime integers, each of which divides $n$, then $ab$ divides $n$.

   **Solution.** $a$ and $b$ are relatively prime, so there exist integers $s, t$ such that $as+bt = 1$. It clearly follows then that $nas + nbt = n$, and since $a|n$ and $b|n$, then there exist integers $x, y$ such that $ax = n$, and $by = n$.

   Substituting for $n$, we get that

   $$byas + axbt = n$$
   $$ab(ys + xt) = n$$

   $ys + xt$ is an integer. Therefore it is clear that $ab|n$, and the proof is complete.